

Special Session in memory of F.W. Lawvere  
CT 2023 Louvain-la-Neuve  
Anders Kock,  
Dept. of Math., University of Aarhus



Thank you to the  
**organizers;**

and to

**M.M. Clementino and J. Picado**

for making the interview with L. in 2007 (Braga, Portugal),  
including the picture:

and to

**Fatima and Danilo Lawvere**

for friendship and help

# CT 1987 in Louvain-la-Neuve and Leuven

“I dedicate this talk to the fundamental unity of the Indo-European languages”

# Unification

The unification of mathematics is an important strategy for learning, developing and using mathematics

(L. and Schanuel, in “Conceptual Mathematics” 2009)

# From topology to algebra; steps in the unification

Algebraic topology; homology functors

$\text{Top.Spaces} \rightarrow \text{Abelian groups}$

Later

$\text{Simplicial sets} \rightarrow \text{Abelian groups};$

A simplicial set itself as a functor  $\Delta^* \rightarrow \text{Sets};$  adjoint functors.

# From Categories to Category Theory

Lawvere:

The Category of Categories as a **Foundation** for Mathematics  
(La Jolla 1965)

The past 100 years' tradition of “foundations” as justification has not helped mathematics very much. In my own education I was fortunate to have two teachers who used the term “foundations” in a common-sense way . . . :

Foundations of Algebraic Topology, published in 1952 by Eilenberg (with Steenrod), and the Mechanical Foundations of Elasticity and Fluid Mechanics, published in the same year by Truesdell.

# Category of Sets

... I reasoned that Grothendieck's theory of Abelian categories should have a non-linear analogue whose examples would include categories of sheaves of sets; ... and noted that, on the basis of my work on the category of sets, such a theory would have a greater autonomy than the Abelian one could have.



Mac Lane books:

Homology 1963

Categories for the Working Mathematician 1971.

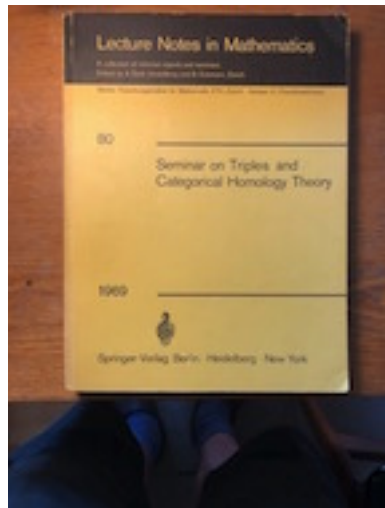
The title of the latter indicates that in 1971, category theory is seen not just as a tool for Algebraic Topology/Homological Algebra, but for every working mathematician.

The slogan is “Adjoint functors arise everywhere”.

(Mac Lane 1971)

Zürich 1966

Eckmann's seminar 1966-67



(Triple = Monad =  $(T, \eta, \mu)$ )



# 1967-1973

1967-1969: Chicago

1969-1971: Halifax. Not renewed due to political activities

1971-1972: Aarhus

1972-1973: Perugia

1973-2023: Buffalo

# Categorical Dynamics 1967

Tangent bundle of  $M$ :

$$T(M) = M^D$$

$M$  a object (space) in an unspecified Cartesian closed category,  
 $D$  a postulated infinitesimal object.

Since the late 1970s “Synthetic Differential Geometry”: postulate further a commutative ring  $R$ , (“the line”), and requires  $R \times R \rightarrow R^D$  to be an iso; then  $R$  is called of “line type”. and develop axiomatically;  
in a topos, with the insight brought about from Elementary Toposes: just argue as if the topos were SETS



Hanne, Anders, Bill, Fatima



Marie, Danilo, Silvana, Joachim, Hans

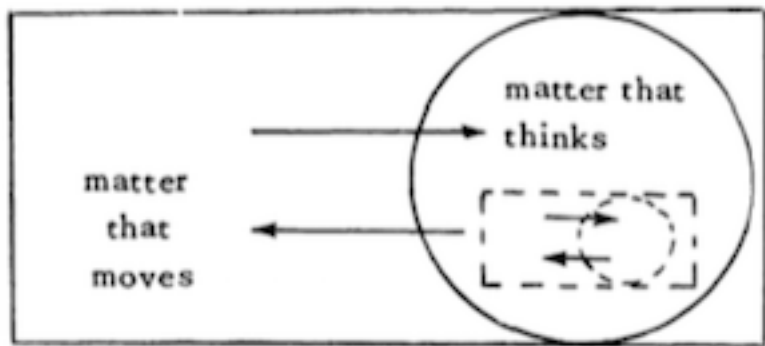
## Aarhus Open House, Summer of 1973, 1978, 1983

“TOPOS THEORETIC METHODS IN GEOMETRY”, Aarhus Math. Institute, Various Publication Series No. 30 (1979).

Among the scheduled events were “Discussion on philosophy”, with Joyal and Lawvere (780517), as well as an elaboration (780524). Some of L.’s contribution in this discussion are published in Cahiers 1980:



# Scientific world picture

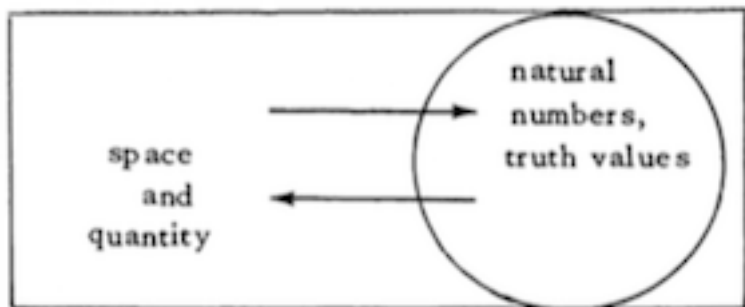


(Cahiers 1980)

# Mathematical world picture

Mathematics: studying space and quantity and their relations

Logic of mathematics: constructing concept and reasoning for this



I agree with Brouwer that  $\mathbb{N}$  is subjective, in contrast to the continuum which is objective (quote from 780517)

In the circle may add: continuum.

# The continuum

the continuum .. [is] primarily a **concept** derived **directly** from our historical-scientific experience with the world of matter-in-motion

**not** only a mental from construction  $\mathbb{N}$  and  $\Omega$ : (the subjective idealizations of iteration and truth respectively)

A model for the continuum *may* be constructed (Dedekind, Cauchy, . . .) from  $\mathbb{N}$  and  $\Omega$  and the denoted  $\mathbb{R}$ . Thus: subjective, and leads to to space filling curves, which do not exists.  
[But] space is here now (quote from 780517)

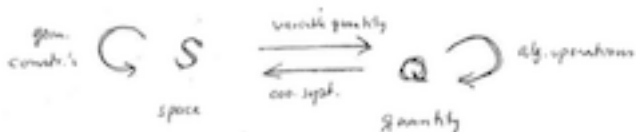
# Space vs. Quantity

(From 780524)

The axioms of Euclid - are not so bad.

Problems come when making them rigorous. Nobody would use Hilbert's book for a high school text book.

Note: study of space forms and quantitative relations, and their relationship. Cannot we take this idea directly and arrive at axioms for mathematics. - Slightly schematically



(Basic variable quantity: distance).

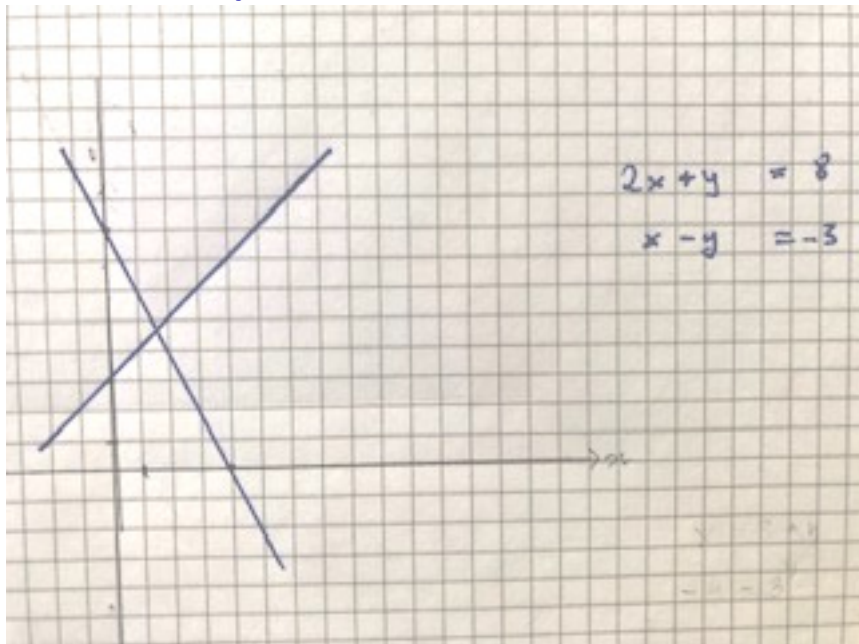
This problem is not good enough to be a cat. Because from an, say, distinct space forms.

Roughly:

**S** a topos of Spaces

**Q** a Picard closed additive category of physical dimensions (with objects types of quantities: length, area, volume, mass, time-lapse, ... linear maps

## Space vs. Quantity



# Conceptual Construction vs Formal Deduction

Inside the logic of mathematics, have a non-antagonistic contradiction,

LCC: Logic of Conceptual Construction

LFD: Logic of Formal Deduction

Nearly every piece of mathematics involves usually an advance in both.

LFD is not the whole thing: this idea “*waves the flag of LFD to oppose LCC*”, especially the purpose of math., (clarifying, guiding,) (from 780517).

LFD is overestimated, LCC underestimated



# Concepts

The existence of Euclidean Geometry is more important than any specific Theorem in it.

The fact that there must be a Continuum is more important than the way you “construct” it.

Euclidean geometry was from the outset based on the **concepts** plane, line, point, - derived from experience from the real world, say, architecture. “Line” a concept derived from taut strings, typically made of linen, whence the name.

# Grothendieck

Grothendieck was in Buffalo in the spring of 1973, and gave a series of colloquium talks.

L. coming back from Perugia only in time for the last of these. According to Michael Wright:

Alas, none of G's talks to the Buffalo Math Colloquium (as distinct from the Lecture Courses) were recorded and Bill attended and made notes only of the last talk. . . . Bill gave an account of this Talk in the course of the discussions held in Fougères in 2005, which are now (finally) in the course of being transcribed.

The talk contained

. . . an overarching view of how G saw Narrow-sense logic as fitting within (broad-sense) Geometry what Bill referred to in passing as "Grothendieck's Vision of how to by-pass Logic".

Good concepts imply fewer proofs.

LCC over LFD.

# Schanuel



Photo: R.Walters, Sydney 1988

S. and L. worked together on a daily and private basis for decades. In the Buffalo winters, they (with Fatima) went together to Oaxaca in Mexico. - L. writes in an obituary for Schanuel 2014:

We studied deep into the nights; sometimes Fatima heard us giggle, because we had discovered how simply some results could be proved. In the morning she typed the notes that I had left on her table.

Danilo:

She [Fatima] made sure that dad went to conferences with notes or drafts, and she ensured that deadlines were met, letters were answered as well as all the extensive class notes we have. *Conceptual Mathematics* [with Schanuel, 2009] was in part conceived by her, and she was the driving force in getting it done, and then published.

# Archives; Danilo Lawvere

Danilo:

There is a lot to do with respect to the archive, but my mom [Fatima L.] sure did prepare the way.

First, I want to say that the plans for archiving and making available all of my dads work is already underway, and it is something I will be working on for several years.

Some of the first steps will be ensuring that the materials on the website have links, then move toward unpublished works, and then comments to TAC, selected correspondence etc. I may be able to get some assistance from the math department here and am looking at the possible places to put the papers (possibly University of Buffalo).

Dads website we are planning to update

<https://www.acsu.buffalo.edu/~wlawvere/>

## Archives; Michael Wright, and links (url)

Michael Wright has recorded and scanned a lot of material from Lawvere and others, including scannings of notes I took from L.'s lectures over the years, in particular from L.'s many visits to Aarhus. These scannings are about to be made public on his

<https://archmathsci.org/category/mathematics/>

I have made some of these scannings of my notes available, e.g.

<https://users-math.au.dk/kock/780524.pdf>

where "780524" is the date of the lecture, here May 24 1978 (from Aarhus Open House 1978). Also 780516, 780517, 780518, 780519

Braga Interview:

<http://www.mat.uc.pt/~picado/lawvere/interview.pdf>

Best source:

<https://www.acsu.buffalo.edu/~wlawvere/>

Thank you !

“... out of fashion...”

*“Nevertheless there are areas which are out of fashion, for instance category theory and set theoretic topology, also mathematical logic, partially, judging from the number of posts”*

Vasco Alexander Schmidt, interviewing Matthias Kreck, in the chapter “The Unity of Mathematics”,  
in Springer’s “Mathematics Unlimited - 2001 and Beyond”