A COMBINATORIAL THEORY OF CONNECTIONS

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For a set M equipped with a reflexive and symmetric relation $x \sim y$, we study a purely combinatorial notion of <u>connection</u>, and <u>group-valued</u> 1-<u>form</u>. Curvature, torsion, and integrability are defined in a very primitive combinatorial manner, and we give conditions (of "technical" nature) under which connections on M, which have zero curvature and torsion, are integrable, provided closed 1-forms on M are exact. The proviso "closed group-valued 1-forms on M are exact" is of course not just a "technical" condition, but of more objective nature, and it plays the same role as Frobenius' integrability Theorem does in the corresponding differential-geometry theory.

The integrability theorem for connections, proved here in the rude combinatorial context, does actually have the corresponding differential-geometric theorem as a consequence, which can be seen via synthetic differential geometry; the relation $\sim$ becomes that subobject of $M \times M$ which is usually called "the first neighbourhood of the diagonal". We sketch this in §3.

I owe credit to André Joyal who was the first to consider connections as "actions of the graph of the 1st neighbourhood of the diagonal", and also used this graph for considering differential forms.

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§1. Connections, framings, and forms

For any group G, we can consider the ternary operation λ on G given by

$$\lambda(x,y,z) := y\cdot x^{-1}\cdot z.$$

It evidently satisfies

(1.1) $$\lambda(x,y,x) = y$$

and its "dual" (i.e., with the role of the second and third variable interchanged)

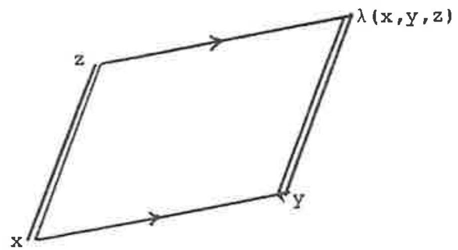
(1.2) $$\lambda(x,x,z) = z.$$

It furthermore satisfies

(1.3) $$\lambda(y,x,\lambda(x,y,z)) = z$$

and its dual, and $\lambda(y,z,\lambda(x,y,t)) = \lambda(x,z,t)$ and its dual, and this in fact is a complete list of equations satisfied by λ, and defines the notion of pregroup, [7]*. If $G = (R^n,+)$, $\lambda(x,y,z)$ is the fourth corner in a parallelogram whose three other corners are x, y and z:

(1.4)



We have used different signatures for the segments xy and xz, so that the picture can be used as a representation of $\lambda(x,y,z)$ even when G is not commutative.

* This notion is equivalent to the notion of "Schar" of Prüfer, [10], except that he also assumes a commutative law $\lambda(x,y,z) = \lambda(x,z,y)$.

Let now M be an object equipped with a reflexive and symmetric relation $\sim$. It is not assumed transitive. We introduce the notation

$$M(x) := \{y \in M \mid y \sim x\}$$

for the set of points $\sim$-related to x, and call it the monad around x.

A connection on M is a law λ which to any triple x,y,z of elements of M with $x\sim y$ and $x\sim z$ associates an element $\lambda(x,y,z)$ with

(1.5) $$y \sim \lambda(x,y,z)$$

and such that the equations (1.1) and (1.2) and (1.3) hold. (In the application, (1.3) and its dual, as well as (1.5) and its dual, are consequences of (1.1).)

We say that λ is torsion free if it is symmetric in the second and third argument

$$\lambda(x,y,z) = \lambda(x,z,y)$$

(where $x\sim y$ and $x\sim z$).

To talk about curvature of λ, it is convenient to reinterpret it as follows. For $x\sim y$, $\lambda(x,y,-)$ establishes a map

(1.6) $$\lambda(x,y,-)\colon M(x) \to M(y)$$

which sends x to y by (1.1) and is the identity map if $x = y$, by (1.2). Also, the map (1.6) has $\lambda(y,x,-)$ as inverse, by (1.3). It should be thought of as "parallel transport from x to y". Now, if $x_1,x_2,x_3 \in M$ form a "$\sim$-triangle" (by which we mean $x_1\sim x_2$, $x_2\sim x_3$ and $x_3\sim x_1$), we may consider the composite map $M(x_1) \to M(x_1)$

given by going once round the diagram

(1.7)
$$\begin{tikzcd} M(x_1) \arrow[rr, "\lambda(x_1,x_2,-)"] & & M(x_2) \arrow[dl, "\lambda(x_2,x_3,-)"] \\ & M(x_3) \arrow[ul, "\lambda(x_3,x_1,-)"] & \end{tikzcd}$$

We say λ is *curvature free* if this map is the identity map of $M(x_1)$, for any $\sim$-triangle x_1,x_2,x_3.

The extent to which a given connection λ fails to be curvature free can be described in terms of a certain *group-valued 2-form* on M, provided M is equipped with some *framing*, so we proceed to describe these two concepts.

Let $\underline{D}$ be a set with a given base point denoted $\underline{O}$ (in the application, $\underline{D} = D(n) \subseteq R^n$, well-known standard object in synthetic differential geometry). A $\underline{D}$-*frame* at $x \in M$ is a bijective map

$$k: \underline{D} \to M(x)$$

with $k(\underline{O}) = x$. A $\underline{D}$-*framing of* M is a map

$$k: M \times \underline{D} \to M$$

such that for each $x \in M$, $k(x,-): \underline{D} \to M$ is a $\underline{D}$-frame at x. So, equivalently, a $\underline{D}$-framing is a law which to each $x \in M$ associates a $\underline{D}$-frame $k_x = k(x,-)$ at x.

Let G be a group (written multiplicatively) with neutral element e. A 1-*form on* M, *with values in* G, is a law which to every pair $x \sim y$ in M associates an element $\omega(x,y) \in G$ such that

(1.8) $$\omega(x,x) = e \qquad \forall x$$

and

(1.9) $$\omega(x,y)\cdot\omega(y,x) = e \qquad \forall x \sim y.$$

(In the application, (1.9) is a consequence of (1.8).)

For any $f: M \to G$, we get a 1-form df on M by putting

$$(df)(x,y) = f(y)\cdot f(x)^{-1}.$$

1-forms which arise this way are called *exact*. A 1-form ω is called *closed* if "$d\omega = 0$", by which we mean that

(1.10) $$\omega(z,x)\cdot\omega(y,z)\cdot\omega(x,y) = e$$

for any $\sim$-triangle x,y,z. Clearly an exact 1-form is closed.

A 2-*form* is similarly a law which to each $\sim$-triangle x,y,z associates an element of G, and associates e if two of the vertices are equal. Clearly, the left hand side of (1.10) defines a 2-form $d\omega$.

Given a connection λ on M, we would like to define its curvature as a 2-form with values in a suitable fixed group G. For that, we need to transfer the automorphism of $M(x_1)$ displayed in (1.7) so as to become an element of some fixed group G, independent of x_1. So we shall assume that M can be equipped with a $\underline{D}$-framing, for some pointed object $\underline{D} = (\underline{D},\underline{O})$. Then choose one such framing h ("the reference framing"), and let $G = \mathrm{Aut.}(\underline{D})$, the group of $\underline{O}$-preserving invertible maps $\underline{D} \to \underline{D}$. The *curvature* ρ of λ, with respect to the framing h, is then defined to be the 2-form which to a $\sim$-triangle x_1,x_2,x_3 associates the composite

$$\underline{D} \xrightarrow{h_{x_1}} M(x_1) \longrightarrow M(x_1) \xrightarrow{h_{x_1}^{-1}} \underline{D}$$

where the middle map is the one displayed as the composite triangel in (1.7). Of course, ρ depends on the reference framing.

Any $\underline{D}$-framing k on M gives rise to a connection $\lambda = \lambda_k$, namely by putting, for $x \sim y$ and $x \sim z$,

$$\lambda_k(x,y,z) = k_y(k_x^{-1}(z))$$

or equivalently,

$$\lambda_k(x,y,-) = k_y \circ k_x^{-1} \colon M(x) \to M(y). \tag{1.11}$$

From this latter description, we infer that λ_k is curvature-free (in fact this is formally the same calculation which shows that an exact 1-form is closed).

Assuming, as above, existence of some "reference" $\underline{D}$-framing for M, we have

Proposition 1. Assume closed G-valued 1-forms on M are exact (where $G = \mathrm{Aut.}(\underline{D})$). Then a connection λ on M is of form λ_k for some $\underline{D}$-framing k on M if and only if it is curvature free.

Proof. We have already seen that connections of form λ_k are curvature-free. So, conversely, let λ be a curvature-free connection. Choose an arbitrary reference framing h; using h and λ, we construct a G-valued 1-form θ on M by putting $\theta(x,y)$ (for $x{\sim}y$) equal to the composite

$$\underline{D} \xrightarrow{h_x} M(x) \xrightarrow{\lambda(x,y,-)} M(y) \xrightarrow{h_y^{-1}} \underline{D}.$$

It sends $\underline{0}$ to $\underline{0}$, by (1.1), and equals $e \in G$ (= the identity map of $\underline{D}$) if $x = y$, by (1.2); and (1.9) holds in virtue of (1.3).

We claim that to say λ curvature-free is equivalent to saying θ closed. For, let x_1, x_2, x_3 be a $\sim$-triangle and consider the diagram obtained from (1.7) by pasting the commutative squares that express λ in terms of θ and h on it:

$$\begin{tikzcd}
\underline{D} \arrow[rr, "\theta(x_1,x_2)"] \arrow[drrr] & & \underline{D} \arrow[drr, "h_{x_2}"] \arrow[ddl, "\theta(x_2,x_3)"] & & \\
& & & M(x_1) \arrow[r] & M(x_2) \arrow[dl, "\lambda(x_2,x_3,-)"] \\
& \underline{D} \arrow[uul, "\theta(x_3,x_1)"] \arrow[rr, "h_{x_3}"'] & & M(x_3) \arrow[u] &
\end{tikzcd}$$

then the right hand triangle (= (1.7)) is the identity map of $M(x_1)$ if λ is curvature-free, and the left hand triangle is the identity if θ is closed, proving the equivalence of these two conditions.

So from the assumption on λ, we infer that θ is closed, and then by the main assumption of the Proposition, that θ is exact, $\theta = dg$ for some $g\colon M \to G = \mathrm{Aut.}(\underline{D})$, so that for $x \sim y$

$$\theta(x,y) = g(y) \circ g(x)^{-1}. \tag{1.12}$$

Combining this with the defining equation for θ

$$\theta(x,y) = h_y^{-1} \circ \lambda(x,y,-) \circ h_x,$$

and rearranging, we get

$$h_y \circ g(y) \circ g(x)^{-1} \circ h_x^{-1} = \lambda(x,y,-)$$

which expresses that λ is λ_k for the framing k given by

$$k_x = h_x \circ g(x).$$

This proves the Proposition.

(Of course, an inspection of some of the arguments in this proof shows that for any connection λ on M, and any $\underline{D}$-framing h on M, the θ constructed above is an $\mathrm{Aut.}(\underline{D})$-valued 1-form such that $d\theta$ equals the curvature of λ when the curvature is now viewed as a 2-form with values in the fixed group $\mathrm{Aut.}(\underline{D})$, related to the M-indexed family of groups $\mathrm{Aut.}(\mathcal{M}(x))$ via the frames h_x, $x \in M$.)

§2. Additive theory and torsion free connections

In this section we consider an abelian group A (to be thought of as that affine space that gives rise to the idea of affine connection). We consider two subsets $\underline{D}$, $\underline{D}'$ with $\underline{0} \in \underline{D} \subseteq \underline{D}' \subseteq A$, $\underline{0}$ being the zero element of A. We assume that if $\underline{u},\underline{v} \in \underline{D}$, then $\underline{u}+\underline{v} \in \underline{D}'$. We shall consider $\underline{D}$-framings on M as in the previous section, but now we need two assumptions relating M, $\underline{D}$ and $\underline{D}'$:

(2.1) If $k: \underline{D} \to M$ is a frame at x, and $\underline{u} \in D$, $\underline{v} \in D$ have the property that $k(\underline{u}) \sim k(\underline{v})$, then $\underline{v}-\underline{u} \in \underline{D}$

(2.2) If $\tau: \underline{D}\times\underline{D} \to M$ is a map with $\tau(\underline{u},\underline{v}) = \tau(\underline{v},\underline{u})$ $\forall \underline{u},\underline{v} \in \underline{D}$, then $\exists!\ t: \underline{D}' \to M$ with $t'(\underline{u}+\underline{v}) = \tau(\underline{u},\underline{v})$ $\forall \underline{u},\underline{v} \in \underline{D}$.

(In the application, $A = R^n$, $\underline{D} = D(n)$, $D' = D_2(n)$, where we use standard notation of synthetic differential geometry [6]. The assumptions (2.1) and (2.2) will hold for any n-dimensional formal manifold.)

We call a map $f: M \to A$ an <u>open immersion</u> if for each $x \in M$, $f|_{\mathcal{M}(x)}$ maps $\mathcal{M}(x)$ bijectively onto $f(x) + \underline{D}$. Clearly, such f gives rise to a $\underline{D}$-framing k, namely

$$k_x(\underline{u}) = \text{the unique } y \in \mathcal{M}(x) \text{ with } f(y) = f(x) + \underline{u}$$

for any $\underline{u} \in \underline{D}$, or equivalently,

$$k_x^{-1}(y) = f(y) - f(x) \tag{2.3}$$

for any $y \sim x$. The framing k thus constructed in turn gives rise to a connection λ_k, as described in §1. We can relate $\lambda = \lambda_k$ directly to f as follows. Let $x \sim y$ and $x \sim z$, say $y = k_x(\underline{u})$, $z = k_x(\underline{v})$, or equivalently $f(y) - f(x) = \underline{u}$, $f(z) - f(x) = \underline{v}$. Then

$$\lambda(x,y,z) = k_y\,(k_x^{-1}(z)) = k_y(\underline{v}),$$

so

$$f(z) - f(x) = \underline{v} = k_y^{-1}(\lambda(x,y,z)) = f(\lambda(x,y,z)) - f(y),$$

so that we have

$$f(\lambda(x,y,z)) = f(z) - f(x) + f(y). \tag{2.4}$$

This in fact completely describes λ, for since $\lambda(x,y,z) \sim y$, and $f|_{\mathcal{M}(y)}$ is monic, only <u>one</u> $\lambda(x,y,z)$ can satisfy (2.4)

Because (2.4) is symmetric in y and z (because A is abelian), we immediately see that the connection associated to (the framing associated to) an open immersion is torsion free.

Framings, as well as connections, arising in this way from open immersions, we call <u>integrable</u>.

Under the assumptions (2.1) and (2.2) made, we have

Proposition 2. Assume closed A-valued 1-forms on M an exact. Then, a $\underline{D}$-framing k on M is integrable if and only if the connection λ_k associated to k is torsion free.

Proof. We have already seen that connections of form λ_k with k integrable are torsion free. So assume conversely that the framing k has the property that λ_k is torsion free. Consider the map

$$\lambda_k(x,k_x(-),k_x(-)) : \underline{D}\times\underline{D} \to M.$$

Because it is symmetric, we get by (2.2) a map

$$k'_x : \underline{D}' \to M$$

with

$$\lambda_k(x,k_x(\underline{u}),k_x(\underline{v})) = k'_x(\underline{u}+\underline{v}), \qquad \forall(\underline{u},\underline{v}) \in \underline{D}\times\underline{D}.$$

From (1.1) and (1.2) follows that k'_x extends k_x. Let now x,y,z be a ~-triangle. Since $x \sim y$, $x \sim z$,

$$y = k_x(\underline{u}) \qquad z = k_x(\underline{v})$$

for some $\underline{u},\underline{v} \in \underline{D}$. Since further $y \sim z$, we get by (2.1) that $\underline{v}-\underline{u} \in \underline{D}$. We claim

$$k_y(\underline{v}-\underline{u}) = z. \tag{2.6}$$

For

$$\begin{aligned} z = k_x(\underline{v}) &= k'_x(\underline{v}) = k'_x(\underline{u} + (\underline{v}-\underline{u})) \\ &= \lambda_k(x,k_x(\underline{u}),k_x(\underline{v}-\underline{u})) \qquad \text{(by (2.5))} \\ &= \lambda_k(x,y,k_x(\underline{v}-\underline{u})) \\ &= k_y(k_x^{-1}(k_x(\underline{v}-\underline{u}))) \qquad \text{(by (1.11))} \\ &= k_y(\underline{v}-\underline{u}). \end{aligned}$$

We now construct an A-valued 1-form ω on M by putting

$$\omega(x,y) = k_x^{-1}(y).$$

We shall prove that λ_k torsion free implies that ω is closed. Let x,y,z be a ~-triangle, as above. Then

$$\begin{aligned} (d\omega)(x,y,z) &= \omega(x,y) + \omega(y,z) + \omega(z,x) \\ &= k_x^{-1}(y) + k_y^{-1}(z) + k_z^{-1}(x). \end{aligned}$$

The first term is $\underline{u}$. The second term is $\underline{v} - \underline{u}$, by (2.6). The third term is $-\underline{v}$, by the same argument which gave (2.6) (or, as a substitution instance of it: write $\underline{v}$ for $\underline{u}$ and $\underline{0}$ for $\underline{v}$, z for y and x for z). So the sum is zero, proving that ω is closed. By the main assumption of the Proposition, we get ω is exact, $\omega = df$ for some $f: M \to A$, so that, for $x \sim y$, $\omega(x,y) = f(y) - f(x)$. Combining this with the defining equation for ω, we have

$$k_x^{-1}(y) = f(y) - f(x), \qquad \forall x \sim y,$$

which is (2.3). The fact that f is any open immersion follows because k_x is a framing. So f witnesses that k is integrable, and the Proposition is proved.

With the assumptions (2.1) and (2.2) on M, $\underline{D}$, $\underline{D}'$, A, we have, by combining Propositions 1 and 2:

Corollary 3. Assume that closed Aut.($\underline{D}$)-valued and closed A-valued 1-forms on M are exact, and that M admits some $\underline{D}$-framing. Then, for a connection λ on M, the following two conditions are equivalent:

(i) λ is curvature-free and torsion free

(ii) λ is integrable.

§3. Application to differential geometry

Since all constructions and arguments in the preceding sections were of purely equational-combinatorial nature, they apply to objects in any finitely complete category $\mathcal{E}$; an object M with a binary relation $\sim$ is now an object M in which there is given a subobject of $M\times M$,

$$M_{(1)} \rightarrow M\times M;$$

the "set" of $\sim$-triangles is now to be understood as a certain intersection of pull-backs of the given maps from $M_{(1)}$ to M, etc. This kind of interpretation is familiar since the early sixties.

The application to differential geometry comes about when specializing M to be a smooth manifold, and $M_{(1)}$ to be the so-called "first neighbourhood of the diagonal", familiar in modern differential geometry as a certain "ringed space", or as a

subobject of $M\times M$ when M is considered as an object of one of the models of synthetic differential geometry, see e.g. [6], and notably the forthcoming [2]. We shall work in this context, so we consider a topos $\mathcal{E}$ containing the category Mf of smooth manifolds as a full subcategory, $\mathrm{Mf} \rightarrow \mathcal{E}$, in such a way as to make $\mathcal{E}$ into a fully well-adapted model for synthetic differential geometry, [1], [4], [6],§III.3-4. We shall utilize that any manifold $M \in \mathrm{Mf} \subseteq \mathcal{E}$ becomes a formal manifold in $\mathcal{E}$ in the sense of [3], or [6],§I.17. For such, a relation $\sim_1$ of "1-neighbour" is defined, cf. loc. cit.

Considering a formal manifold, and denoting $\sim_1$ by $\sim$, we shall analyse the notions and prove the assumptions made in §1 and 2. We use the "naive" mode of speaking about $\mathcal{E}$, which essentially means that when we say "element" of M, we mean a <u>generalized</u> or <u>parametrized</u> element, i.e. a map $X \rightarrow M$ for some parameter object X, which may vary, cf. [6], Part II.

Let us analyse the notion of <u>connection</u>. Since this is a local notion, we may assume M to be a formal-étale subset of R^n (which is just $\mathbb{R}^n$ when viewed in $\mathcal{E}$). For given $x \in M$, the function

$$\lambda(x,-,-): M(x) \times M(x) \rightarrow M \subset \mathbb{R}^n$$

gets identified with a map $D(n) \times D(n) \rightarrow R^n$, via the canonical identification $D(n) \cong M(x)$ given by $\underline{u} \rightarrow x+\underline{u}$, where

$$D(n) = [\![(x_1,\dots,x_n) \in R^n \mid x_i \cdot x_j = 0 \quad \forall_{i,j}]\!].$$

It takes value x on the axes $\{\underline{O}\} \times D(n)$ and $D(n) \times \{\underline{O}\}$ by (1.1) and (1.2), so by [5], Proposition 2.2, it is of the form $x + B_x(-,-)$ for a unique bilinear $B_x\colon R^n \times R^n \rightarrow R^n$. This is the usual "coordinate"-description of the notion of affine connection in the tangent bundle of $M \subset R^n$. It can, alternatively (cf. [8]) be viewed as a map

$$TM \times_M TM \to M^{D \times D} = T(TM)$$

$$(\underline{u},\underline{v}) \to [(d_1,d_2) \to x + d_1 \cdot \underline{u} + d_2 \cdot \underline{v} + d_1 \cdot d_2 \cdot B_x(\underline{u},\underline{v})]$$

where $\underline{u}$ and $\underline{v}$ denote vectors in R^n, as well as the tangent vectors at x with principal part $\underline{u}$ and $\underline{v}$, respectively.

For the notion of $\underline{D}$-frame (where $\underline{D} = D(n)$), note that, by [6], Theorem I.8.1, say, a bijective map $D(n) \to M(x)$ (with $\underline{0} \to x$) is the same as a bijective linear map $R^n \to T_x M$ which is thus an element over x of the usual frame-bundle of M. A framing is then a cross-section (in E) of the frame bundle of M, which in turn, by fullness of $Mf \to E$ is the same as a <u>smooth</u> cross-section of the usual frame bundle of M.

Finally, for the notion of group valued form on a formal manifold, we refer the reader to [5], or [6], §I.18. It should be noted that the assumption "closed G-valued 1-forms on M are exact" holds for the case where G is a Lie group, M a connected simply connected manifold, and the model $Mf \to E$ is sufficiently good, like the "Cahiers Topos" of Dubuc [1], cf. [6]. On the other hand, the two applications made of "closed 1-forms are exact" in Propositions 1 and 2, respectively (and with value group $Aut.(D(n)) = GL(n,R)$, and R^n, respectively) replace the two uses of Frobenius, Theorem in the classical proof of the main Theorem (Corollary 3).

For the assumptions (2.1) and (2.2), we take $\underline{D} = D(n)$ as above, and $\underline{D}' = D_2(n)$, where as usual (cf. [6], §I.6)

$$D_2(n) = [\![(x_1,\ldots,x_n) \in R^n \mid x_i \cdot x_j \cdot x_k = 0 \quad \forall i,j,k]\!] .$$

Clearly, if $\underline{d} \in D(n)$ and $\underline{\delta} \in D(n)$, then $\underline{d} + \underline{\delta} \in D_2(n)$, by simple calculation. To prove (2.1), we may assume $M \subset R^n$ a formal-ètale subset, whence $x \sim y$ iff $x-y \in D(n)$. A frame $k: D(n) \to M$ is then the restriction of a unique affine bijective map $K: R^n \to R^n$, so that (2.1) follows from: $x-y \in D(n)$ iff $K(x) - K(y) \in D(n)$. Finally, for the assumption (2.2), this is a version of the "symmetric functions property" for formal manifolds, cf. [6], Proposition I.17.6.

By applying a sufficiently good fully well-adapted model $Mf \to E$ (one in which curve integration of Lie group valued forms can be performed, cf. [5]), and by putting the above arguments together with Corollary 3, we get the classical theorem that if M is a connected and simply connected smooth n-dimensional manifold whose frame bundle admits a cross section and λ is a smooth affine connection in the tangent bundle of M, then it comes about from an open immersion $f: M \to R^n$ via (2.4) iff it has zero curvature and zero torsion.

The notion of connection, as presented here, also makes sense for graphs since a binary reflexive symmetric relation $M_{(1)}$ on a set M can be viewed as the set of edges of an unoriented graph. Consider for instance the graph $M_{(1)}, M$ consisting of the edges and vertices of an octahedron. Taking geometric inspiration from the Riemannian connection on the sphere S^2, we define a connection on $M_{(1)}$, by declaring

$$\lambda(x,y,z) = z$$

whenever xy and xz are two of the edges of one of the surface triangles of the octahedron, and

$$\lambda(x,y,z) = \lambda(x,y,y) = \text{vertex opposite to } x$$

whenever y and z are not adjacent to each other. This connection has non-vanishing curvature, in fact the holonomy group (image of curvature form) is $\mathbb{Z}_4$. Unlike Riemannian connections, the connection here also has torsion.

A combinatorial version of the Riemannian connection on the sphere may also be found in [9].

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