Wavelength Routing of Symmetric Communication Requests in Directed Fiber Trees

(Research Paper)

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Abstract

We focus on a special version of the problem of allocating bandwidth for a set of directed communication requests in tree–shaped optical networks. We consider patterns of requests that are symmetric, e.g. for each request originating from a node v_1 and destined to a node v_2, there also exists its symmetric, a request that originates from v_2 and destines to v_1. The problem can be viewed as coloring of undirected paths in trees, but we cannot hope for optimal solutions even when the network is a binary tree, a case for which the undirected problem is in P. In this paper we investigate the relation of this special case with both the undirected problem and the general (non–symmetric) version of the directed problem. We prove that the problem is NP–hard for arbitrary tree topologies and present lower and upper bounds on the number of wavelengths.

1 Introduction

Optics is emerging as a key technology in state-of-the-art communication networks. A single optical wavelength supports rates of gigabits-per-second (which in turn support multiple channels of voice, data, and video [12,22]. Multiple laser beams that are propagated over the same fiber on distinct optical wavelengths can increase this capacity much further; this is achieved through WDM (Wavelength Division Multiplexing). However, data rates are limited in opto–electronic networks by the need to convert the optical signals on the fiber to electronic signals in order to process them at the network nodes. Electronic parallel processing techniques are capable, in principle, to meet future high data rate requirements, but the opto–electronic conversion is expensive. It appears likely that, as optical technology improves, simple optical processing will remove the need for opto–electronic conversion. Networks using optical transmission and maintaining optical data paths through the nodes are called all–optical networks.

The model we use for the underlying fiber network is that of a graph. Connectivity requests are ordered pairs of nodes, to be thought of as transmitter-receiver pairs. For networks with unique transmitter-receiver paths (such as trees), the load of a fiber link is the number of paths going through the link. WDM technology establishes connectivity by finding transmitter-receiver paths, and assigning a wavelength to each path, so that no two paths going through the same link use the same wavelength. Optical bandwidth is the number of available wavelengths. As state-of-the-art optics technology allows for a limited number of wavelengths (even in the laboratory) the engineering question to be solved can be expressed as follows: "Given fixed W–wavelength technology, what type of requests can we route?". Alternatively phrased, for unique transmitter-receiver path networks

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(like trees) the question becomes: 'What is the minimum number of necessary wavelengths to route communication requests of maximum load l per fiber link?'

We consider tree topologies, with each edge of the tree consisting of two opposite directed fiber links. Previous work considered trees with undirected fiber links and undirected paths. However, it has since become apparent that optical amplifiers placed on fiber will be directed devices [6]. Thus, directed graphs are essential to model the current optics technology. At the rest of this paper we will use the terms directed and undirected problem to distinguish between the two different network models.

The undirected optimization problem (routing requests using the minimum number of wavelengths) is NP–hard for arbitrary trees [7] while a polynomial time algorithm for binary tree networks is presented in [7]. Raghavan and Upfal [25] gave the first approximation for the problem. They proved that routing requests of maximum load l per link of undirected arbitrary trees can be satisfied using at most $3l/2$ wavelengths. An improvement of $9l/8$ is implied in [21] and Erlebach and Jansen [7] present a 1.1–approximation algorithm using a result for edge–coloring due to Nishizeki and Kashiwagi [23].

Several NP–completeness results for the directed version of the optimization problem are presented in [7,18]. The problem is NP-hard for binary trees and trees of constant depth. The arguments of [25] extend to give a $2l$ bound for the directed case. Mihail et al. [21] address the directed problem as well. Their main result is a $15l/8$ upper bound. This is done by reduction to a bipartite graph edge-coloring, which is achieved in phases by obtaining matchings of the bipartite graph, and coloring them in pairs using detailed potential and averaging arguments. The algorithm in [21] is a greedy algorithm. Greedy algorithm are important as they are very simple and, more importantly, they are amenable of being implemented in a distributed enviroment.

Kaklamanis and Persiano [15] improve the upper bound for directed trees to $7l/4$. The main idea of their algorithm is similar to the one of [21] but new techniques are used for partitioning the bipartite graph matchings into groups that can be colored and accounted for independently. Improving these techniques and solving optimally the constrained bipartite edge coloring problem, Kaklamanis et al. [16] present a greedy algorithm that routes a set of requests of maximum load l using at most $5l/3$. They also prove that no greedy algorithm can go below $5l/3$. Their lower bound holds for binary trees also. Caragiannis et al. [4] present a simple greedy algorithm that achieves the same upper bound for binary trees. They also prove that the lower bound of $5l/3$ for greedy algorithms holds for the case of leaf–to–leaf communication patterns.

A good survey on results for both problems (and other problems on optical networks) can be found in [2]. In this paper we study a special case of the directed wavelength routing problem in trees. We consider symmetric patterns of requests, e.g. for any request originating from a node v_1 and destined to a node v_2, there also exists its symmetric, originating from v_2 and destined to v_1. Our motivation lies in the fact that many services (like video–conference) that are expected to be supported by high performance optical networks in the future require bidirectional reservation of bandwidth. We believe that restricting these services to the use of one wavelength in both directions (as the undirected model requires) cannot guarrantee high bandwidth utilization.

1.1 Outline of the paper

The rest of the paper is structured as follows. In section 2 we prove that the special case of allocating bandwidth for symmetric communication requests is hard for arbitrary trees. Lower bounds on the total number of wavelengths necessary for wavelength routing are presented in section 3. In the same section we prove that there is also a lower bound on the number of colors seen by a fiber link for the non–symmetric problem. We also present a lower bound for greedy wavelength routing algorithms. A discussion on upper bounds and open questions are presented in section 4.

2 Arbitrary tree networks

In this section we prove that the problem is hard for arbitrary networks, even if the load is a small constant. We use a similar reduction with the one that was used for proving the NP–completeness of the non–symmetric problem [7].

Theorem 1 *Bandwidth allocation for symmetric connection requests of load at least 3 in directed trees is NP-hard.*

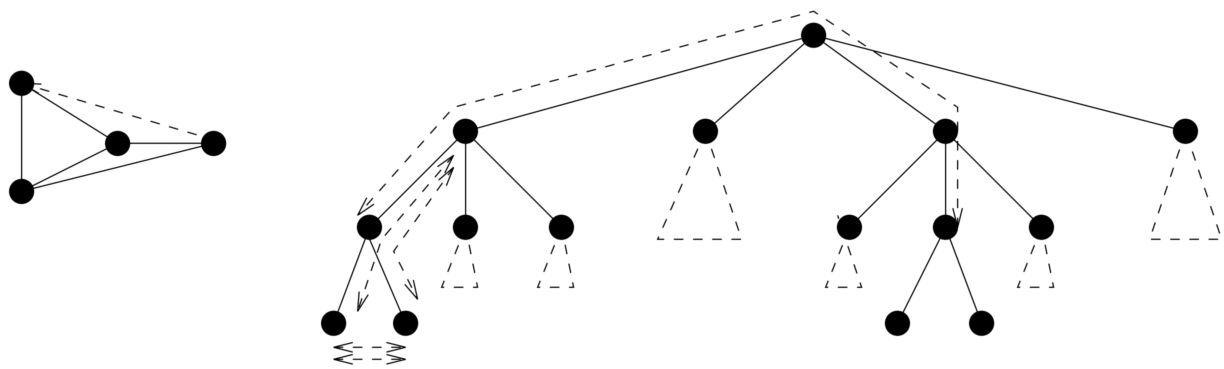

Figure 1: The reduction from EDGE–COLORING.

Proof: We use a reduction from EDGE–COLORING. Let $G(U, E)$ be a 3-regular graph. It is known that it is NP-complete to decide whether the edges of G can be colored with 3 colors. Any instance of the EDGE–COLORING problem can be transformed to an instance of our problem as follows:

We construct a tree network $T(V, E(T))$ such that the root w of T has $|V|$ children (one for each vertex u_i of G). Each one of them has three binary subtrees (one for each edge adjacent to u_i). Formally:

$$V' = \{w\} \cup \{v_i | 1 \leq i \leq |V|\} \cup \{v_{ij}, v_{ij1}, v_{ij2} | 1 \leq i \leq |V|, j = 1, 2, 3\}$$

$$\begin{aligned} E' &= \{(w, v_i) | 1 \leq i \leq |V|\} \cup \{(v_i, v_{ij}) | 1 \leq i \leq |V|, j = 1, 2, 3\} \\ &\cup \{(v_{ij}, v_{ij1}), (v_{ij}, v_{ij2} | 1 \leq i \leq |V|, j = 1, 2, 3\} \end{aligned}$$

For every edge $(u_i, u_j) \in E$, the multiset of requests R contains 5 symmetric requests:

$$R = \{[v_{ik_1}, v_{jk_2}], [v_i, v_{ik_1 1}], [v_i, v_{ik_1 2}], [v_{ik_1 1}, v_{ik_1 2}], [v_{ik_1 1}, v_{ik_1 2}] | (u_i, u_j \in E\}$$

and the k_1, k_2 are choosen such that no other request of R corresponding to some $(u_i, s), (u_j, s) \in E$ $(s \neq u_i, u_j)$ uses the links (v_i, v_{ik_1}) and (v_j, v_{jk_2}). The transformation for one edge is depicted in figure 1.

Now we claim that there exists a proper coloring of requests with 3 colors iff there exists a 3–coloring of the edges of G. If we have a 3–coloring of G, we can assign the colors used for every edge $(u_i, u_j) \in E$ to both the corresponding symmetric requests $[v_{ik_1}, v_{jk_2}]$. The rest of the requests in the subtrees of v_{ik_1} can be properly colored with the two colors that are available. Also, coloring the requests that corresponds to other edges of G does not create any conflicts and the 3–coloring of requests is legal.

If there exists a 3–coloring of requests in T, the requests in the binary subtree of v_{ik_1} make sure that both the symmetric requests $[v_{ik_1}, v_{jk_2}]$ that traverse the root w are colored with the same color. By assigning this color to the edge (u_i, u_j) of G, we obtain a proper 3–coloring for G. Thus, the decision problem is NP-complete and the optimization problem is NP-hard. ■

3 Lower bounds

It is known [19] that there exist patterns of non–symmetric requests with load l that require at least $5l/4$ total colors. In the following we prove that the same bound holds for the symmetric version of the problem. Thus one must not hope for a bandwidth utilization higher than 80% in WDM optical tree networks.

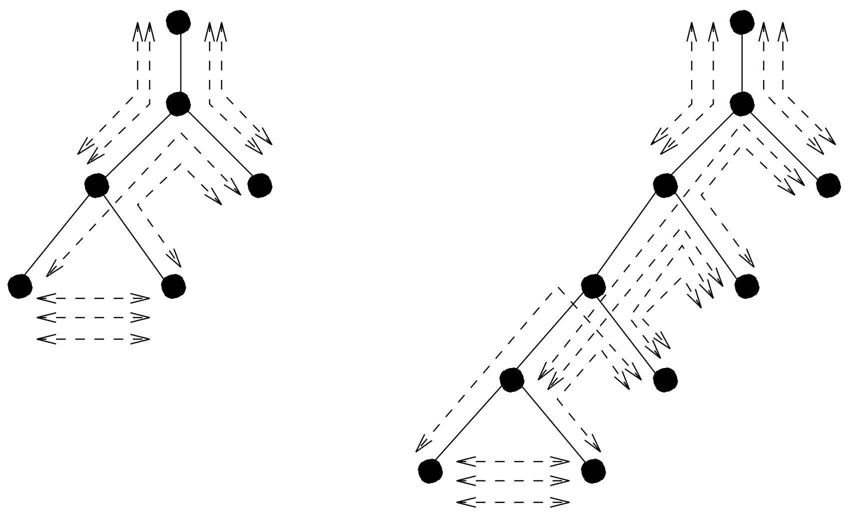

Figure 2: The networks T_0 and T_1 of the proof of theorem 2.

Theorem 2 *For each l and $\epsilon > 0$, there exists a tree T and a pattern R of symmetric communication requests of (maximum) load l per directed fiber link such that $(5/4 - \epsilon)l$ colors are necessary to color R.*

Proof: Consider the network T_0 with the pattern of symmetric communication requests R_0 shown at the left of figure 2. Each arc represents a set of $l/4$ pairs of symmetric communication requests. There exists a total of $9l/2$ requests and the maximum independent set of requests (the maximum number of requests that can be colored with the same color) is 4. Thus R_0 requires at least $9l/8$ total colors.

Consider now the network T_1 with the pattern of requests R_1 shown at the right of figure 2. Generally, we define the sequence of networks T_i by replacing the leftmost node of T_{i-1} with a binary tree network of 5 nodes. The pattern R_i is also produced by R_{i-1} as shown in figure 2. We can verify inductively that the pattern R_n consists of $\frac{9+5n}{2}l$ requests while the maximum independent set of requests has size $4 + 2n$. Thus the pattern of requests R_n requires at least $\left(\frac{5}{4} - \frac{1}{8+4n}\right) l$ colors. The theorem follows. ■

Although the previous theorems give the intuition that the directed path coloring and its symmetric seem to be similar problems, there is an inheritant difference between them. Obviously, we can color each pattern of symmetric requests of load l in such way that each link sees at most l colors in both directions. This is trivial since we can consider each pair of symmetric requests as an undirected request, and assign to both of them the same color. This is not the case in the non–symmetric version of the problem, as the next theorem states.

Theorem 3 *For any l, there exists a tree T and a pattern of communication requests of (maximum) load l per directed fiber link that cannot be colored in such way that any link of T sees (strictly) less than $9l/8$ colors.*

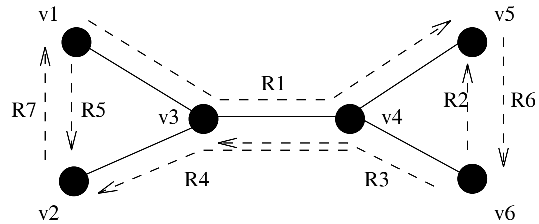

Figure 3: A network and pattern of communication requests that requires $9l/8$ colors per link.

Proof: Consider the network and the pattern of communication requests shown in figure 3. Each one of the arcs $R_1, ..., R_5$ represents a set of $l/2$ requests while R_6 and R_7 represent sets of l requests. At first we present a proper coloring of the requests with $5l/4$ total colors in such a way that no link sees more than $9l/8$ colors in both directions.

Let W be the set of total colors consisting of five disjoint sets of colors $W = A \cup B \cup C \cup D \cup E$. Each one of the sets A, B, C, D, E has cardinality $l/4$. We use the colors of A and B to color the requests of R_1, the colors of C and D for the requests of R_2, the colors of E and A for the requests of R_3, the colors of B and C for the requests of R_4, and the colors of D and E for the requests of R_5. The requests of R_6 are colored using the colors of A, C and D, half the colors of B and half the colors of E while the requests of R_7 are colored using the colors B, D and E, half the colors of A and half the colors of C. The coloring is proper and, furthermore, the links (v_1, v_3), (v_2, v_3), (v_4, v_5), and (v_4, v_6) see exactly $9l/8$ colors.

Assume that requests R_1, R_4, R_5, and R_7 are colored such that both links (v_1, v_3) and (v_2, v_3) see strictly less than $9l/8$ colors. This means that there are more than $l/4$ common colors between requests R_1 and R_4. Since R_3 must be colored with different colors from R_4 there will be less than $l/4$ common colors between R_1 and R_3. Thus, after coloring R_2 and R_6 at least one of the links (v_4, v_5) and (v_4, v_6) sees more than $9l/8$ colors. ■

Current approaches of the wavelength routing problem in trees use greedy algorithms [21,15,16, 4,14,19]. A greedy algorithm visits the nodes of the network in a top to bottom manner and at each node v colors all requests that touch vertex v and are still uncolored. Moreover, once a requests has been colored it is never colored again. Greedy algorithms are important as they are very simple, and they are amenable of being implemented in a distributed environment.

The following theorem introduces limitations on the performance of greedy bandwidth allocation algorithms when they are applied to symmetric patterns of requests. Its proof derives by slightly modifying the proof of the lower bound for the non–symmetric problem[16].

Theorem 4 *For each l and for each greedy algorithm A there exists a tree and a pattern of symmetric communication requests of maximum load l for which A uses at least $3l/2$ wavelengths.*

4 Upper bounds

The problem of assigning wavelengths to communication requests can be viewed as an undirected problem. Each pair of symmetric requests is considered as one undirected request and is colored with one color. Although the algorithm of Erlebach and Jansen [7] guarrantees a 1.1 approximation of the optimal wavelength assignment for the undirected problem, it may produce a coloring with $3l/2$ colors where l is the maximum load of the communication pattern. Such a coloring can also be produced by the algorithm of Raghavan and Upfal [25] which is simpler.

As an alternative (for binary tree networks), note that the conflict graph (the graph that has one vertex for each request and an edge between two vertices iff the corresponding requests share a link) of a pattern of undirected communication requests is chordal [9] (any cycle of length greater than

3 has a chord). Chordal graphs can be colored in polynomial time using a number of colors equal to the size of the largest clique [11], which can be at most $3l/2$ where l is the maximum load of the communication pattern.

The techniques described above imply that the algorithms know which requests are symmetric. This may be difficult to implement in a distributed setting. Greedy algorithms have only local knowledge of the requests that are colored in each step. It can be proved that the greedy algorithm presented in [16] colors any pattern of symmetric communication requests using at most $3l/2$ colors, without the knowledge of which requests are symmetric. Furthermore, the algorithm need not to maintain any invariant on the number of colors seen by each fiber link at each step, and is much simpler than the standard one.

Theorem 5 *There exist a greedy algorithm that colors each pattern of symmetric communication requests using at most $3l/2$ colors.*

Two issues are still open.

- Is the problem of allocating optical bandwidth for symmetric communication requests in binary tree–shaped optical networks in P?
- Implement an approximation algorithm for arbitrary directed trees that achieves better approximation than 3/2.

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