39

Minimum Energy Communication in Ad Hoc Wireless Networks

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39.1 Introduction

Wireless networks have received significant attention during the recent years. Especially, *ad hoc wireless networks* emerged owing to their potential applications in environmental monitoring, sensing, specialized ad hoc distributed computations, emergency disaster relief, battlefield situations, and so forth [37, 41]. Unlike traditional wired networks or cellular wireless networks, no wired backbone infrastructure is installed for ad hoc networks.

A node (or station) in these networks is equipped with an omnidirectional antenna that is responsible for sending and receiving signals. Communication is established by assigning to each station a transmitting power. In the most common power attenuation model [37], the signal power falls proportionally to $1/r^{\alpha}$, where r is the distance from the transmitter and α is a constant that depends on the wireless environment (typical values of α are between 1 and 6). So, a transmitter can send a signal to a receiver if $P_s/(d(s,t)^{\alpha}) \geq \gamma$, where P_s is the power of the transmitting signal, $d(s,t)$ is the Euclidean distance between the transmitter and the receiver, and γ is the receiver's power threshold for signal detection, which is usually normalized to 1.

Communication from a node s to another node t may be established either directly if the two nodes are close enough and s uses adequate transmitting power or by using intermediate nodes. Observe that owing to the nonlinear power attenuation, relaying the signal between intermediate nodes may result in energy conservation.

A crucial issue in ad hoc networks is to support communication patterns that are typical in traditional networks. These include broadcasting, multicasting, and gossiping (all-to-all communication). Since establishing a communication pattern strongly depends on the use of energy, the important engineering question to be solved is to guarantee a desired communication pattern, minimizing the total energy consumption. In this chapter, we consider a series of *minimum energy communication* problems in ad hoc wireless networks, which we formulate below.

We model an ad hoc wireless network by a complete directed graph $G = (V, E)$, where $|V| = n$, with a nonnegative edge cost function $c : E \to R^+$. Given a nonnegative node weight assignment $w : V \to R^+$, the *transmission graph* G_w is the directed graph defined as follows. It has the same set of nodes as G and a directed edge (u, v) belongs to G_w if the weight assigned to node u is at least the cost of the edge (u, v) (i.e., $w(u) \geq c(u, v)$). Intuitively, the weight assignment corresponds to the energy levels at which each node operates (i.e., transmits messages), while the cost between two nodes indicates the minimum energy level necessary to send messages from one node to the other. Usually, the edge cost function is symmetric (i.e., $c(u, v) = c(v, u)$). An important special case, which usually reflects the real-world situation, henceforth called geometric case, is when nodes of G are points in a Euclidean space and the cost of an edge (u, v) is defined as the Euclidean distance between u and v raised to a fixed power α ranging from 1 to 6 (i.e., $c(u, v) = d(u, v)^\alpha$). Asymmetric edge cost functions can be used to model medium abnormalities or batteries with different energy levels [32].

The problems we study in this work can be stated as follows. Given a complete directed graph $G = (V, E)$, where $|V| = n$, with nonnegative edge costs $c : E \to R^+$, find a nonnegative node weight assignment $w : V \to R^+$ such that the transmission graph G_w maintains a connectivity property and the sum of weights is minimized. Such a property is defined by a requirement matrix $R = (r_{ij}) \in \{0, 1\}$, where r_{ij} is the number of directed paths required in the transmission graph from node v_i to node v_j. Depending on the connectivity property for the transmission graph, we may define a variety of problems.

Several communication requirements are of interest. In minimum energy steiner subgraph (MESS), the requirement matrix is symmetric. Alternatively, we may define the problem by a set of nodes $D \subseteq V$ partitioned into p disjoint subsets $D_1, D_2, ..., D_p$. The entries of the requirement matrix are now defined as $r_{ij} = 1$ if $v_i, v_j \in D_k$ for some k and $r_{ij} = 0$, otherwise. The minimum energy subset strongly connected subgraph (MESSCS) is the special case of MESS with $p = 1$ while the minimum energy strongly connected subgraph (MESCS) is the special case of MESSCS with $D = V$ (i.e., the transmission graph is required to span all nodes of V and to be strongly connected). Althaus et al. [1] and Călinescu et al. [11] study MESCS under the extra requirement that the transmission graph contains a bidirected subgraph (i.e., a directed graph in which the existence of a directed edge implies that its opposite directed edge also exists in the graph), which maintains the connectivity requirements of MESCS. By adding this extra requirement to MESS and MESSCS, we obtain the bidirected MESS and bidirected MESSCS, respectively, that is, the requirement for the transmission graph in bidirected MESS (resp., bidirected MESSCS) is to contain as a subgraph a bidirected graph satisfying the connectivity requirements of MESS (resp., MESSCS). Minimum energy communication problems with symmetric requirement matrices are usually referred to as *group communication* problems.

In minimum energy multicast tree (MEMT), the connectivity property is defined by a root node v_0 and a set of nodes $D \subseteq V - \{v_0\}$ such that $r_{ij} = 1$ if $i = 0$ and $v_j \in D$ and $r_{ij} = 0$, otherwise. The minimum energy broadcast tree (MEBT) is the special case of MEMT with $D = V - \{v_0\}$. By inverting the connectivity requirements, we obtain the following two problems: the minimum energy inverse multicast tree (MEIMT) where the connectivity property is defined by a root node v_0 and a set of nodes $D \subseteq V - \{v_0\}$ such that $r_{ij} = 1$ if $v_i \in D$ and $j = 0$ and $r_{ij} = 0$, otherwise, and the minimum energy inverse broadcast tree (MEIBT), which is the special case of MEIMT with $D = V - \{v_0\}$.

TABLE 39.1 Abbreviations for Problems Used in This Chapter

Abbreviation	Problem
MESS	Minimum Energy Steiner Subgraph
MESSCS	Minimum Energy Subset Strongly Connected Subgraph
MESCS	Minimum Energy Strongly Connected Subgraph
MEMT	Minimum Energy Multicast Tree
MEBT	Minimum Energy Broadcast Tree
MEIMT	Minimum Energy Inverse Multicast Tree
MEIBT	Minimum Energy Inverse Broadcast Tree
SF	Steiner Forest
ST	Steiner Tree
DST	Directed Steiner Tree
MSA	Minimum Spanning Arborescence
NWSF	Node-Weighted Steiner Forest
NWST	Node-Weighted Steiner Tree

The terminology used in this chapter is the same as the one in Reference 14. Table 39.1 summarizes the abbreviations used for the problems studied, as well as for other combinatorial problems used several times in the rest of this chapter.

In the following sections, we usually refer to classical combinatorial optimization problems and show the relations of minimum energy communication problems to them. For completeness, we present their definitions here. The steiner forest (SF) problem is defined as follows. Given an undirected graph $G = (V, E)$ with an edge cost function $c : E \rightarrow R^+$ and a set of nodes $D \subseteq V$ partitioned into p disjoint sets $D_1, \ldots, D_p$, compute a subgraph H of G of minimum total edge cost such that any two nodes v_i, v_j belonging to the same set D_k for some k are connected through a path in H. Steiner Tree (ST) is the special case of SF with $p = 1$. An instance of the Directed Steiner Tree (DST) is defined by a directed graph $G = (V, E)$ with an edge cost function $c : E \rightarrow R^+$, a root node $v_0 \in V$, and a set of terminals $D \subseteq V - \{v_0\}$. Its objective is to compute a tree of minimum edge cost that is directed out of v_0 and spans all nodes of D. The special case of DST, with $D = V - \{v_0\}$ is called Minimum Spanning Arborescence (MSA). The Node-Weighted Steiner Forest (NWSF) problem is defined as follows. Given an undirected graph $G = (V, E)$ with a node cost function $c : V \rightarrow R^+$ and a set of nodes $D \subseteq V$ partitioned into p disjoint sets $D_1, \ldots, D_p$, compute a subgraph H of G of minimum total node cost such that any two nodes v_i, v_j belonging to the same set D_k for some k are connected through a path in H. Node-Weighted Steiner Tree (NWST) is the special case of NWSF with $p = 1$.

We study symmetric wireless networks in Section 39.2. In this setting, communication problems with symmetric connectivity requirements admit algorithms with constant approximation ratio. Multicasting and broadcasting are inherently more difficult, admitting only logarithmic approximations. Almost all communication problems become even harder in the more general case of asymmetric wireless networks. For example, MEMT is equivalent to DST in terms of hardness of approximation; this implies similar complexity for other communication problems as well. Surprisingly, broadcasting still admits logarithmic approximation in asymmetric wireless networks. Results on the asymmetric model are surveyed in Section 39.3. Table 39.2 summarizes the known results in symmetric and asymmetric wireless networks.

Better results exist for minimum energy communication problems in geometric wireless networks. The linear case where nodes correspond to points on a line has been proved to be tractable while most problems become hard in higher dimensions. We discuss the related results in Section 39.4. A summary of them is presented in Table 39.3.

We conclude this chapter by presenting extensions of the network model and briefly discussing results on related communication problems in Section 39.5.

TABLE 39.2 The Best Known Results for the Problems Discussed in This Chapter in Symmetric and Asymmetric Wireless Networks

	Approximability in asymmetric networks		Approximability in symmetric networks	
Problem	Lower bound	Upper bound	Lower bound	Upper bound
MESS	$\Omega(\log^{2-\epsilon} n)$ [14]		313/312 [21]	4 [14]
MESSCS	$\Omega(\log^{2-\epsilon} n)$ [14]		313/312 [21]	3.1 [14]
MESCS	$\Omega(\log n)$ [9]	$O(\log n)$ [9,14]	313/312 [21]	2 [30]
Bidirected MESS	$\Omega(\log \|D\|)$ [1]	$O(\log \|D\|)$ [14]	96/95 [14]	4 [14]
Bidirected MESSCS	$\Omega(\log \|D\|)$ [1]	$O(\log \|D\|)$ [14]	96/95 [14]	3.1 [14]
Bidirected MESCS	$\Omega(\log n)$ [1]	$O(\log n)$ [9,14]	313/312 [21]	5/3 [1]
MEMT	$\Omega(\log^{2-\epsilon} n)$ [9,14]	$O(\|D\|^{\epsilon})$ [32]	$\Omega(\log n)$ [18]	$O(\log n)$ [14]
MEBT	$\Omega(\log n)$ [18]	$O(\log n)$ [9,14]	$\Omega(\log n)$ [18]	$O(\log n)$ [6,13]
MEIMT	$\Omega(\log^{2-\epsilon} n)$ [14]	$O(\|D\|^{\epsilon})$ [14]	96/95 [14]	1.55 [14]
MEIBT	1	1	1	1

TABLE 39.3 The Best Known Results under the Geometric Model

	Linear networks	Multidimensional networks	
	Complexity	Complexity	Approximability
MESCS	P [30]	NP-hard [21,30] APX-hard ($d \geq 3$) [21]	2 [30]
MEMT	P [13,19]	NP-hard [18]	9.3 ($\alpha = d = 2$) [1]
MEBT	P [13,19]	NP-hard [18]	6 ($\alpha = d = 2$) [2] $3^d - 1$ [23]

39.2 Symmetric Wireless Networks

In this section, we consider ad hoc wireless networks with symmetric edge cost functions in the underlying subgraph. Almost all the minimum energy communication problems discussed in the previous section are NP-hard in these networks. Observe that the geometric model is a special case of symmetric networks, hence, the hardness results for the geometric case hold in this case as well. We postpone the discussion on the particular hardness results until Section 39.4 where we discuss the results on geometric networks in more detail. Here, we only state the stronger inapproximability bounds that hold owing to the generality of symmetric wireless networks. On the positive side, we present approximation algorithms for each problem.

39.2.1 Symmetric Connectivity Requirements

We start by presenting a constant approximation algorithm for MESS; this is the most general problem falling in this category. The algorithm constructs a solution to MESS by exploiting the solution of a corresponding instance for problem SF. The reduction presented in the following appeared in Reference 14. It can be thought of as a generalization of an algorithm in Reference 30, which uses minimum spanning trees (MST) to approximate instances of MESCS.

Consider an instance I_{MESS} of MESS that consists of a complete directed graph $G = (V, E)$, a symmetric edge cost function $c : E \rightarrow R^+$, and a set of terminals $D \subseteq V$ partitioned into p disjoint subsets $D_1, \ldots, D_p$. Construct the instance I_{SF} of SF that consists of the complete undirected graph $H = (V, E')$, the edge cost function $c' : E' \rightarrow R^+$, defined as $c'(u, v) = c(u, v) = c(v, u)$ on the undirected edges of E', and the set of terminals D together with its partition into the sets $D_1, \ldots, D_p$. Consider a solution for I_{SF} that consists of a subgraph $F = (V, A)$ of H. Construct the weight assignment w to the nodes of V

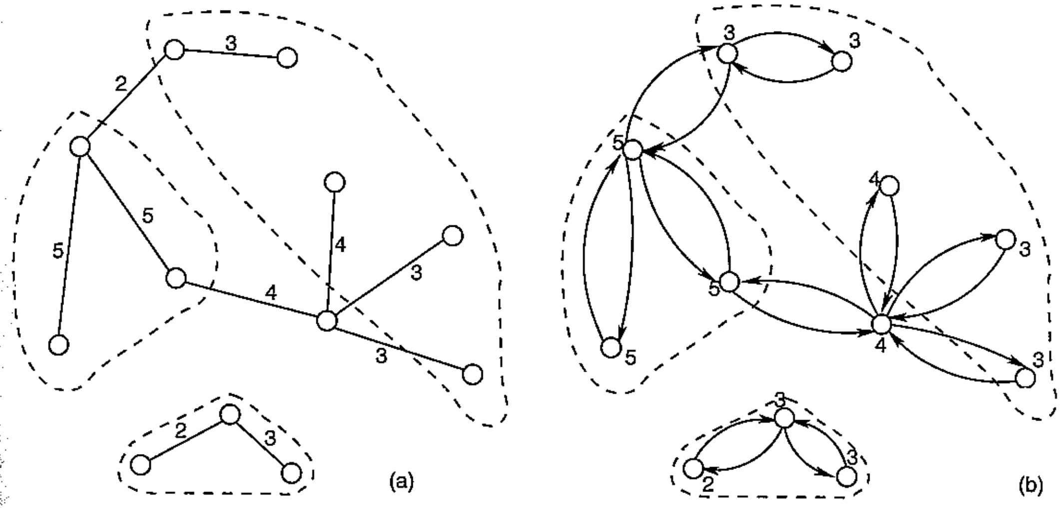

FIGURE 39.1 Transforming a solution for I_{SF} (a) to a solution for I_{MESS} (b). In (a), numbers on the edges of the solution denote their cost. In (b), numbers are associated with the nodes and denote their weight. In both cases, the dashed closed lines indicate the subsets in which the set of terminals is partitioned.

by setting $w(u) = 0$, if there is no edge touching u in A and $w(u) = \max_{v:(u,v)\in A}\{c'(u,v)\}$, otherwise. An example of this construction is presented in Figure 39.1. The following statement has been proved in Reference 14 (see also Reference 30).

Lemma 39.1 *If F is a ρ-approximate solution for I_{SF}, then w is a 2ρ-approximate solution for I_{MESS}.*

We can solve I_{SF} using the 2-approximation algorithm of Goemans and Williamson [25] for SF. When $p = 1$ (i.e., when I_{MESS} is actually an instance of MESSCS), the instance I_{SF} is actually an instance of ST that can be approximated within $1 + \frac{1}{2}\ln 3 \approx 1.55$, using an algorithm of Robins and Zelikovsky [38]. We obtain the following result.

Theorem 39.1 *There exist a 4-, a 3.1-, and a 2-approximation algorithm for MESS, MESSCS, and MESCS in symmetric wireless networks, respectively.*

Clearly, the transmission graph constructed by the above technique contains a bidirected subgraph that maintains the connectivity requirements of MESS, and thus, the algorithms for MESS, MESSCS, and MESCS actually provide solutions to bidirected MESS, bidirected MESSCS, and bidirected MESCS, respectively. The analysis presented in Reference 14 still holds; thus, the approximation guarantees of Theorem 39.1 hold for bidirected MESS and bidirected MESSCS in symmetric wireless networks as well.

Next, we present a simple approximation-preserving reduction from ST to bidirected MESSCS in symmetric wireless networks.

Given an instance I_{ST} of ST that consists of an undirected graph $G = (V, E)$ with edge cost function $c : E \to R^+$, and a set of terminals $D \subseteq V$, construct the instance $I_{bMESSCS}$ as follows. $I_{bMESSCS}$ consists of a complete directed graph $H = (U, A)$ with symmetric edge cost function $c' : A \to R^+$, and a set of terminals $D' \subseteq U$. The set of nodes U contains a node h_v for each node v of V and two nodes $h_{(u,v)}$ and $h_{(v,u)}$ for each edge (u, v) of E. The edge cost function c' is defined as $c'(h_u, h_{(u,v)}) = c'(h_{(u,v)}, h_u) = c'(h_{(v,u)}, h_v) = c'(h_v, h_{(v,u)}) = 0$ and $c'(h_{(u,v)}, h_{(v,u)}) = c'(h_{(v,u)}, h_{(u,v)}) = c(u, v)$, for each edge (u, v) of E, while all other directed edges of A have infinite cost. The construction is presented in Figure 39.2. The set of terminals is defined as $D' = \{h_u \in U | u \in D\}$. It is not difficult to see that a ρ-approximate solution for instance $I_{bMESSCS}$ reduces in polynomial time to a ρ-approximate solution

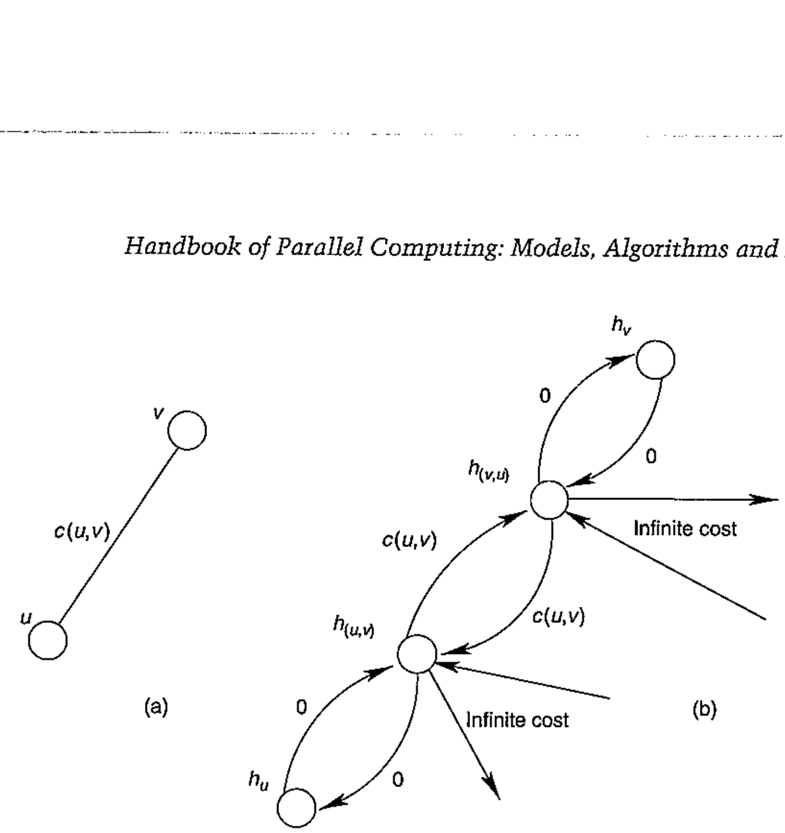

FIGURE 39.2 An edge in I_{ST} (a) and the corresponding structure in $I_{bMESSCS}$ (b). In (b), all edges that are incident to $h_{(u,v)}$ but are not incident to either $h_{(v,u)}$ or h_u, as well as all edges that are incident to $h_{(v,u)}$ but are not incident to either $h_{(u,v)}$ or h_v have infinite cost.

for instance I_{ST}. Thus, using an inapproximability result for ST presented in Reference 17, we obtain the following:

Theorem 39.2 *For any $\epsilon > 0$, bidirected MESSCS in symmetric wireless networks is not approximable within $96/95 - \epsilon$, unless P = NP.*

Clearly, this result also applies to bidirected MESS. For MESCS, a weaker inapproximability result of 313/312 follows by adapting a reduction of Reference 20 to Vertex Cover in bounded-degree graphs and using a known inapproximability result for the latter problem [5].

39.2.2 Multicasting and Broadcasting

In this section, we present logarithmic approximation algorithms for MEMT and MEBT in symmetric wireless networks. Such results have been obtained independently in References 6, 7, 13, and 14. The algorithms in References 6 and 7 use set covering techniques, while the algorithm in References 13 reduces the problem to Node Weighted Connected Dominating Set. We present the algorithm from Reference 14 here, which is probably the simplest one; it reduces the problem to NWST.

Consider an instance I_{MEMT} of MEMT, which consists of a complete directed graph $G = (V, E)$, a symmetric edge cost function $c : E \rightarrow R^+$, a root node $v_0 \in V$, and a set of terminals $D \subseteq V - \{v_0\}$.

Construct an instance I_{NWST} of NWST, which consists of an undirected graph $H = (U, A)$, a node weight function $c' : U \rightarrow R^+$, and a set of terminals $D' \subseteq U$. For a node $v \in V$, we denote by n_v the number of different edge costs in the edges directed out of v, and, for $i = 1, \ldots, n_v$, we denote by $X_i(v)$ the ith smallest edge cost among the edges directed out of v. The set of nodes U consists of n disjoint sets of nodes called *supernodes*. Each supernode corresponds to a node of V. The supernode Z_v corresponding to node $v \in V$ has the following $n_v + 1$ nodes: an *input node* $Z_{v,0}$ and n_v *output nodes* $Z_{v,1}, \ldots, Z_{v,n_v}$. For each pair of nodes $u, v \in V$, the set of edges A contains an edge between the output node $Z_{u,i}$ and the input node $Z_{v,0}$ such that $X_i(u) \geq c(u, v)$. Also, for each node $v \in V$, A contains an edge between the input node $Z_{v,0}$ and each output node $Z_{v,i}$, for $i = 1, \ldots, n_v$. The cost function c' is defined as $c'(Z_{v,0}) = 0$ for the input nodes and as $c'(Z_{v,i}) = X_i(v)$ for $i = 1, \ldots, n_v$, for the output nodes. The set of terminals D' is defined as $D' = \{Z_{v,0} \in U | v \in D \cup \{v_0\}\}$. An example of this reduction is depicted in Figure 39.3.

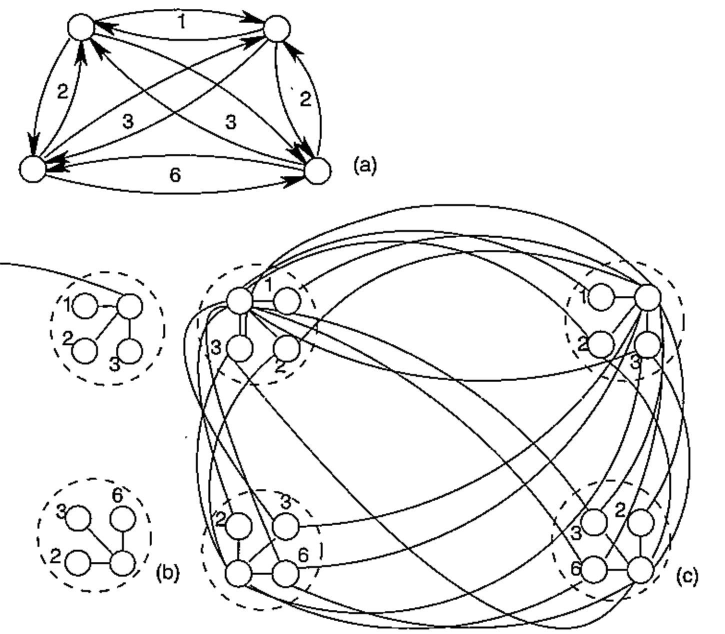

FIGURE 39.3 The reduction to Node-Weighted Steiner Tree. (a) The graph G of an instance of MEMT. (b) The graph H of the corresponding instance of NWST. Each large cycle indicates a supernode. Only the edges incident to the node of weight 2 of the upper left supernode are shown. These edges are those that correspond to edges in (a) of cost at most 2, directed out of the left upper node. (c) The graph H of the corresponding instance of NWST.

Consider a subgraph $F = (S, A')$ of H, which is a solution for I_{NWST}. We compute a spanning tree $T' = (S, A'')$ of F and, starting from $Z_{v_0,0}$, we compute a breadth-first search (BFS) numbering of the nodes of T'. For each $v \in S$, we denote by $m(v)$ the BFS number of v. We construct a tree $T = (V, E')$ that for each edge of F between a node $Z_{u,i}$ of supernode Z_u and a node $Z_{v,j}$ of another supernode Z_v such that $m(Z_{u,i}) < m(Z_{v,j})$, contains a directed edge from u to v. The output of our algorithm is the weight assignment w defined as $w(u) = \max_{(u,v)\in T} c(u, v)$ if u has at least one outgoing edge in T, and $w(u) = 0$, otherwise. The following lemma is proved in Reference 14 and relates the quality of the two solutions.

Lemma 39.2 *If F is a ρ-approximate solution to I_{NWST}, then w is a 2ρ-approximate solution to I_{MEMT}.*

Guha and Khuller [26] present a $1.35 \ln k$-approximation algorithm for NWST, where k is the number of terminals in the instance of NWST. Given an instance I_{MEMT} of MEMT with a set of terminals D, the corresponding instance I_{NWST} has $|D| + 1$ terminals. Thus, the cost of the solution of I_{MEMT} is within $2 \cdot 1.35 \ln(|D| + 1) = 2.7 \ln(|D| + 1)$ of the optimal solution. We remind that MEBT is the special case of MEMT with $D = V - \{v_0\}$.

Theorem 39.3 *There exist a $2.7 \ln(|D| + 1)$- and a $2.7 \ln n$-approximation algorithm for MEMT and MEBT in symmetric wireless networks, respectively.*

39.3 Asymmetric Wireless Networks

In general, minimum energy communication problems are more difficult in the asymmetric model. For example, MEIMT is equivalent to DST. This is a rather disappointing result since DST has polylogarithmic inapproximability while the best known algorithm has polynomial approximation ratio. The following simple reduction from References 9 and 14 demonstrates the equivalence of MEIMT and DST.

Assume that we have an instance I_{MEIMT} of MEIMT defined by a complete directed graph $G = (V, E)$, an edge cost function $c : E \to R^+$, a root node $v_0 \in V$, and a set of terminals $D \subseteq V - \{v_0\}$. Consider the instance I_{DST} of DST that consists of G, the edge cost function $c' : E \to R^+$ defined as $c'(u, v) = c(v, u)$ for any edge $(u, v) \in E$, the set of terminals D, and the root node v_0. Also, we may start by an instance I_{DST} of DST and construct I_{MEIMT} in the same way. Then, it is not difficult to see that any ρ-approximate solution for I_{DST} reduces (in polynomial time) to a ρ-approximate solution for I_{MEIMT} while any ρ-approximate solution for I_{MEIMT} also reduces to a ρ-approximate solution for I_{DST}.

As corollaries, using the approximability and inapproximability results presented in References 16 and 29, we obtain that MEIMT is approximable within $O(|D|^{\epsilon})$ and inapproximable within $O(\ln^{2-\epsilon} n)$, for any constant $\epsilon > 0$. Note that DST in symmetric wireless networks is equivalent to ST. Going back to symmetric networks, using the approximability and inapproximability results presented in References 38 and 17, we obtain that MEIMT in symmetric wireless networks is approximable within 1.55 and inapproximable within $96/95 - \epsilon$, for any $\epsilon > 0$. Also, instances of DST having all nonroot nodes as terminals are actually instances of MSA that is known to be computable in polynomial time [22]. Thus, MEIBT can be solved in polynomial time (even in asymmetric wireless networks).

39.3.1 Multicasting, Broadcasting, and Group Communication

MEMT and MESSCS are as hard to approximate as DST. Consider an instance I_{DST} of DST that consists of a directed graph $G = (V, E)$ with an edge cost function $c : E \to R^+$, a root node v_0, and a set of terminals $D \subseteq V - \{v_0\}$. Without loss of generality, we may assume that G is a complete directed graph with some of its edges having infinite cost.

We construct the instance I_{MEMT} of MEMT that consists of a complete directed graph $H = (U, A)$ with edge cost function $c' : A \to R^+$, a root node $v'_0 \in U$, and a set of terminals $D' \subseteq U - \{v'_0\}$. The set of nodes U has a node h_v for each node $v \in V$ and a node $h_{(u,v)}$ for each directed edge (u, v) of E. For each directed edge (u, v) of E, the directed edge $(h_u, h_{(u,v)})$ of A has zero cost and the directed edge $(h_{(u,v)}, h_v)$ of A has cost $c'(h_{(u,v)}, h_v) = c(u, v)$, while all other edges of A have infinite cost. This construction is presented in Figure 39.4. The set of terminals is defined as $D' = \{h_u \in U | u \in D\}$, while $v'_0 = h_{v_0}$. It is not difficult to see that a ρ-approximate solution to I_{MEMT} reduces in polynomial time to a ρ-approximate solution to I_{DST} [14].

A similar reduction can be used to show inapproximability of MESSCS. We construct the instance I_{MESSCS} of MESSCS, which consists of the graph G, the set of terminals $D \cup v_0$, and an edge cost function $c'' : E \to R^+$ defined as follows. For each directed edge (u, v) of E such that $u \neq v_0$, it is $c''(u, v) = c(v, u)$, while all edges of E directed out of v_0 have zero cost. An example of this construction

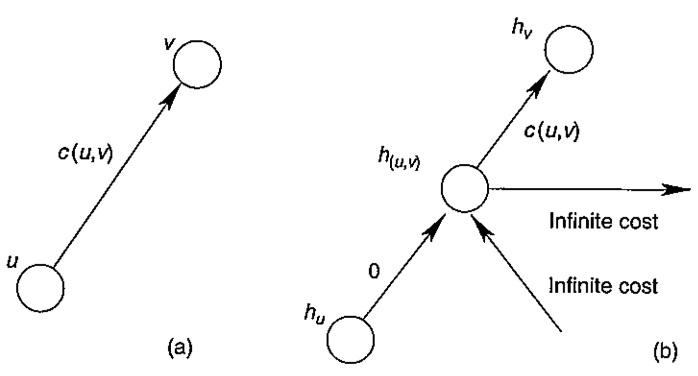

FIGURE 39.4 An edge in I_{DST} (a) and the corresponding structure in I_{MEMT} (b). In (b), edges directed out of $h_{(u,v)}$ that are not incident to h_v, as well as edges that are not incident to h_u and are destined for $h_{(u,v)}$ have infinite cost.

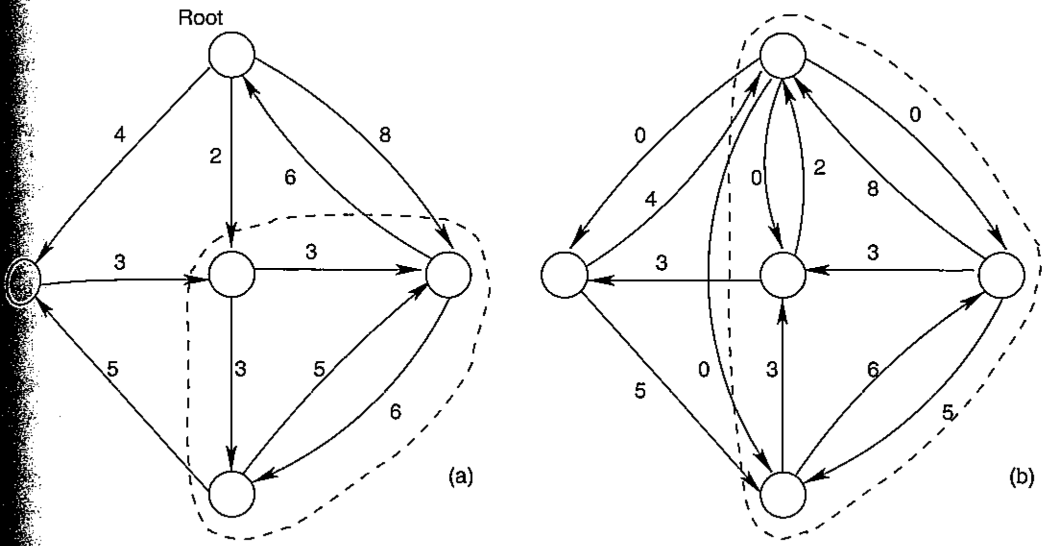

FIGURE 39.5 Transforming an instance of I_{DST} (a) to an instance of I_{MESSCS} (b). Dashed closed lines indicate the sets of terminals.

is presented in Figure 39.5. Again, any ρ-approximate solution to I_{MESSCS} reduces in polynomial time to a ρ-approximate solution to I_{DST} [14].

Using the inapproximability result for DST [29], we obtain the following:

Theorem 39.4 *For any $\epsilon > 0$, MEMT and MESSCS in asymmetric wireless networks are not approximable within $O(\ln^{2-\epsilon} n)$, unless NP $\subseteq$ ZTIME$(n^{polylog(n)})$.*

On the positive side, Liang [32] presents an intuitive reduction for transforming an instance I_{MEMT} of MEMT into an instance I_{DST} of DST in such a way that a ρ-approximate solution for I_{DST} implies a ρ-approximate solution for I_{MEMT}.

We describe this reduction here. Assume that I_{MEMT} consists of a complete directed graph $G = (V, E)$ with an edge cost function $c : E \rightarrow R^+$ and a root node $r \in V$. Then, the instance I_{DST} consists of a directed graph $H = (U, A)$ with an edge cost function $c' : A \rightarrow R^+$, a root node $r' \in U$, and a set of terminals $D \subseteq U - \{r'\}$. For a node $v \in V$, we denote by n_v the number of different edge costs in the edges directed out of v, and, for $i = 1, \ldots, n_v$, we denote by $X_i(v)$ the ith smallest edge cost among the edges directed out of v. For each node $v \in V$, the set of nodes U contains $n_v + 1$ nodes $Z_{v,0}, Z_{v,1}, \ldots, Z_{v,n_v}$. For each directed edge $(v, u) \in E$ and for $i = 1, \ldots, n_v$, the set of edges A contains a directed edge of zero cost from $Z_{v,i}$ to $Z_{u,0}$ if $X_i(v) \geq c(v, u)$. Also, for each node $v \in V$, and $i = 1, \ldots, n_v$, the set of edges A contains a directed edge from $Z_{v,0}$ to $Z_{v,i}$ of cost $c'(Z_{v,0}, Z_{v,i}) = X_i(v)$. An example of this construction is presented in Figure 39.6. The set of terminals is defined by $D = \{Z_{v,0} | v \in V - \{r\}\}$ and $r' = Z_{r,0}$.

Now, a solution for the original instance I_{MEMT} of MEMT is obtained by assigning energy to each node v equal to the cost of the most costly outgoing edge of $Z_{v,0}$ that is used in the solution of I_{DST}. If the solution of I_{DST} is ρ-approximate, the solution obtained for I_{MEMT} in this way is ρ-approximate as well. Using the approximation algorithm for DST presented in Reference 16, we obtain the following:

Theorem 39.5 *For any $\epsilon > 0$, there exists an $O(|D|^{\epsilon})$ approximation algorithm for MEMT in asymmetric wireless networks.*

Note that the algorithm of Liang for approximating MEMT actually computes a solution to an instance of DST with $O(n^2)$ nodes. This means that a polylogarithmic approximation algorithm for DST would immediately yield polylogarithmic approximation algorithms for MEMT.

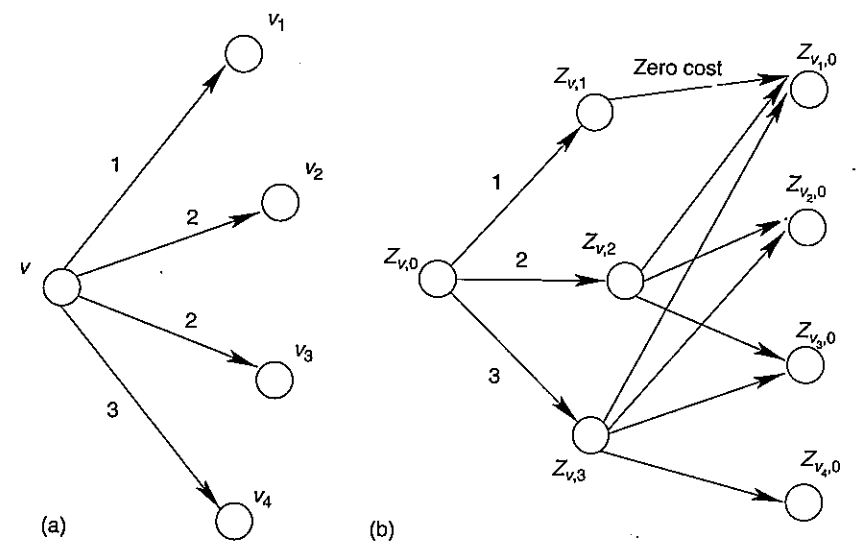

FIGURE 39.6 Liang's reduction of MEMT to DST. A node v and its outgoing edges in I_{MEMT} (a) and the corresponding structure in I_{DST} (b). All edges in (b) directed out of $Z_{v,1}$, $Z_{v,2}$, and $Z_{v,3}$ have zero cost.

Surprisingly, MEBT admits logarithmic approximations. This was independently proved in References 9 and 14. The algorithm presented in Reference 9 uses sophisticated set covering arguments. We briefly discuss the algorithm in Reference 14 that uses Liang's reduction and an algorithm of Zosin and Khuller [43] that efficiently approximates special instances of DST.

The algorithm in Reference 43 approximates I_{DST} by repeatedly solving instances of the minimum density directed tree (MDDT) problem. An instance of MDDT is defined in the same way as instances of DST, and the objective is to compute a tree directed out of the root node such that the ratio of the cost of the tree over the number of terminals it spans is minimized. The algorithm in Reference 43 repeatedly solves instances I^i_{MDDT} of MDDT derived by the instance I_{DST}. The instance I^1_{MDDT} is defined by the graph H with edge cost function c, the set of terminals $D_1 = D$, and the root node $r_1 = r'$. Initially, the algorithm sets $i = 1$. While $D_i \neq \emptyset$, it repeats the following. It finds a solution T to I^i_{MDDT} that consists of a tree $T_i = (V(T_i), E(T_i))$, defines the instance I^{i+1}_{MDDT} by contracting the nodes of T_i into the root node r_{i+1} and by setting $D_{i+1} = D_i \setminus V(T_i)$, and increments i by 1.

Using standard arguments in the analysis of set covering problems, Reference 43 shows that if the solution T_i is a ρ-approximate solution for I^i_{MDDT} in each iteration i, then the union of the trees T_i computed in all iterations is an $O(\rho \ln n)$-approximate solution for I_{DST}. They also show how to find a $(d+1)$-approximate solution for I^i_{MDDT} if the graph obtained when removing the terminals from G has depth d. Observe that, given an instance I_{MEBT} of MEBT, the graph H obtained by applying the reduction of Liang is bipartite, since there is no edge between nodes of $D \cup \{r'\}$ and between nodes of $V - (D \cup \{r'\})$. Thus, the graph obtained by removing the terminals of D from H has depth 1. Following the reasoning presented in Reference 43 and the reduction of Liang, we obtain a logarithmic approximation algorithm for MEBT.

Theorem 39.6 *There exists an $O(\ln n)$-approximation algorithm for MEBT in asymmetric wireless networks.*

We now present a method for approximating MESSCS. Let I_{MESSCS} be an instance of I_{MESSCS} that consists of a complete directed graph $G = (V, E)$ with edge cost function $c : E \rightarrow R^+$ and a set of terminals $D \subseteq V$. Pick an arbitrary node $v_0 \in D$ and let I_{MEMT} and I_{MEIMT} be the instances of MEMT and MEIMT, respectively, consisting of the graph G with edge cost function c, the root node v_0, and the set of terminals $D - \{v_0\}$.

Assume that we have weight assignments w_1 and w_2 to the nodes of V which are solutions for I_{MEMT} and I_{MEIMT}, respectively. Construct the weight assignment w_3 defined as $w_3(u) = \max\{w_1(u), w_2(u)\}$, for every $u \in V$. An example of this construction is presented in Figure 39.7. The following statement is proved in References 9 and 14.

Lemma 39.3 *If the weight assignments w_1 and w_2 are ρ_1 and ρ_2 approximate solutions for I_{MEMT} and I_{MEIMT}, respectively, then the weight assignment w_3 is a $(\rho_1 + \rho_2)$-approximate solution to I_{MESSCS}.*

Hence, we can solve I_{MEMT} and I_{MEIMT} using the reduction of Liang and the $O(|D|^{\epsilon})$-approximation algorithm in Reference 16 for DST, in order to obtain the following result.

Theorem 39.7 *For any $\epsilon > 0$, there exists an $O(|D|^{\epsilon})$-approximation algorithm for MESSCS in asymmetric wireless networks.*

Similarly, we can solve any instance of MESCS by solving an instance of MEBT (using the $O(\ln n)$-approximation algorithm described above) and an instance of MEIBT (this can be done optimally in polynomial time), and then merging the two solutions. In this way, we obtain the following result.

Theorem 39.8 *There exists an $O(\ln n)$-approximation algorithm for MESCS in asymmetric wireless networks.*

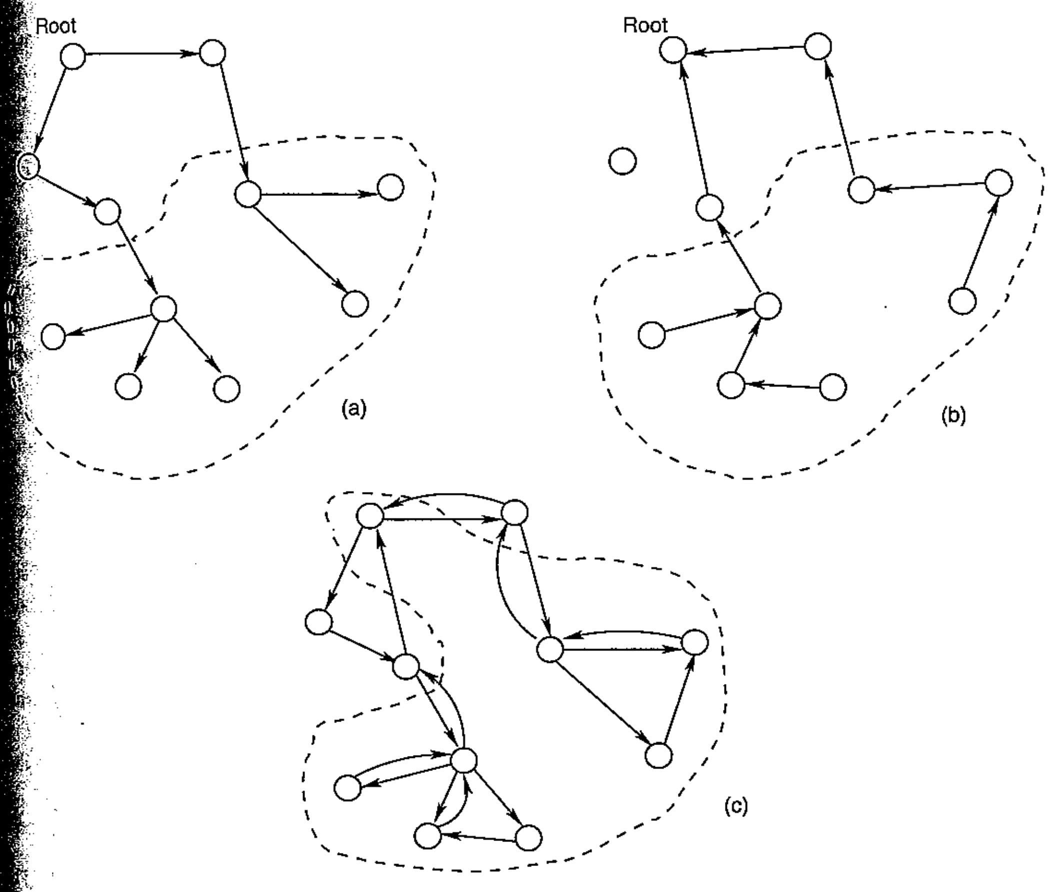

FIGURE 39.7 An example of combining solutions for I_{MEMT} (a) and I_{MEIMT} (b) in order to construct a solution for I_{MESSCS} (c). Dashed closed lines indicate the sets of terminals.

Intuitive reductions of problems MEBT and MESCS to SET COVER show that the results in Theorems 39.6 and 39.8 are tight [18].

39.3.2 Bidirected Connectivity Requirements

In this section, we present a logarithmic approximation algorithm for bidirected MESS in asymmetric wireless networks from Reference 14. We point out that a logarithmic approximation algorithm for the special case of bidirected MESCS based on set covering techniques was obtained independently in Reference 9.

The algorithm in Reference 14 is substantially simpler and uses a reduction of instances of bidirected MESS to instances of NWSF. The main idea behind this reduction is similar to the one used to approximate MEMT in symmetric wireless networks in Section 39.2.2. However, both the construction and the analysis have subtle differences.

Consider an instance $I_{b\text{MESS}}$ of bidirected MESS that consists of a complete directed graph $G = (V, E)$, an edge cost function $c : E \rightarrow R^+$, and a set of terminals $D \subseteq V$ partitioned into p disjoint subsets $D_1, D_2, ..., D_p$. We construct an instance I_{NWSF} of NWSF consisting of an undirected graph $H = (U, A)$, a node weight function $c' : U \rightarrow R^+$, and a set of terminals $D' \subseteq U$ partitioned into p disjoint sets $D'_1, D'_2, \ldots, D'_p$. For a node $v \in V$, we denote by n_v the number of different edge costs in the edges directed out of v, and, for $i = 1, \ldots, n_v$, we denote by $X_i(v)$ the ith smallest edge cost among the edges directed out of v. The set of nodes U consists of n disjoint sets of nodes called *supernodes*. Each supernode corresponds to a node of V. The supernode Z_v corresponding to node $v \in V$ has the following $n_v + 1$ nodes: a *hub node* $Z_{v,0}$ and n_v *bridge nodes* $Z_{v,1}, \ldots, Z_{v,n_v}$. For each pair of nodes $u, v \in V$, the set of edges A contains an edge between the bridge nodes $Z_{v,i}$ and $Z_{u,j}$ such that $X_i(u) \geq c(u, v)$ and $X_j(v) \geq c(v, u)$. Also, for each node $v \in V$, A contains an edge between the hub node $Z_{v,0}$ and each bridge node $Z_{v,i}$, for $i = 1, \ldots, n_v$. The cost function c' is defined as $c'(Z_{v,0}) = 0$ for the hub nodes and as $c'(Z_{v,i}) = X_i(v)$ for $i = 1, \ldots, n_v$, for the bridge nodes. The set of terminals D' is defined as $D' = \cup_i D'_i$, where $D'_i = \{Z_{v,0} \in U | v \in D_i\}$. An example of this reduction is depicted in Figure 39.8.

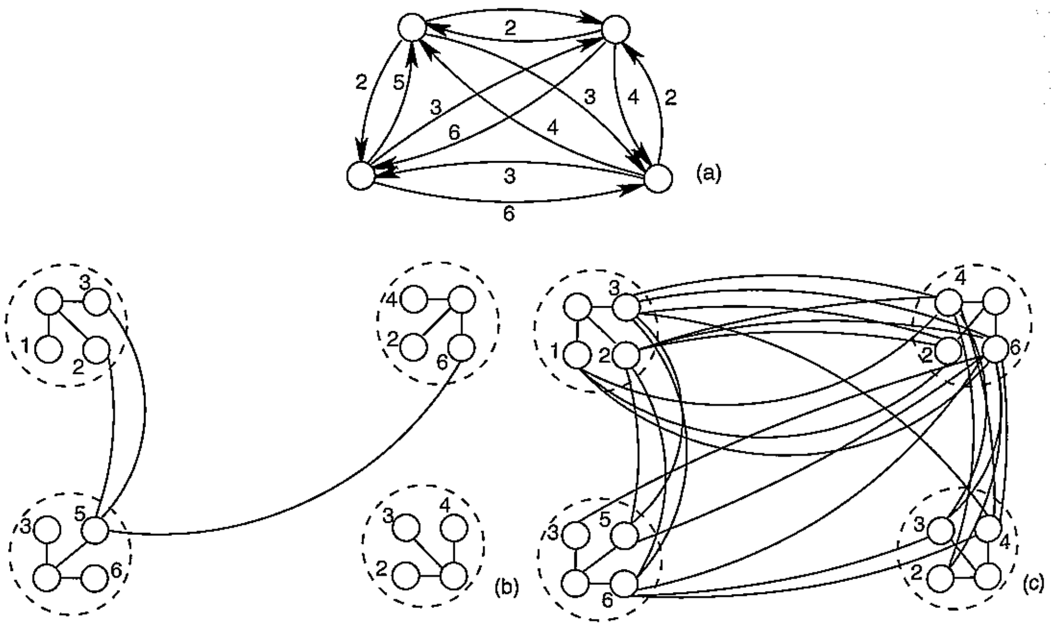

FIGURE 39.8 The reduction to Node-Weighted Steiner Forest. (a) The graph G of an instance of bMESS. (b) The graph H of the corresponding instance of NWSF. Each large cycle indicates a supernode. Only the edges incident to the node of weight 5 of the lower left supernode are shown. (c) The graph H of the corresponding instance of NWSF.

Consider a subgraph $F = (S, A')$ of H, which is a solution for I_{NWSF}. We construct a weight assignment w on the nodes of G by setting $w(v) = 0$, if S contains no node from supernode Z_v, and $w(v) = \max_{u \in (Z_v \cap S)} c'(u)$, otherwise. The next statement is proved in Reference 14 and relates the quality of the two solutions.

Lemma 39.4 *If F is a ρ-approximate solution to I_{NWSF}, then w is a ρ-approximate solution to $I_{b\text{MESS}}$.*

Guha and Khuller [26] present a 1.61 ln k-approximation algorithm for NWSF, where k is the number of terminals in the graph. Using this algorithm to solve I_{NWSF}, we obtain a solution of $I_{b\text{MESS}}$ that is within 1.61 ln $|D|$ of optimal. Moreover, when $p = 1$ (i.e., when $I_{b\text{MESS}}$ is actually an instance of bidirected MESSCS), the instance I_{NWSF} is actually an instance of NWST that can be approximated within 1.35 ln k, where k is the number of terminals in the graph [26]. The next theorem summarizes the discussion of this section.

Theorem 39.9 *There exist an* 1.61 ln $|D|$-, *an* 1.35 ln $|D|$-, *and an* 1.35 ln *n-approximation algorithm for bidirected MESS, bidirected MESSCS, and bidirected MESCS in asymmetric wireless networks, respectively.*

39.4 The Geometric Model

In this section, we survey results in geometric wireless networks. Recall that, in these networks, nodes of G correspond to points in a Euclidean space, and the cost of an edge (u, v) is defined as the Euclidean distance between u and v raised to a fixed power α ranging from 1 to 6, (i.e., $c(u, v) = d(u, v)^{\alpha}$). As we will show in the following, finding an optimal solution becomes easier if nodes are restricted to be placed on a line, while for higher dimensions almost all connectivity requirements lead to hard problems. The most important cases and the best-examined ones are those of MEBT and MESCS.

39.4.1 The Linear Case

A linear wireless network consists of n points on a line having coordinates $x_1 \leq x_2 \leq \cdots \leq x_n$. For MEBT, when we are also given a special node x_r that is the root, the following theorem has been proved independently in References 13 and 19.

Theorem 39.10 *MEBT can be solved in polynomial time in linear wireless networks for any* $\alpha \geq 1$.

The main idea of the corresponding algorithms is the exploitation of structural properties of the optimal solution in order to drastically reduce the search space. More specifically, if we partition the set of nodes into two sets (called *left* and *right*) depending on their position on the line with respect to the root, the transmission graph corresponding to the optimal solution has at most one node that reaches nodes both from the left and the right set. Such a node is called *root-crossing*. The main idea in the proof of this fact is that if there exist $k \geq 2$ root-crossing nodes, then a solution having no greater cost with $k-1$ root-crossing nodes also exists. Examples of situations with two root-crossing nodes are depicted in Figure 39.9. Thus, we can always reduce the number of root-crossing nodes to one without increasing the total cost. So, all possible transmission graphs that are candidates to be the optimal solution can be examined in polynomial time.

For MESCS, an optimal algorithm running in time $O(n^4)$ is presented in Reference 30. This algorithm relies on the use of dynamic programming.

Theorem 39.11 *MESCS can be solved in polynomial time in linear wireless networks for any* $\alpha \geq 1$.

The main idea of the algorithm is that the optimal solution can be computed in a recursive way. Unfortunately, the assumption that starting from an optimal solution for k points $x_1, \ldots, x_k$ we can

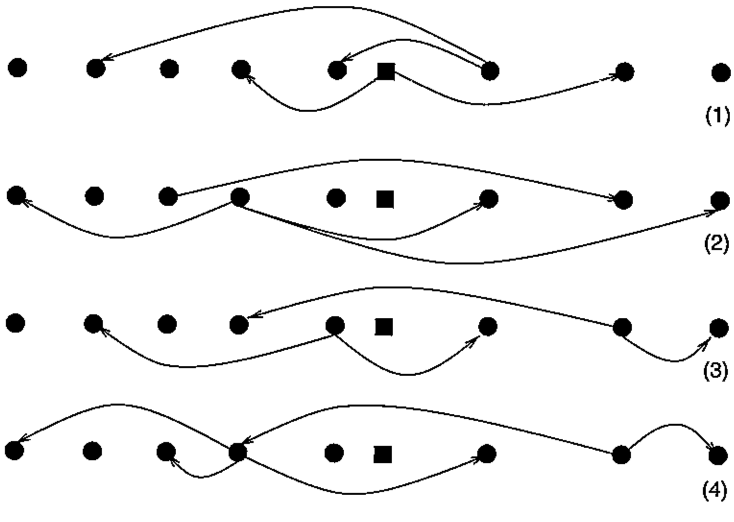

FIGURE 39.9 The four possible cases we have to consider when there are two root-crossing nodes. Only edges directed out of root-crossing nodes are shown. The root is the squared node. In all cases, we can reduce the number of root-crossing nodes to 1.

extend it to include point x_{k+1} does not work, since the energy assigned to point x_{k+1} may lead to a situation where the energy previously assigned to some points in $\{x_1, \ldots, x_k\}$ should be reduced. On the positive side, the following stronger recursive statements are proved in Reference 30. For any $l \geq k$ and any $i \leq k$, there is an assignment that has minimum cost among the assignments with the following properties:

1. There is a path between any pair in $\{x_1, \ldots, x_k\}$ in the transmission graph.
2. x_l is within the reach of a node in $\{x_1, \ldots, x_k\}$.
3. In the transmission graph, any backward edge from x_k to x_i is free of cost. These edges enable connectivity without adding to the cost.

Using the above statement, starting from an empty solution we gradually extend the assignment until covering all nodes, thus obtaining an optimal solution.

39.4.2 Multidimensional Wireless Networks

Almost all connectivity requirements besides MEIBT lead to NP-hard problems when graph G consists of points on a d-dimensional space for $d \geq 2$ and $\alpha \geq d$. Most of the theoretical and experimental work in this framework is for MEMT and MEBT, and especially for the case where the points are located on a Euclidean plane (i.e., $d = 2$). Note that in the case $\alpha = 1$ an optimal solution that consists of assigning sufficient energy to the root so that it reaches the node that is the furthest away can be computed for any d.

The following theorem was presented in Reference 18.

Theorem 39.12 *For any $d \geq 2$ and any $\alpha \geq d$, MEBT is NP-hard.*

Naturally, this result led to the design and analysis of approximation algorithms. The majority of research has focused on the case $\alpha = d = 2$ and, unless stated otherwise, the bounds discussed in the following correspond to this setting. The first algorithms were proposed in the seminal work of Wieselthier et al. [41]. These algorithms are based on the construction of MST and shortest path trees (SPT) on the graph representing the network. The energy assigned to each node is then the minimum required in order to be able to reach its neighbors in the tree. The approach followed in Reference 41 for computing solutions of MEMT was to prune the trees obtained in solutions of MEBT. Another algorithm presented in Reference 41, called *Broadcast Incremental Algorithm* (BIP), constructs a tree starting from the root and gradually augmenting it by adding the node that can be reached with the minimum additional energy from some node already in the tree. BIP can be seen as a node version of Dijkstra's algorithm for computing SPT, with one fundamental difference on the operation whenever a new node is added. Whereas Dijkstra's algorithm updates the node weights (representing distances), BIP updates the edge costs (representing the additional energy required in order to reach a node out of the tree). This update is performed by subtracting the cost of the added edge from the cost of every edge from the source node of the added edge to any node that is not yet included in the tree.

Experimental results presented in Reference 41 showed that BIP outperforms algorithms MST and SPT. In subsequent work, Wan et al. [39] study the algorithms presented in Reference 41 in terms of efficiency in approximating the optimal solution. Their main result is an upper bound of 12 on the approximation ratio of algorithm MST. Slightly weaker approximation bounds for MST have been presented in Reference 18. In Reference 39, it is also proved that the approximation ratio of BIP is not worse than that of MST, and that other intuitive algorithms have very poor approximation ratio. The upper bound in Reference 39 for MST was improved in References 23 and 35. Recently, Ambühl [2] proved an upper bound of 6. This result is tight since there exists a corresponding lower bound [39] presented in Figure 39.10. We should also note that there exists a 13/3 lower bound on the approximation ratio of BIP.

Theorem 39.13 *For* $\alpha = d = 2$, MST *and* BIP *are 6-approximation algorithms for MEBT.*

This result implies a constant approximation algorithm of 6ρ for MEMT as well [40], where ρ is the approximation ratio for ST (currently, $\rho \leq 1 + \ln 3/2$ [38]). For the more general case of arbitrary d and $\alpha \geq d$, the authors of Reference 23 prove a $3^d - 1$ upper bound on the performance of MST that holds independently of α. Very recently, for the case $d = 3$, Navarra [36] presented an 18.8-approximation algorithm that is an improvement over the 26 bound that stems from the aforementioned formula. In this case, there is still a significant gap since the best lower bound is 12.

On the other side, several intuitive algorithms have been experimentally proved to work very well on random instances of MEBT and MEMT. In References 33 and 42, algorithms based on shortest paths are enhanced with the *potential power saving* idea. These algorithms examine whether establishing a new path could also include nodes that had been included in the multicast tree in previous phases, and could now be connected to the multicast tree as children of some node in the path. In this way, the energy of some nodes could be decreased. Čagalj et al. [8] introduced a heuristic called *embedded wireless multicast advantage*

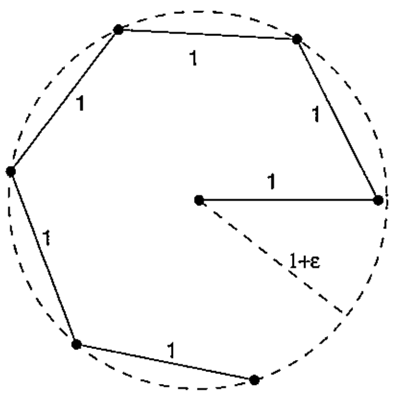

FIGURE 39.10 The lower bound on the performance of MST when $\alpha = d = 2$. The node in the center is the root; an optimal solution would assign energy $(1 + \epsilon)^2$ to it. Solid lines represent the minimum spanning tree that leads to a solution of total energy 6.

(EWMA) for computing efficient solutions to MEBT instances. Starting with a broadcast tree, EWMA "walks" on broadcast trees by performing the following two types of changes in each step: (i) outgoing edges are added to a single node v; this node is said to be *extended* and (ii) all outgoing edges are removed from some descendants of v; in this case we say that the particular descendants of v are *excluded*. Define the *gain* of a node v as the decrease in the energy of the broadcast tree obtained by excluding some of the nodes of the tree in exchange for the increase in node v's energy in order to establish edges to all excluded nodes and their children. Intuitively, EWMA repeatedly examines the nodes of the tree and tries to make use of nodes with maximum gain, so as to make local modifications on the structure of the tree. As it was observed in Reference 4, EWMA can be easily converted to work for MEMT as well. Another heuristic called Sweep was proposed in Reference 41; this also takes as input a tree and transforms it to an energy-efficient tree by performing local improvements. Sweep works as follows. Starting from the broadcast tree, it proceeds in steps; in the ith step it examines node v_i. If for some nodes $v_{i_1}, v_{i_2}, \ldots$ that are not ancestors of v_i the energy of v_i in the broadcast tree is not smaller than the cost of all the edges from v_i to $v_{i_1}, v_{i_2}, \ldots$, Sweep removes the incoming edges of $v_{i_1}, v_{i_2}, \ldots$ and adds edges from v_i to $v_{i_1}, v_{i_2}, \ldots$ in the broadcast tree. The algorithm terminates when all nodes have been examined. Clearly, it can be used on any multicast tree as well.

Another issue of apparent importance is to design algorithms for MEMT that are amenable to implement in a distributed environment (e.g., References 8, 15, and 42). In Reference 4, a characterization of experimental algorithms is presented, while the authors introduced some algorithms that establish *dense shortest paths*, that is, they add to the solution the shortest path that has the lowest ratio of additional energy over the number of newly added nodes. Experimental comparison between these algorithms and already existing ones suggests that density is a useful property.

MESCS has received less attention. A first proof that MESCS is NP-hard for d-dimensional Euclidean spaces appeared in Reference 30 for the case $d \geq 3$. This negative result was strengthened to APX-hardness while the problem was proved to be NP-hard also for $d = 2$ in Reference 21. The above proofs assume that $\alpha \geq 2$, while in Reference 24 MESCS is also proved to be NP-hard for $\alpha = 1$.

Theorem 39.14 *MESCS is NP-hard for any $d \geq 2$ and any $\alpha \geq 1$, and APX-hard for $d \geq 3$ and $\alpha \geq 2$.*

A simple 2-approximation algorithm based on MST was presented in Reference 30. Essentially, this is the algorithm we discussed in Section 39.2.1 in a more general setting. Again, it is not hard to show that the total energy is at most twice the cost of the MST, the latter being a lower bound on cost of the optimal solution.

When the transmission graph is required to contain a bidirected subgraph satisfying the connectivity requirements, Althaus et al. [1] present a 5/3-approximation algorithm, by establishing a connection between bidirected MESCS and k-restricted Steiner trees, and using a 5/3-approximation algorithm for the latter problem. This bound cannot be obtained by the algorithm in Reference 30. As the authors in Reference 1 note, the cost of an optimal solution for MESCS can be half the cost of an optimal solution for bidirected MESCS. Consider a set of $n^2 + n$ nodes, consisting of n groups of $n + 1$ nodes each, that are located on the sides of a regular $2n$-gon. Each group has 2 "thick" nodes in distance 1 of each other and $n - 1$ equally spaced nodes the line segment between them. It is easy to see that an optimal solution for MESCS assigns energy 1 to the one thick node in each group and an amount of energy equal to $\epsilon^2 = (1/n)^2$ to all other nodes in the group. The total energy then equals $n + 1$. For bidirected MESCS it is necessary to assign energy equal to 1 to all but two of the thick nodes, and of ϵ^2 to the remaining nodes, which results in a total energy of $2n - 1 - 1/n + 2/n^2$. An example when $\alpha = 2$ and $n = 3$ is depicted in Figure 39.11.

39.5 Extensions

The connectivity requirements we have considered in this work can be defined by $0 - 1$ requirement matrices. A natural extension is to consider matrices with nonnegative integer entries r_{ij} denoting that

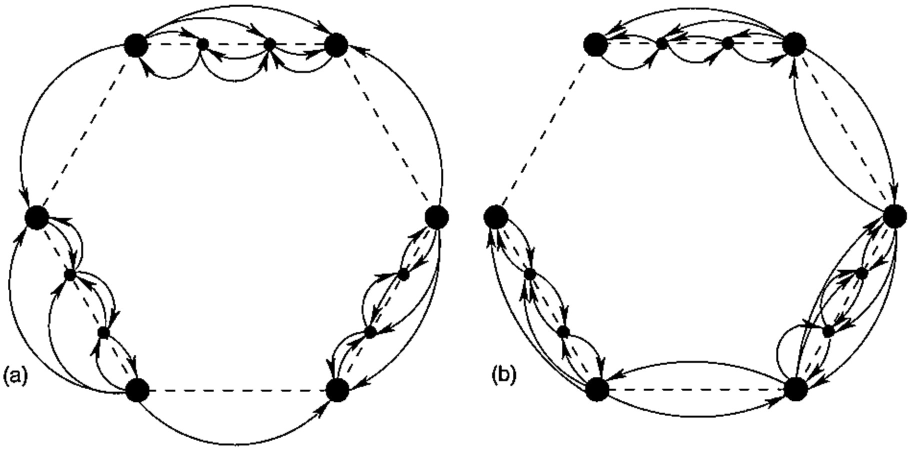

FIGURE 39.11 The transmission graphs of the optimal solution for MESCS (a) and of the optimal solution for bidirected MESCS (b) in the counterexample presented in Reference 1 for $n = 3$. Extending this construction to a regular $2n$-gon proves that the cost of (b) can be twice the cost of (a).

r_{ij} node/edge-disjoint paths are required from node v_i to node v_j. This extension leads to combinatorial problems that capture important engineering questions related to the design of *fault-tolerant* ad hoc wireless networks. In this direction, some results on symmetric wireless networks are presented in References 12, 28, and 31. More specifically, Lloyd et al. [31] examined the problem of establishing a transmission graph containing a bidirected subgraph that is biconnected with respect to both the nodes and the edges. For both cases, they presented 8-approximation algorithms by using algorithms for network design. These results were improved by Călinescu and Wan [12] and were extended to hold also for the case when the transmission graph is not required to contain a bidirected subgraph. In addition, Călinescu and Wan [12] extended the problem to the case of k-connectivity. For the latter case, they presented corresponding $2k$-approximation algorithms, both with respect to nodeconnectivity and edgeconnectivity. Hajiaghayi et al. [28] focus on bidirected MESCS and prove APX-hardness for k-node connectivity and k-edge connectivity and present an $O(\log^4 n)$-approximation algorithm for the first case and an $O(\sqrt{n})$-approximation algorithm for the second case, improving on the previous results for large values of k.

Another direction of research is when the transmission graph is required to have a bounded diameter. Thus, *bounded-hop* versions of the problems studied in this chapter arise. Călinescu et al. [10] examine bounded-hops MEBT and MESCS both for the general and the geometric model. For MEBT, they present an $(O(\log n), O(\log n))$-bicriteria approximation algorithm, that is, the transmission graph has depth at most $O(h \log n)$ and total energy at most $O(\log n)$ times the optimal solution, where h is the bound on the number of hops. For the geometric model, their algorithm can be modified and achieve an approximation ratio of $O(\log^{\alpha} n)$. Ambühl et al. [3] focus on bounded-hop MEBT in the Euclidean plane and present a polynomial time algorithm when $h = 2$, using dynamic programming and a polynomial time approximation scheme (PTAS) for any fixed bound h on the number of hops. Similarly, for MESCS in the general model, Călinescu et al. [10] presents an $(O(\log n), O(\log n))$-bicriteria approximation algorithm, which can be modified to achieve an $O(\log^{\alpha+1} n)$ approximation algorithm for the geometric model. For the geometric model and specifically for linear networks, Clementi et al. [20] present a polynomial algorithm that returns the optimal solution in time $O(n^3)$ when $h = 2$ and a 2-approximation algorithm for arbitrary values of h.

Several other extensions of the model described in this chapter are also interesting. In the *network lifetime* problem [9], the objective is to establish a communication pattern so that the time until the first node exhausts its available energy is maximized. When we drop the assumption that each node is equipped with an omnidirectional antenna, we obtain *directional* equivalents of the discussed problems. Finally, all the cases considered in this work make the assumption that interference [34] is not a concern and no transmission is lost owing to collisions. Excluding this assumption is also worth investigating.

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