The Complexity of Dynamic Data Race Prediction

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Concurrency and Challenges

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- Concurrency bugs arise in production-level software
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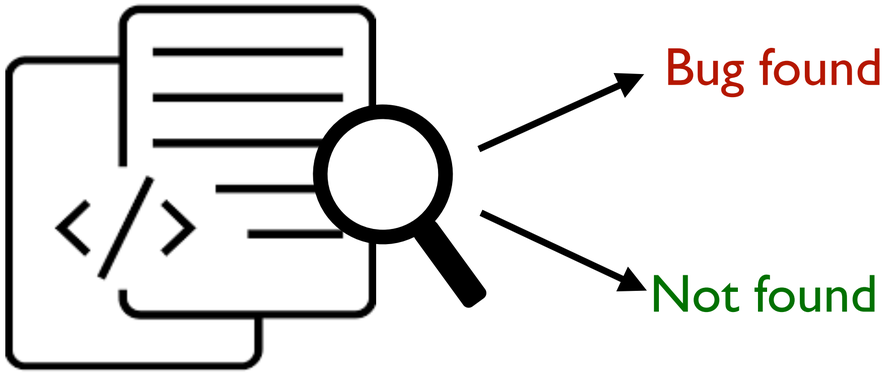
- **Data races** : most common source of concurrency issues

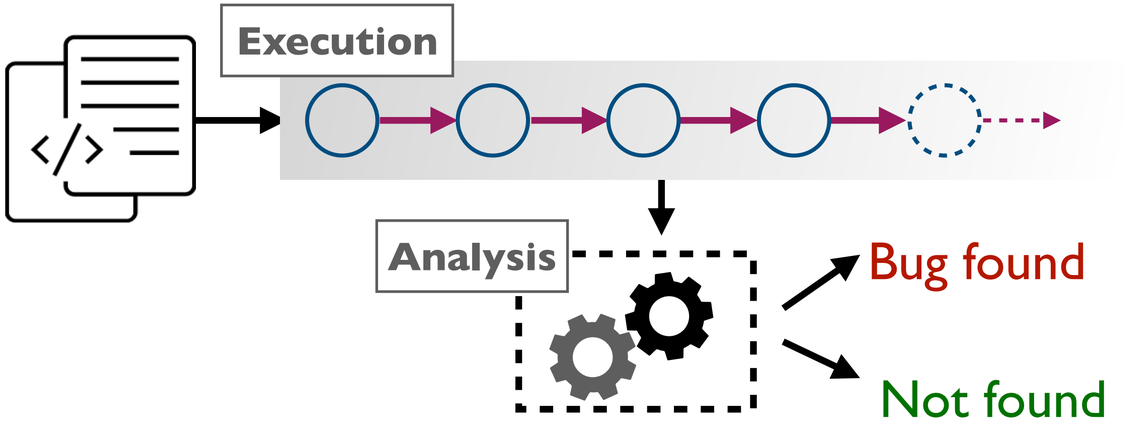
Data Race Detection

Static analyses



- Analyze source code
- Undecidable problem
- Excessive false alarms

Dynamic analyses



- Analyze executions at runtime
- Typically sound
- Widely adopted - TSan, Helgrind, etc.,

Concurrent Program Executions

- Sequences of events
- Event e = <t, op>
  - t is the thread that performs e
  - op is an operation
- Operations:
  - Read/Write to memory locations
  - Acquire and release of locks
- Well formed-ness
  - At most one thread holds a lock at any time

	t1	t2
1	acq(l)	
2	w(x)	
3	w(y)	
4		r(x)
5	rel(l)	
6		w(y)

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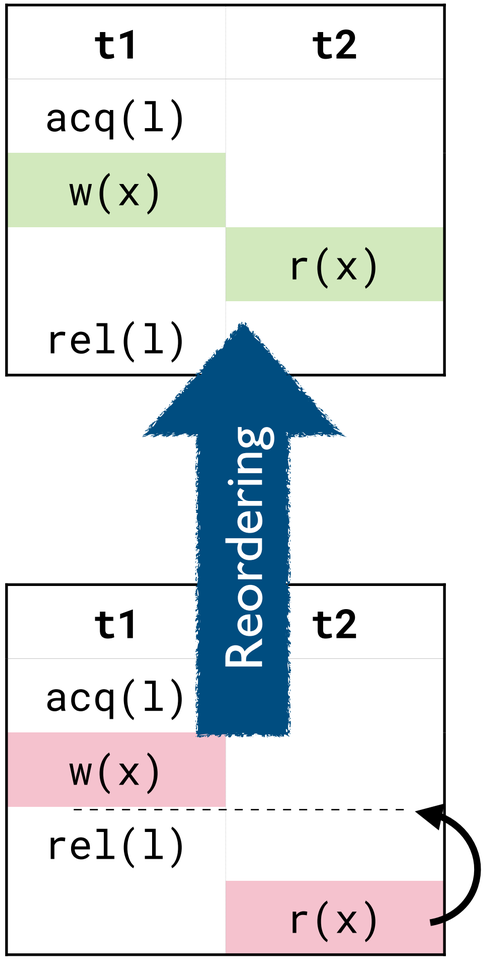

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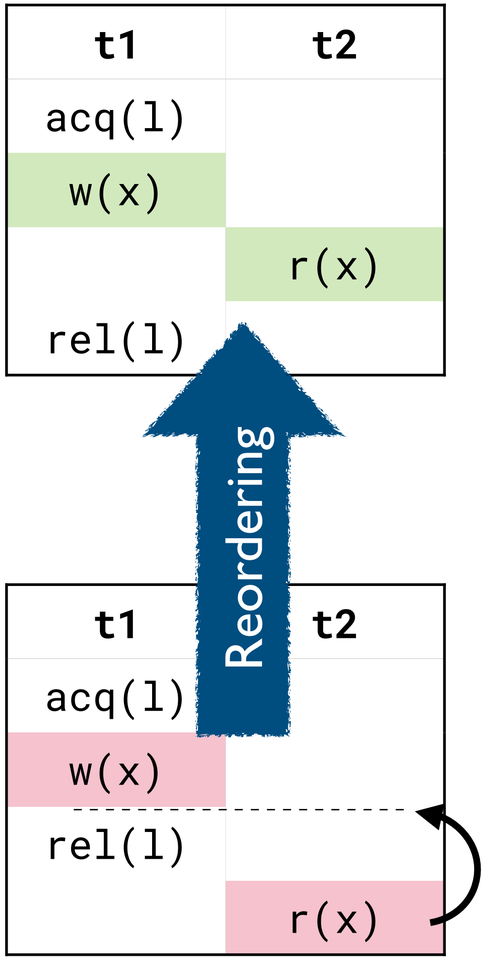

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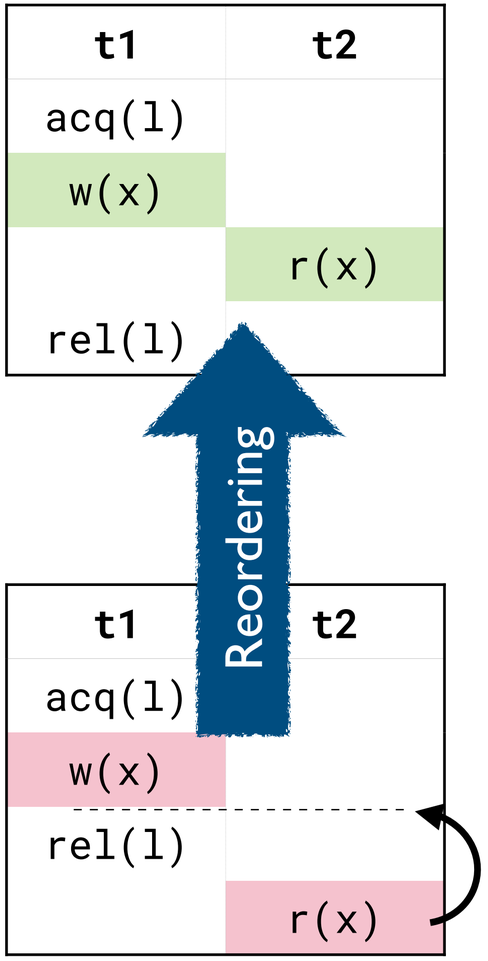

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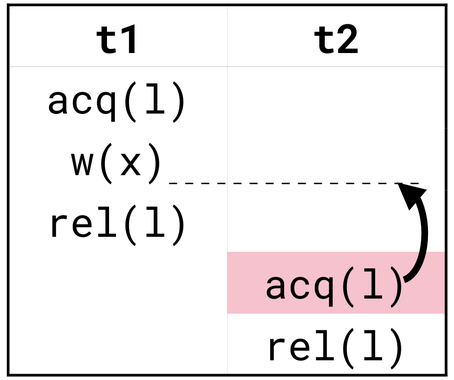

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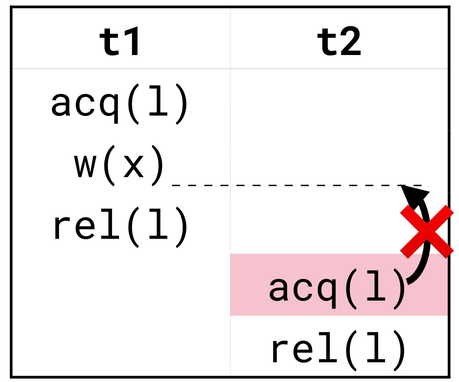

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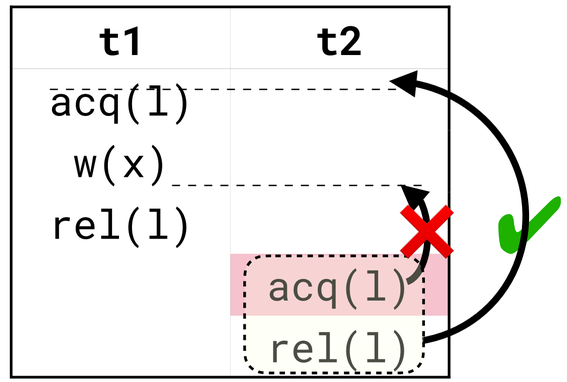

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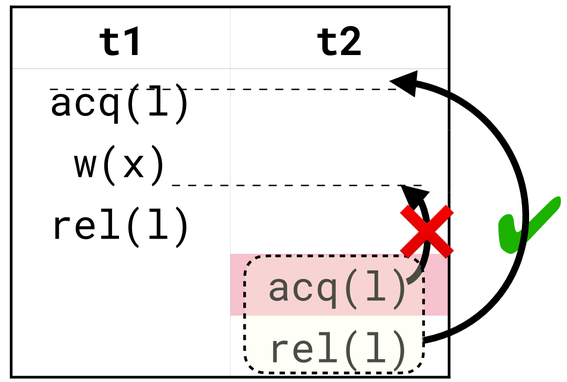

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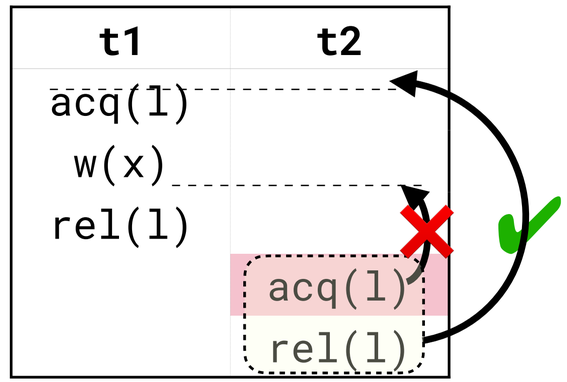

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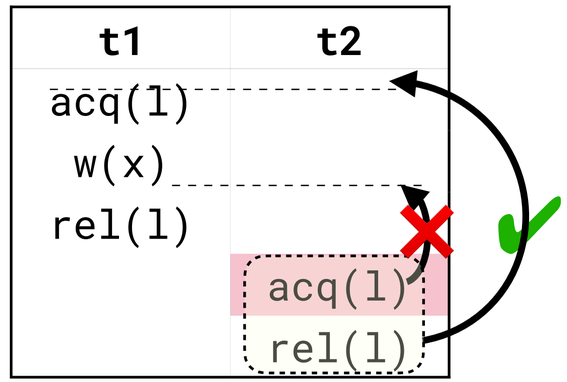

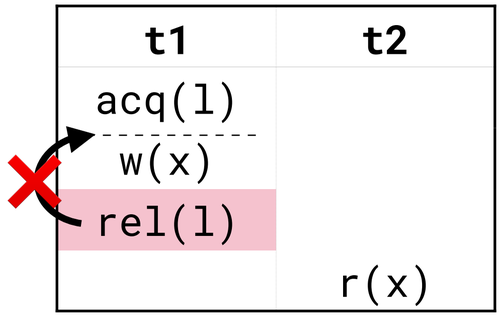

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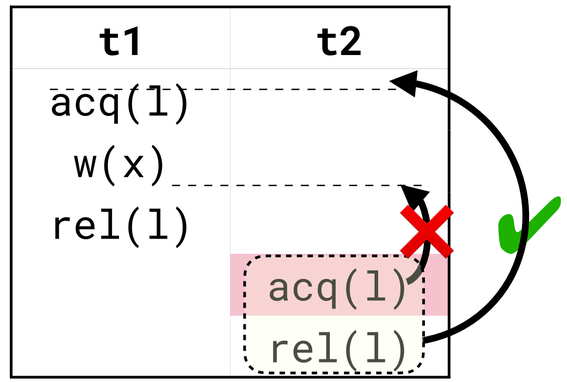

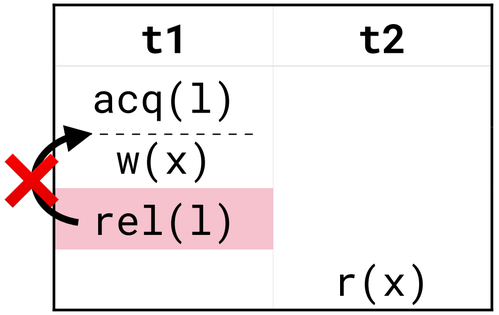

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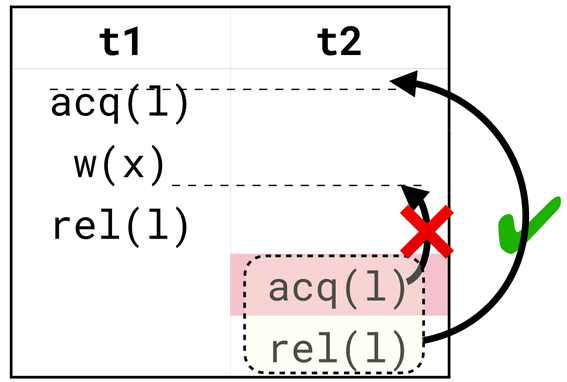

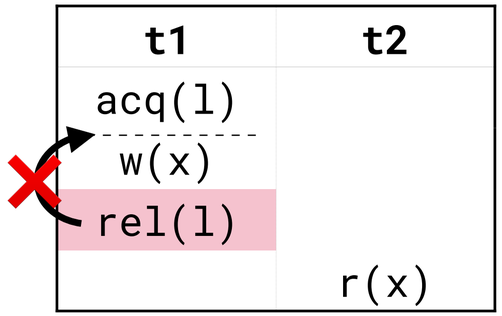

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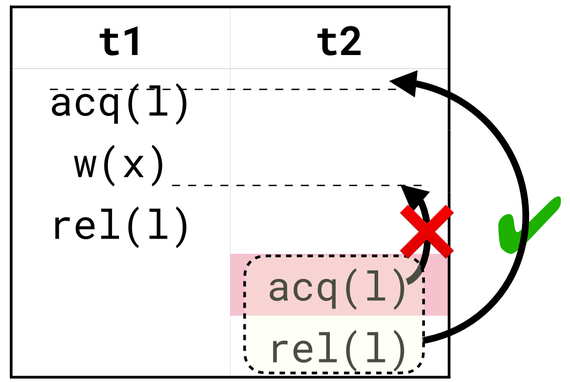

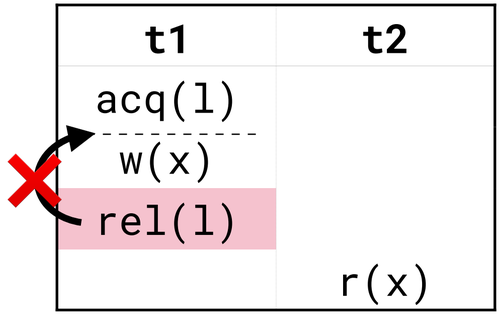

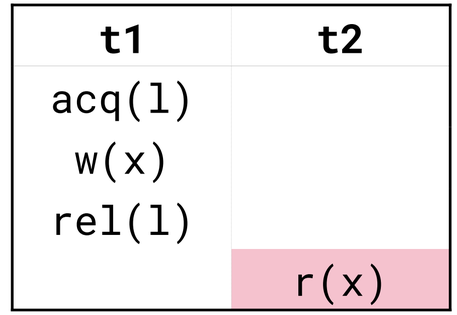

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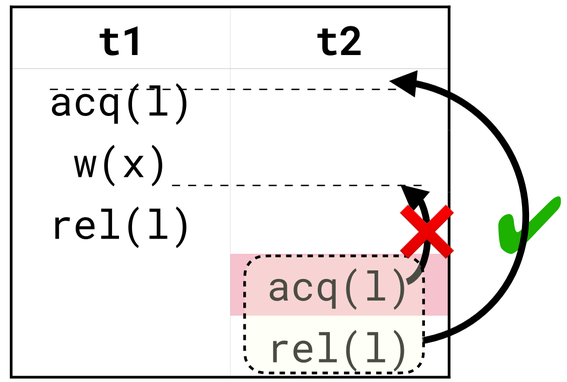

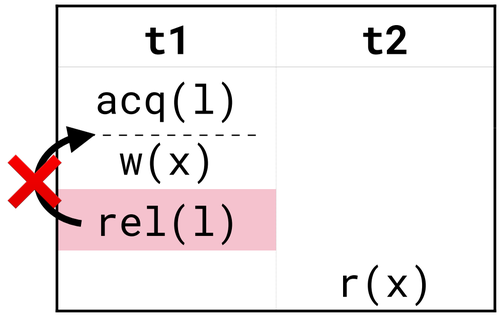

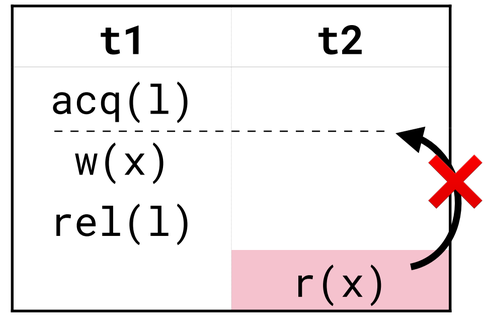

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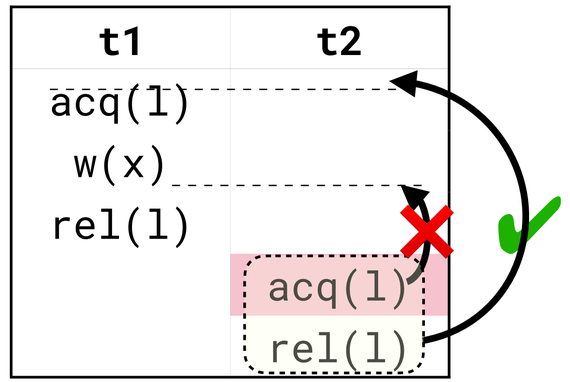

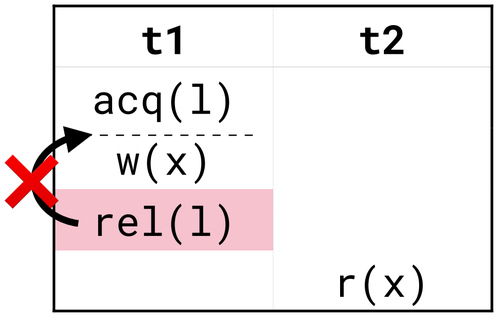

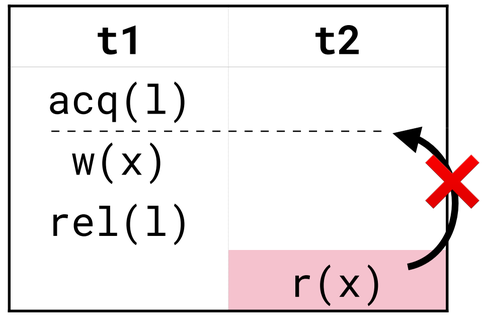

* Herlihy and Wing, Linearizability: A Correctness Condition for Concurrent Objects, TOPLAS 1990

Smaragdakis et al, Sound predictive race detection in polynomial time, POPL 2012

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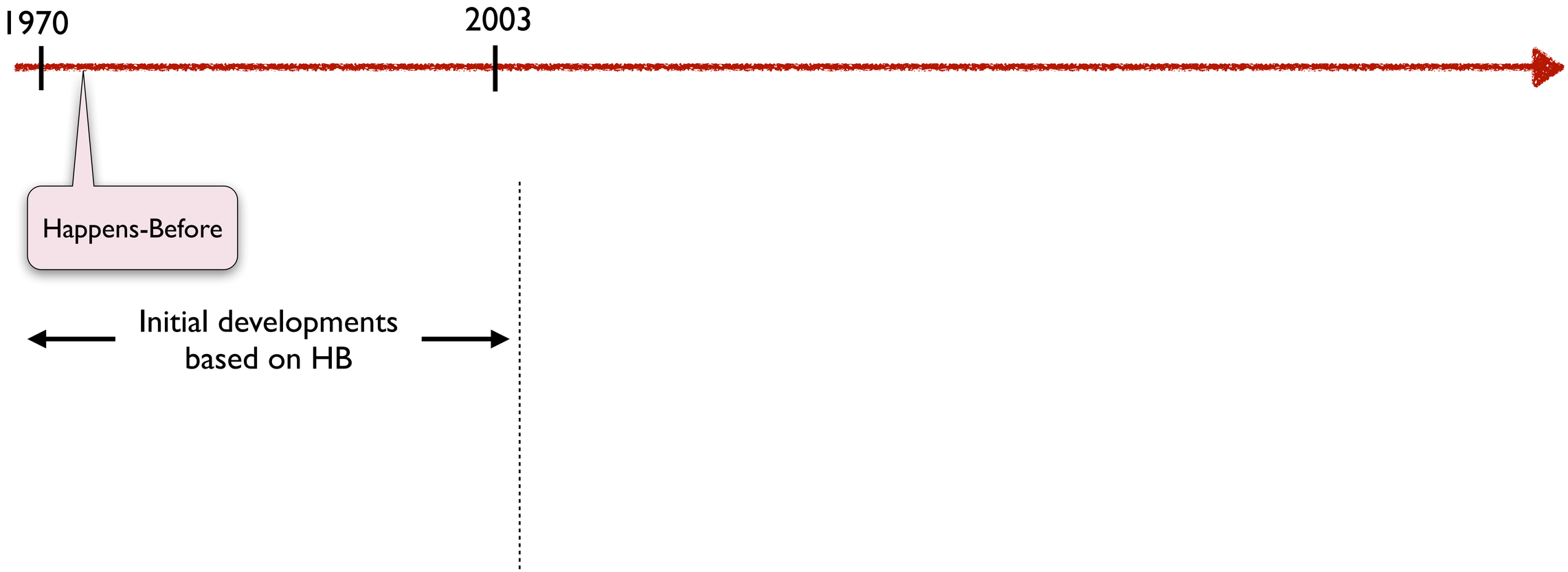

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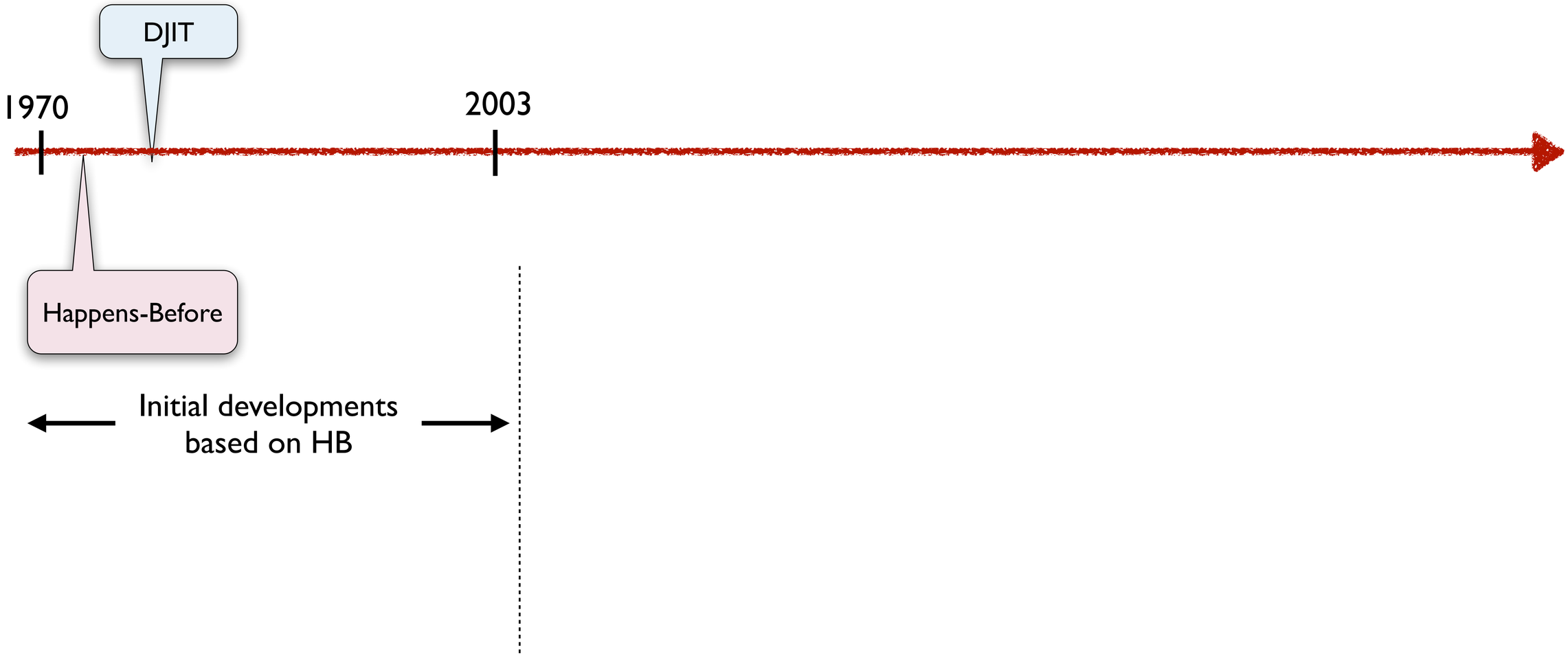

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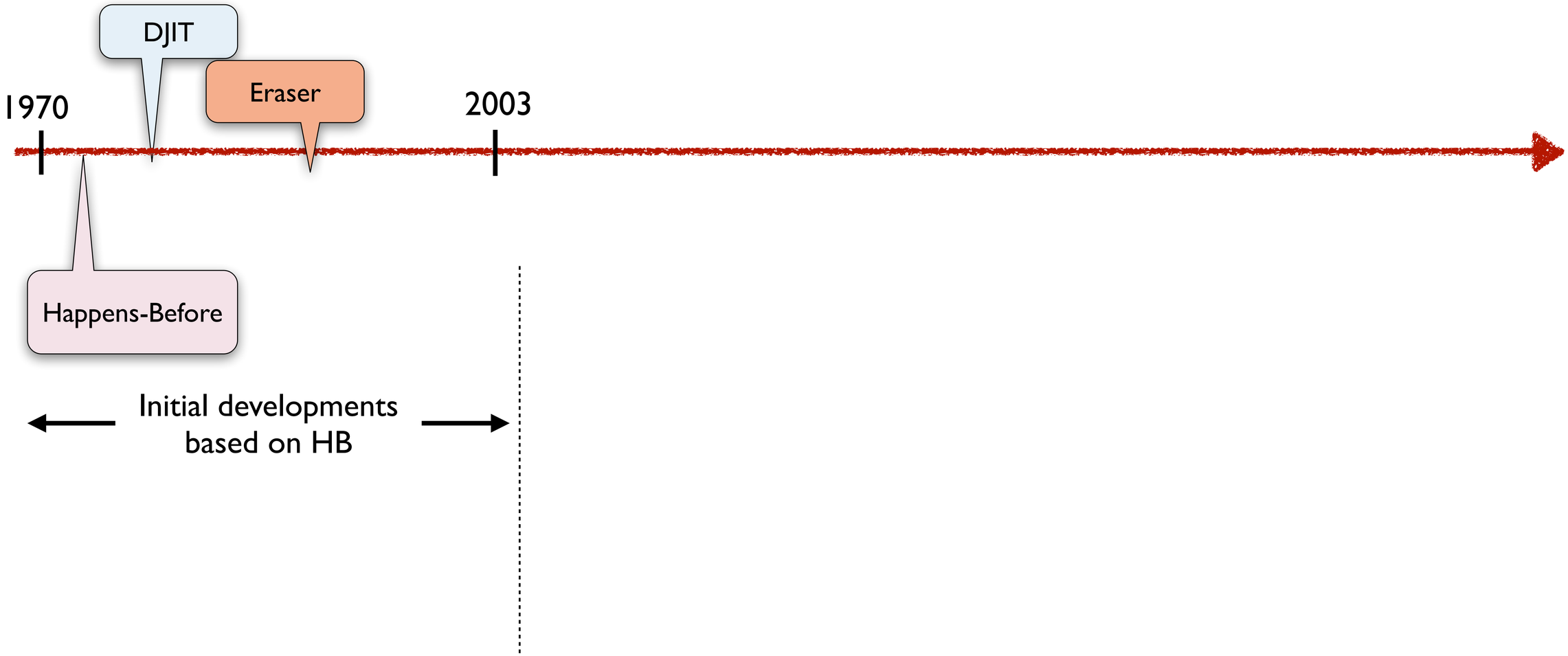

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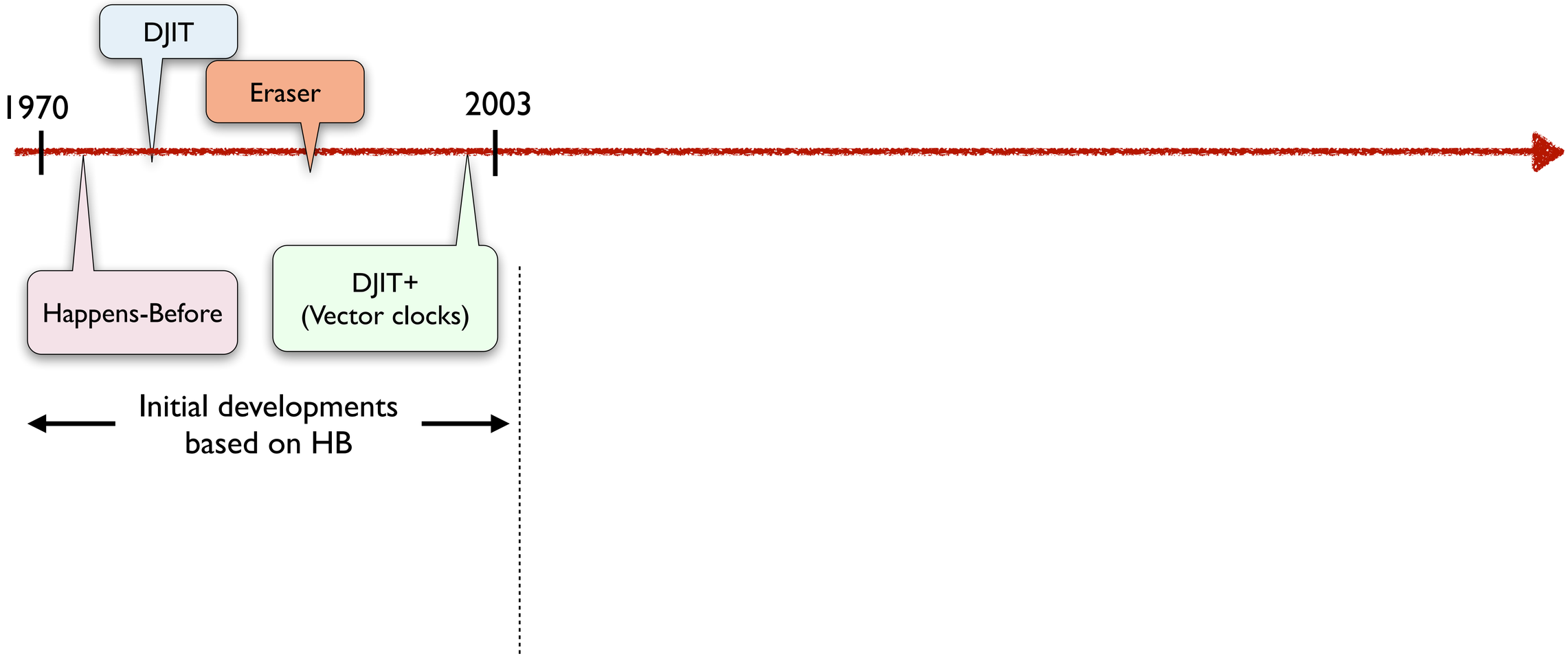

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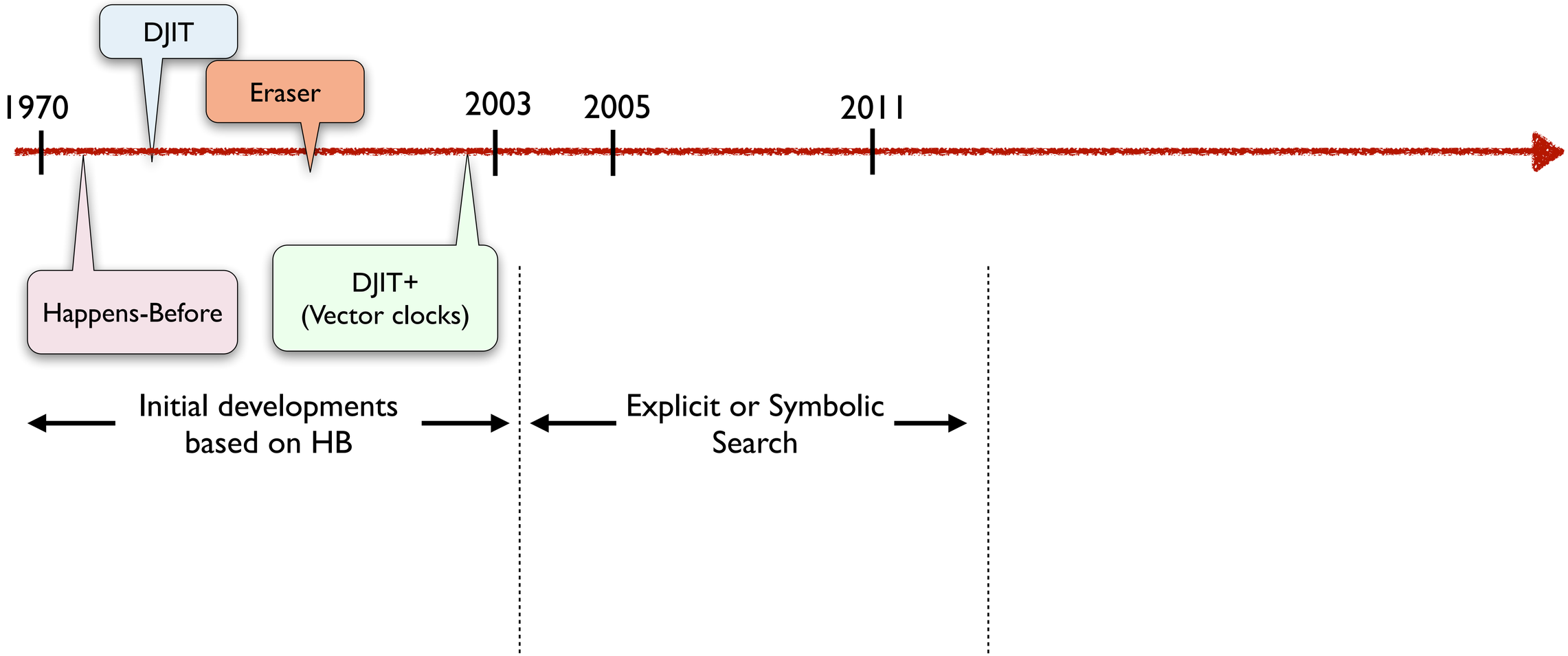

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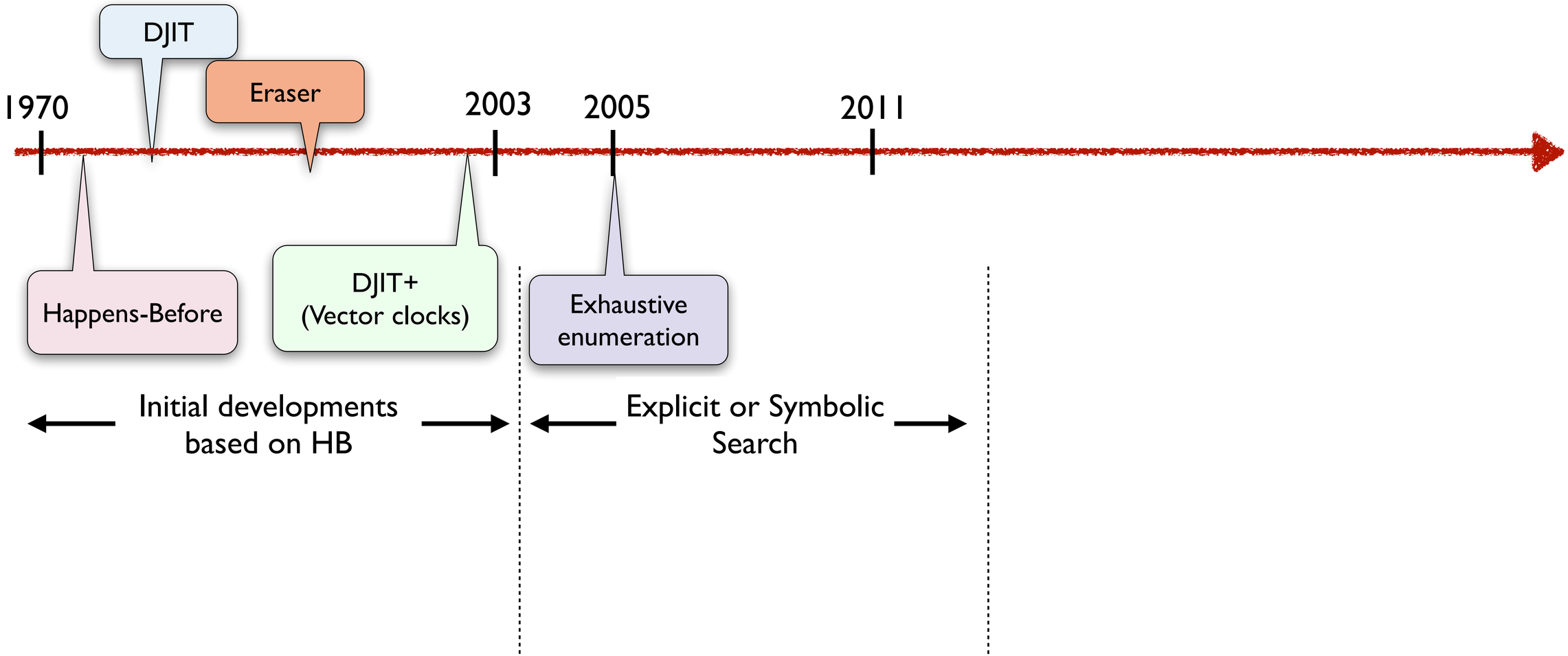

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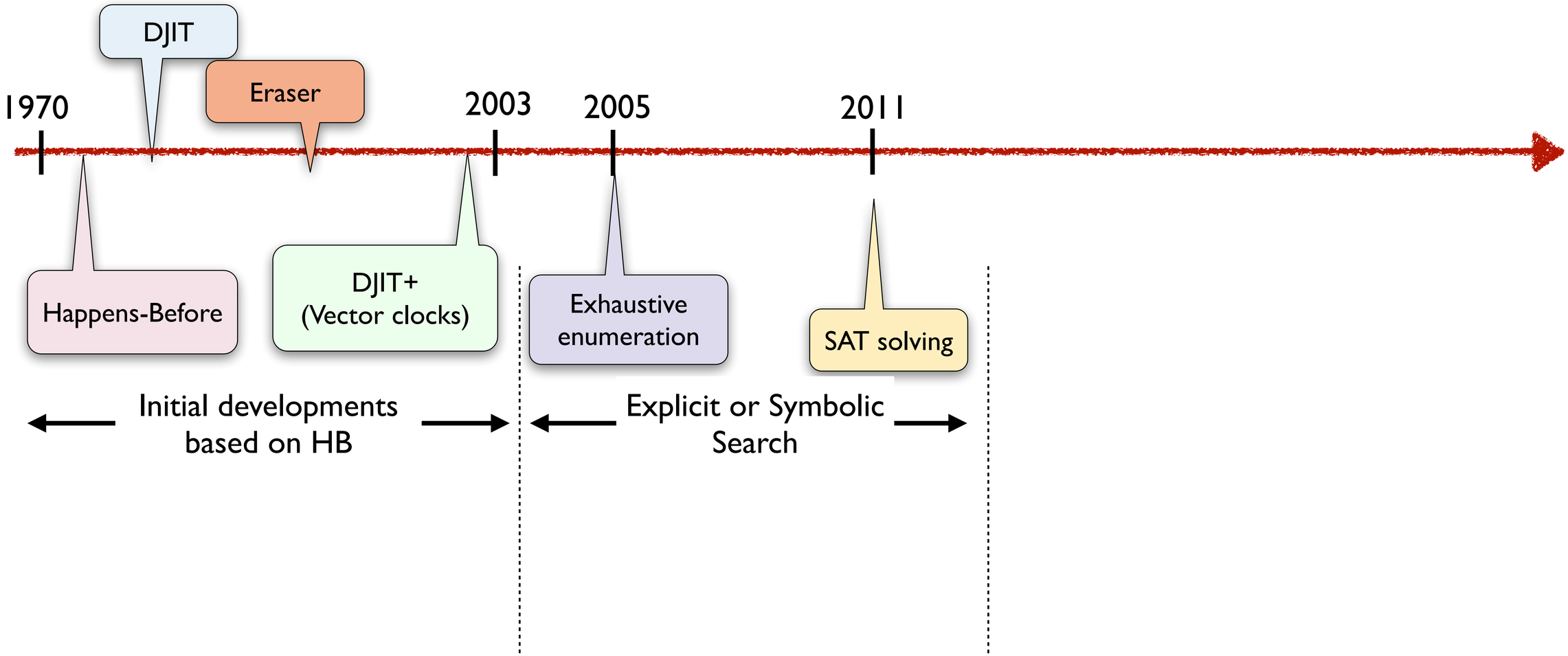

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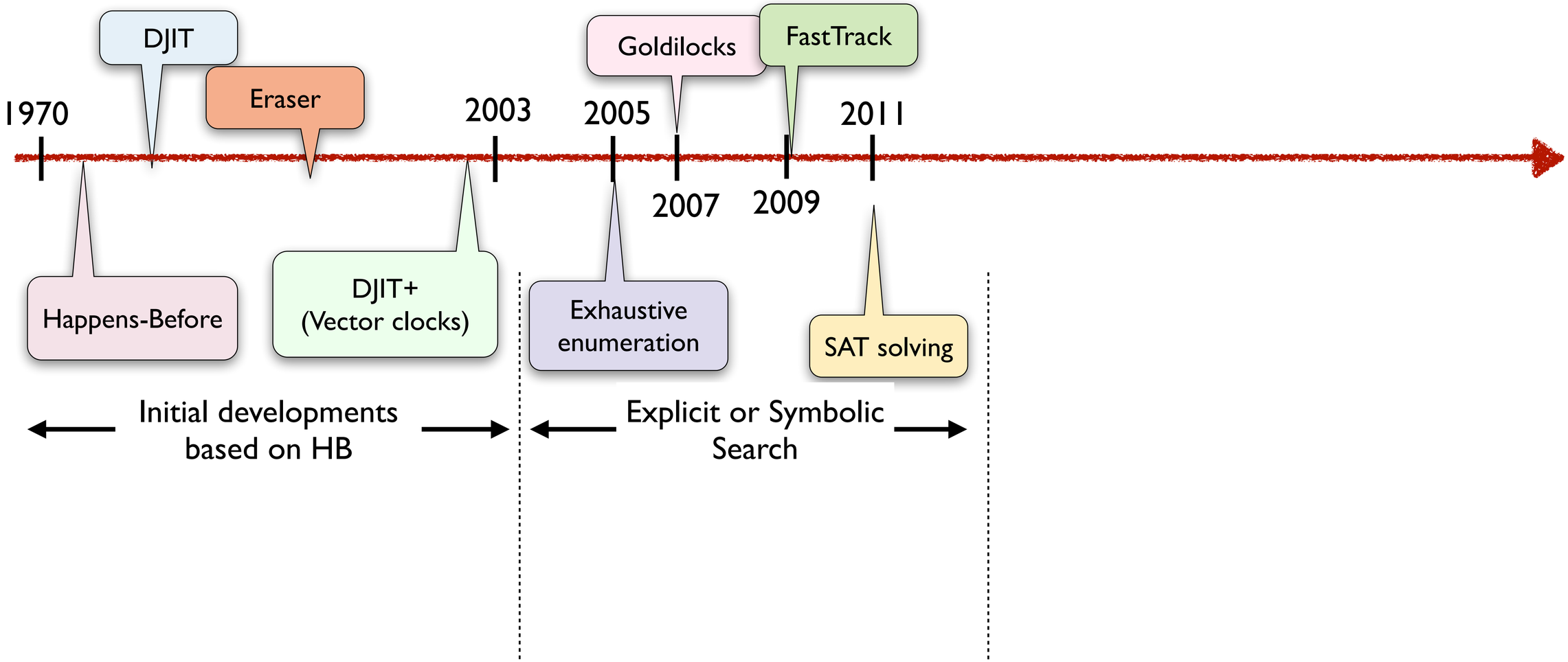

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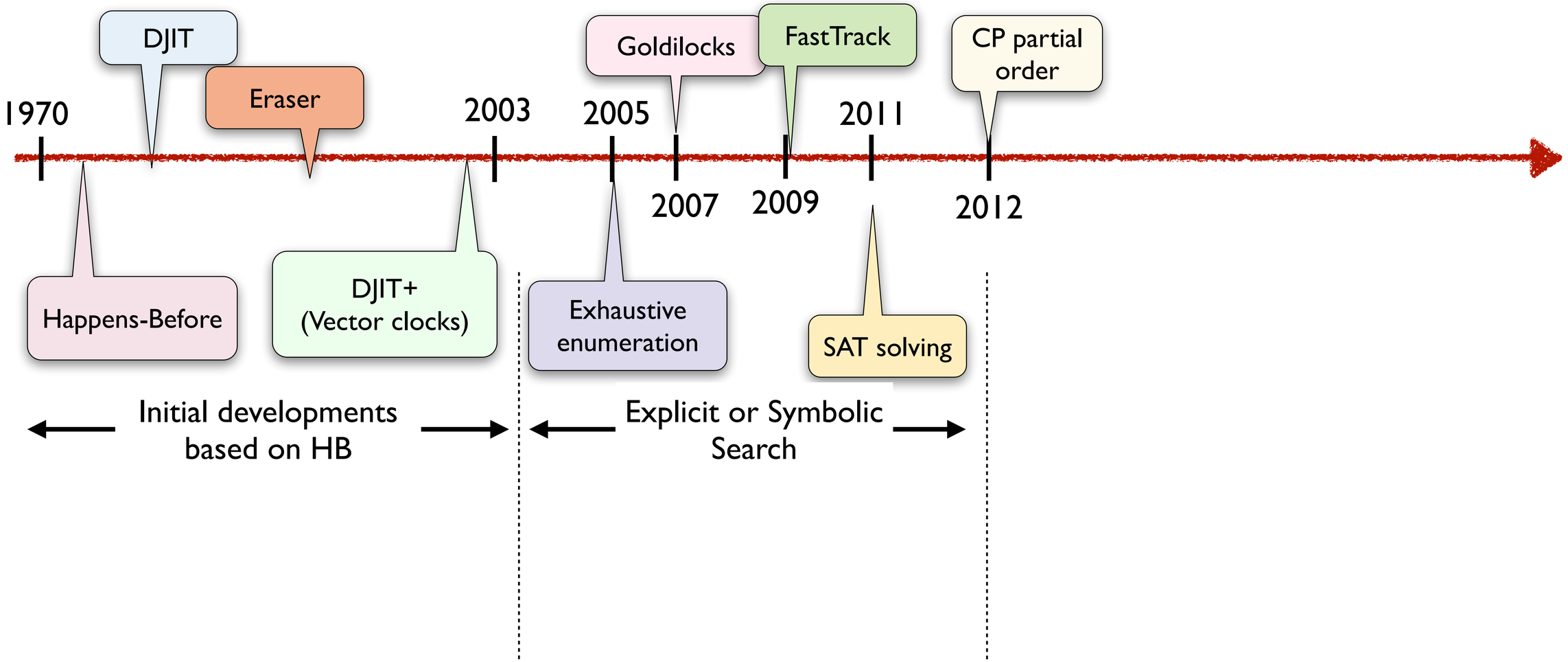

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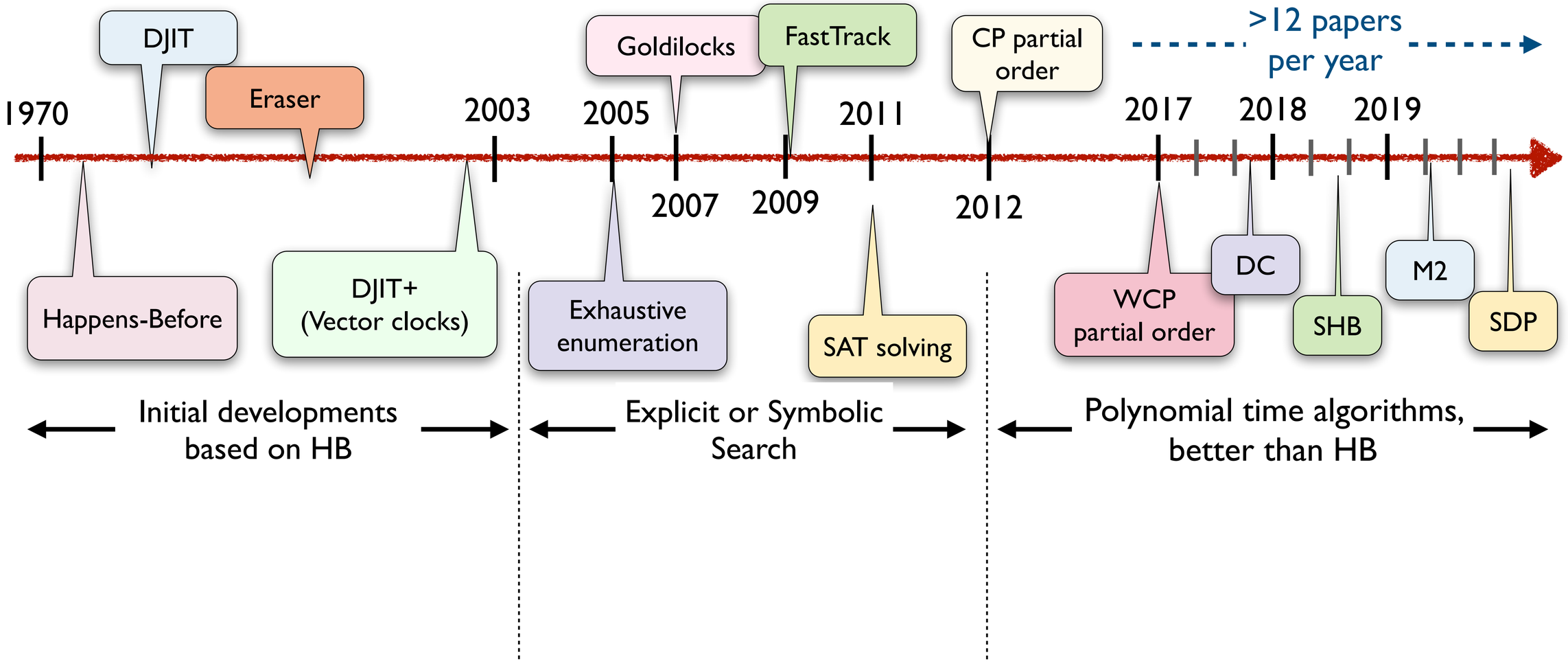

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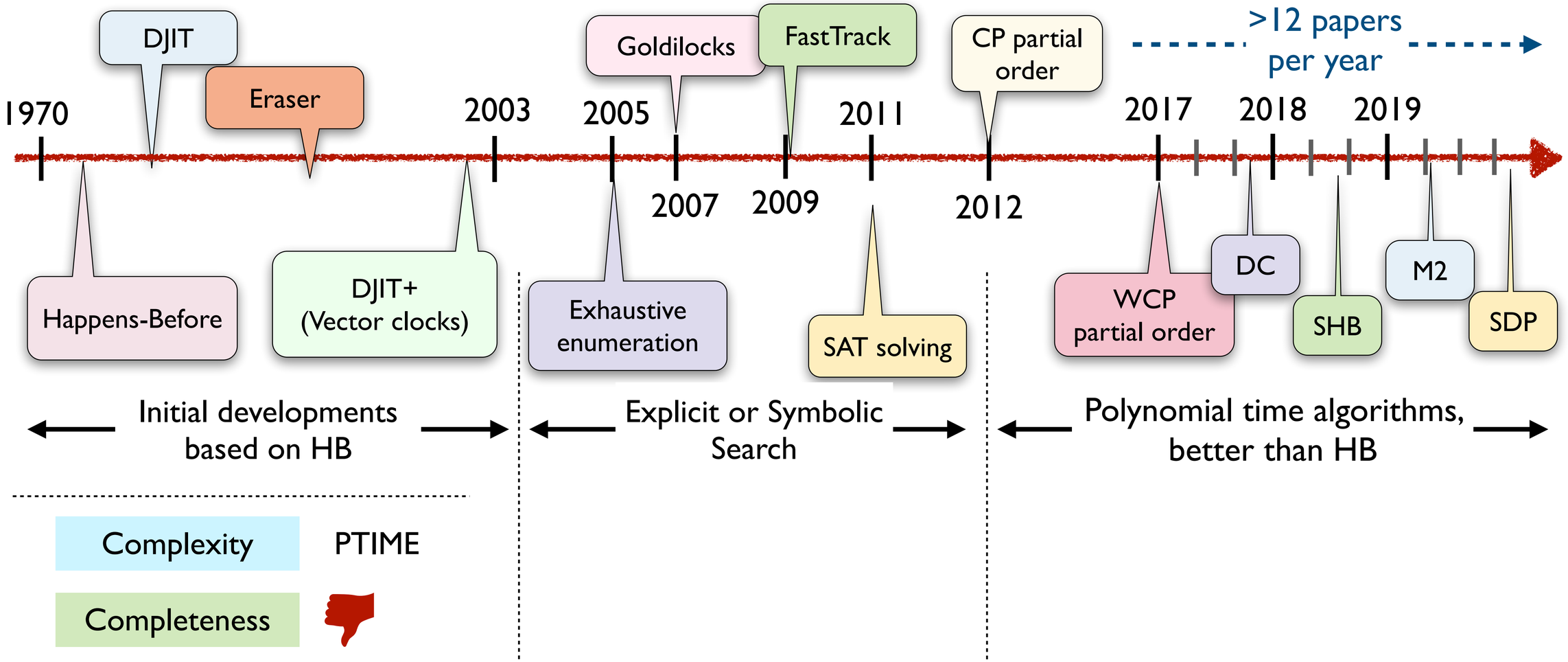

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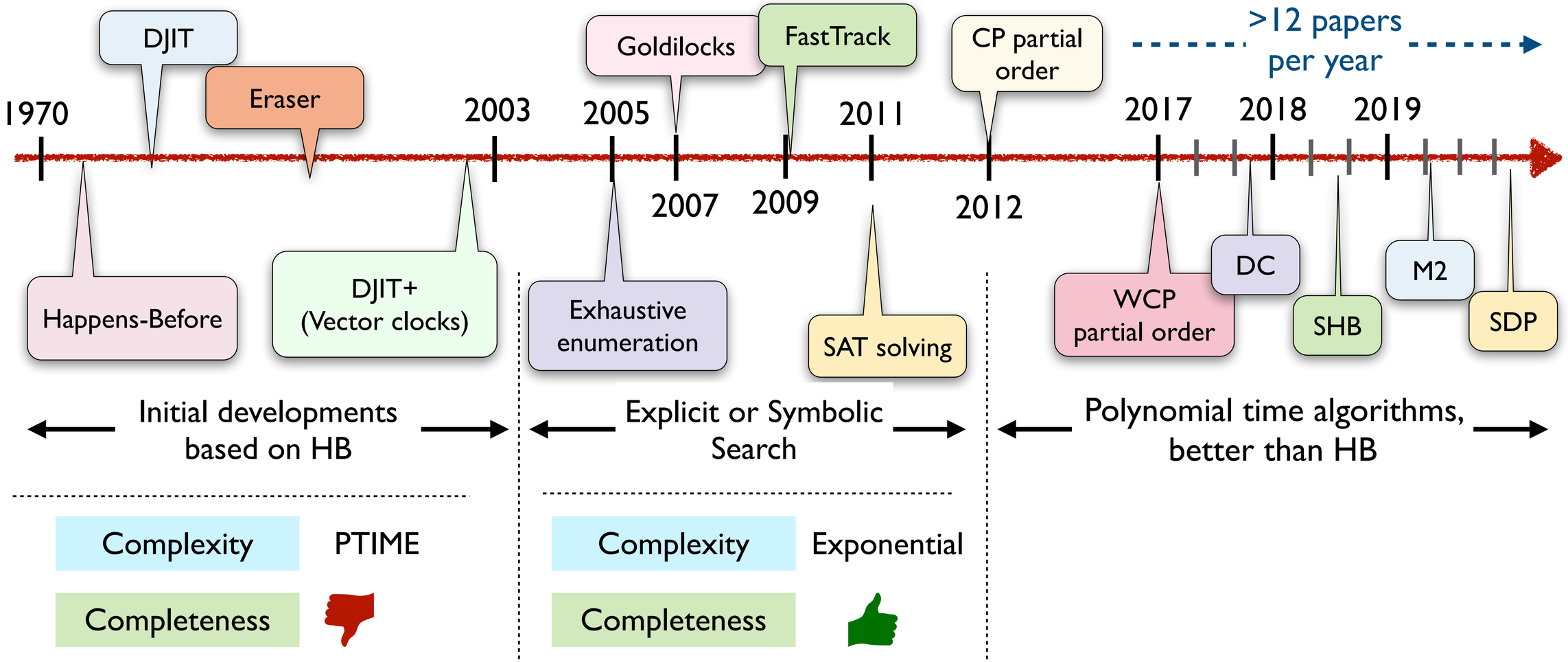

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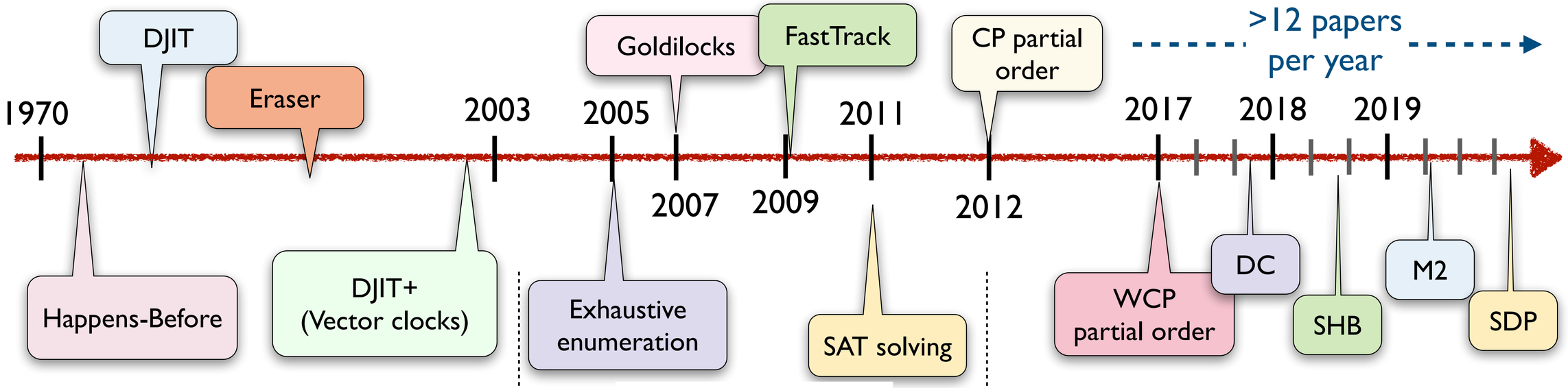


Initial developments based on HB

Explicit or Symbolic Search

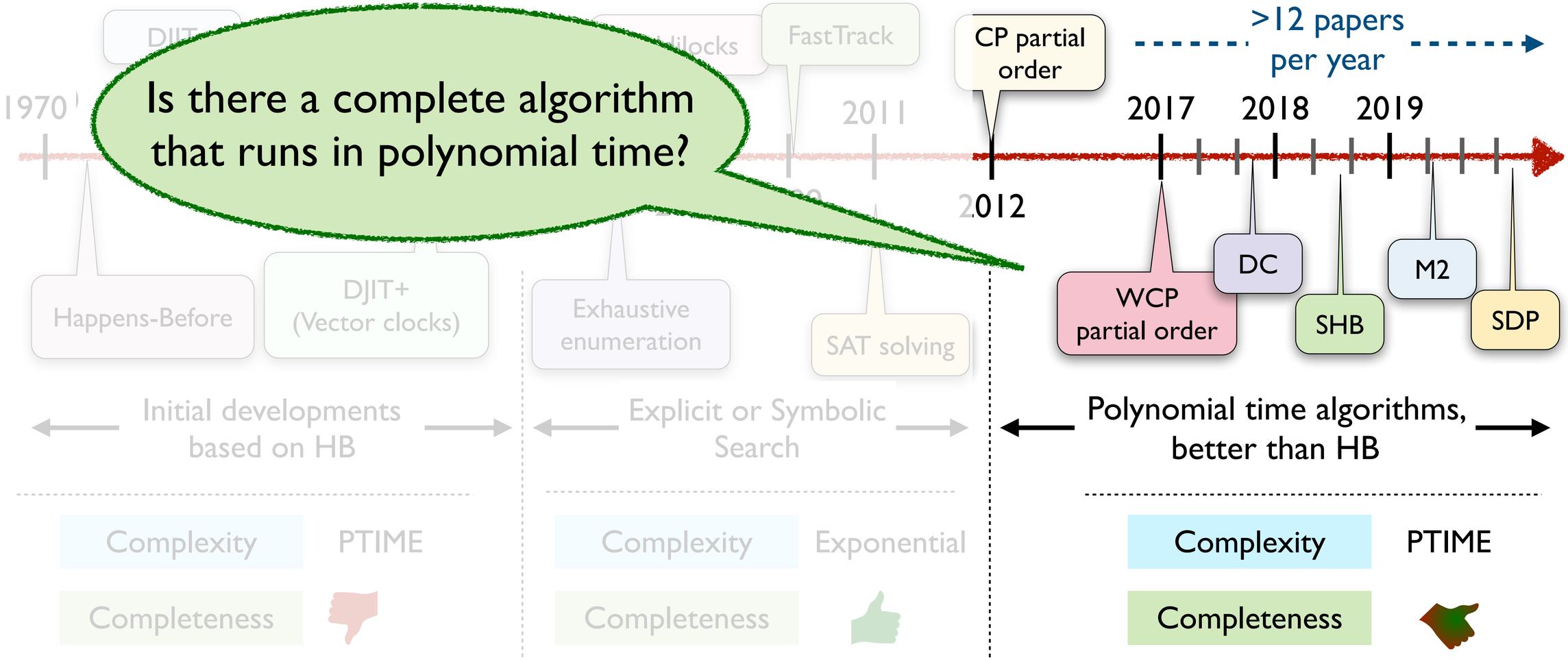
Polynomial time algorithms, better than HB

Complexity	PTIME	Complexity	Exponential	Complexity	PTIME
Completeness	👎	Completeness	👍	Completeness	👉

Some History

Data Race Prediction

Given an execution σ, is there a **correct reordering** of σ with a data race?



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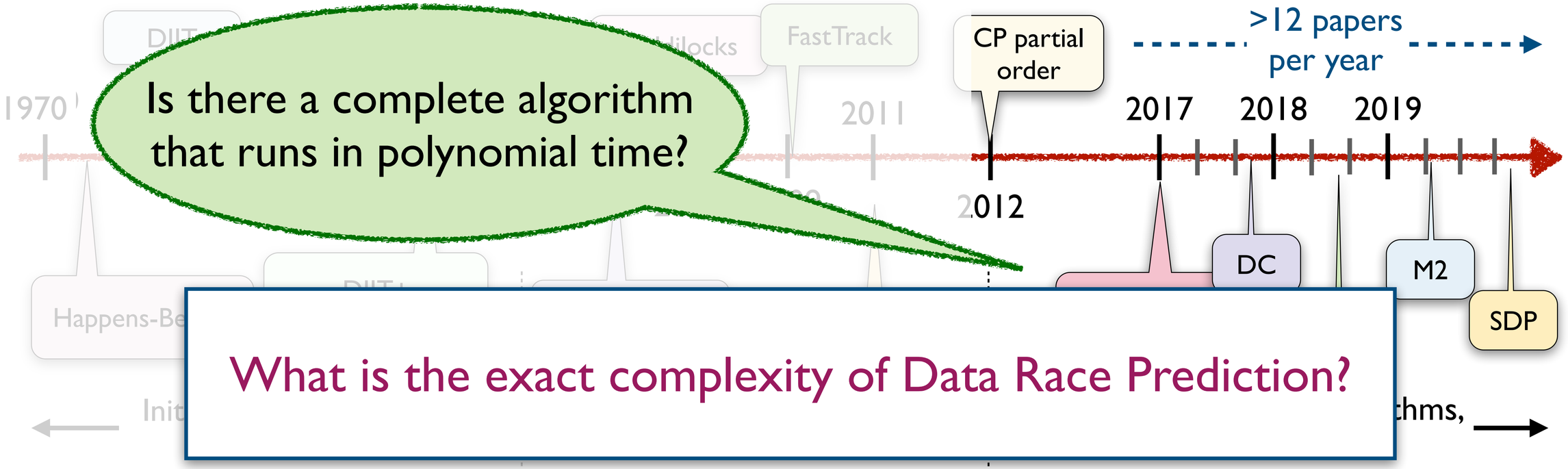

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Lower Bound

- Unknown!
- Is it **NP**-hard? Is enumeration unavoidable?
- Is it polynomial time?

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Closely Related Work

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Contributions

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4. Restricting the space of data races to be reported
   - **Linear** time algorithm

Outline

1. Trace Ideals and Realizability
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Some Notations

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1	acq(l)	
2	rel(l)	
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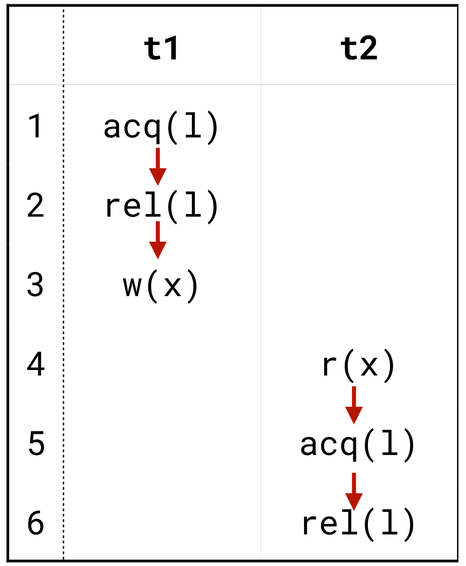

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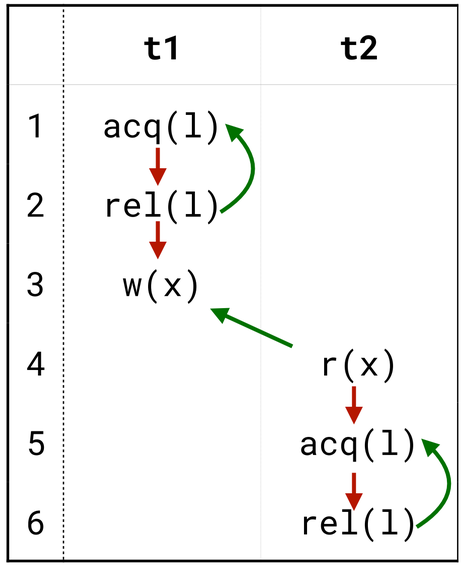
  - For a release event **rel**, RF_σ(**rel**) = **acq**

last write on the same memory location

matching acquire



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- TRF_σ = smallest partial order that includes TO_σ and respects RF_σ

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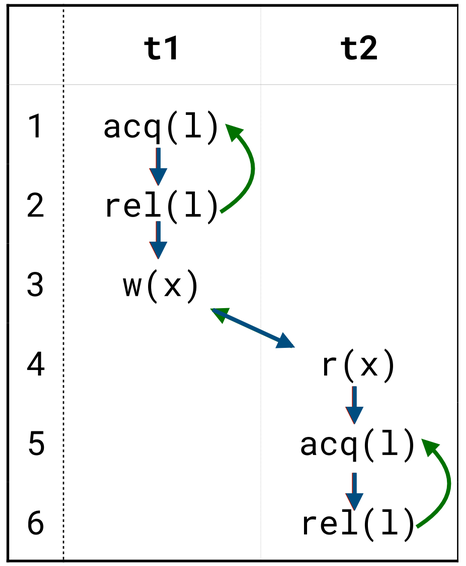

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Let σ be an execution trace.

- TO_σ = intra-thread ordering on events of σ
- RF_σ : reads-from mapping of σ:
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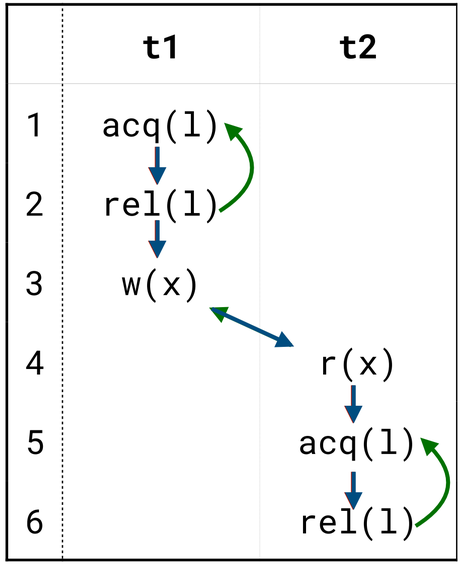

Observation

Let ρ be a sequence of events with Events(ρ) $\subseteq$ Events(σ).

ρ is a correct reordering of an execution trace σ if

(i) For every lock ℓ, there is at most one unmatched acquire of ℓ in ρ

(ii) ρ respects TO_σ

(iii) $RF_\rho \subseteq RF_\sigma$

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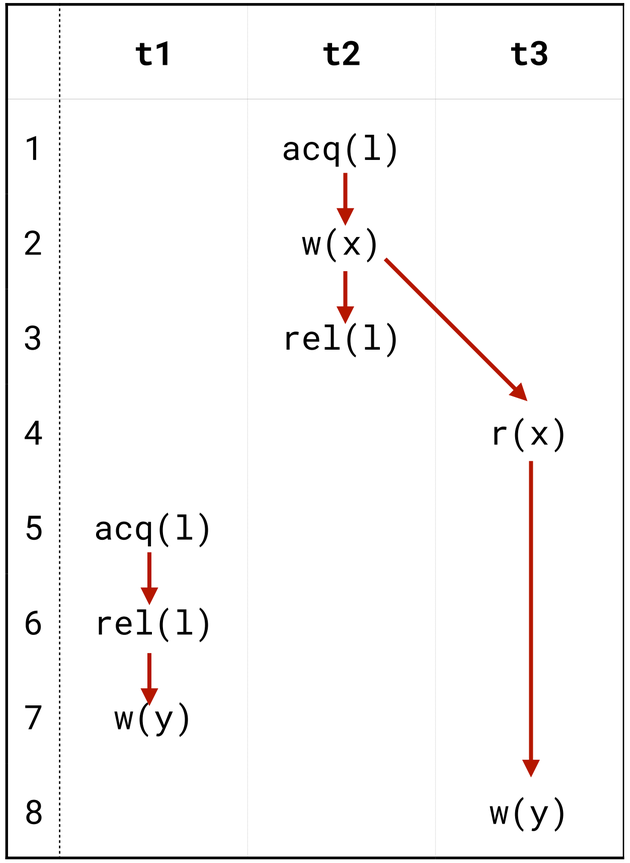

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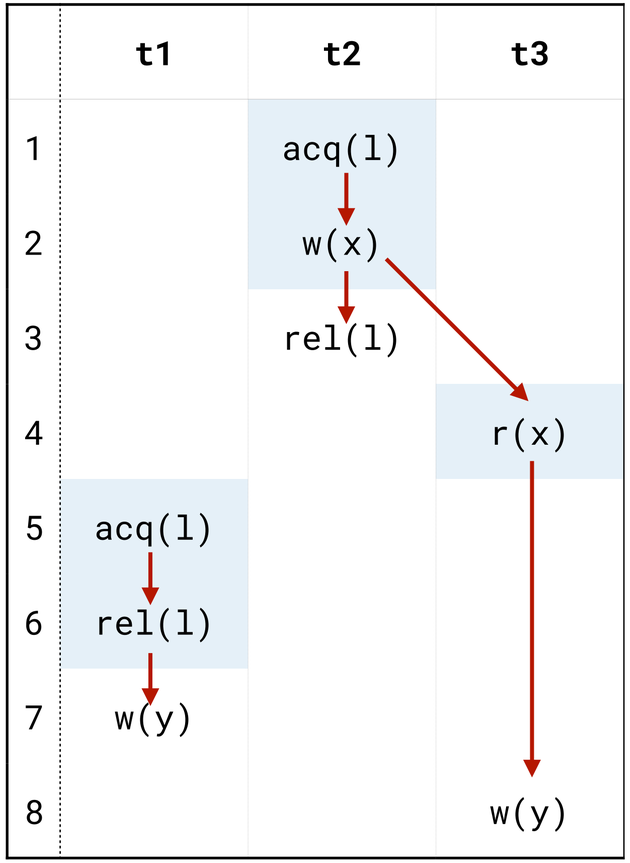

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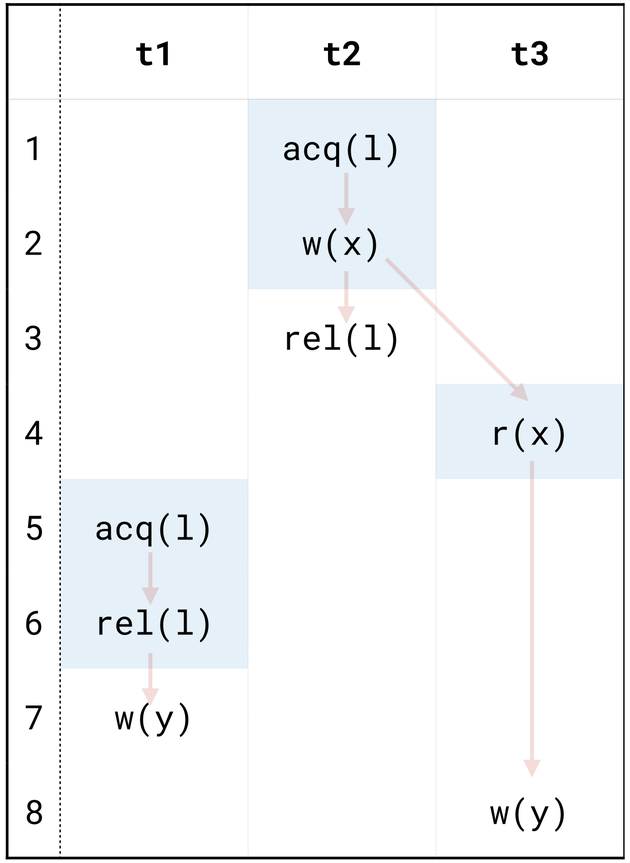

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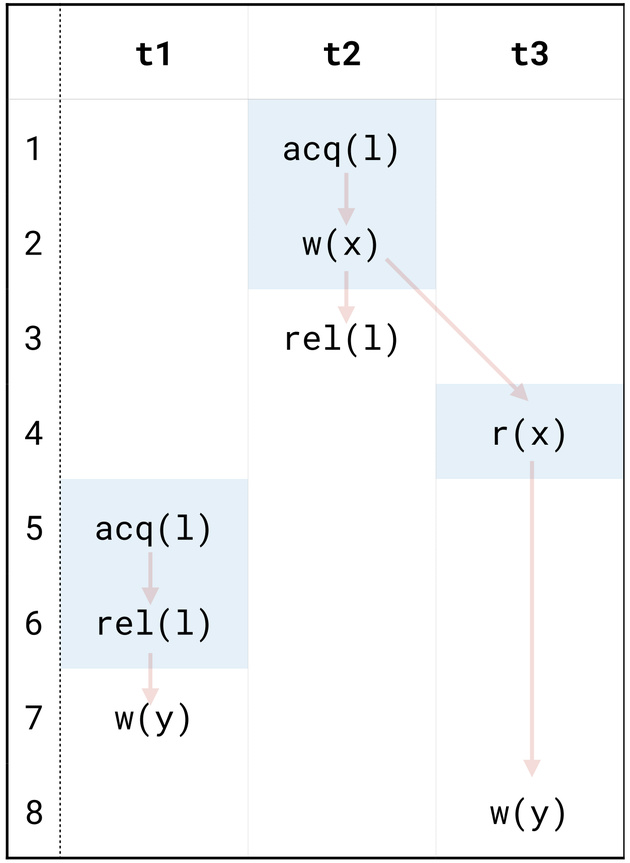

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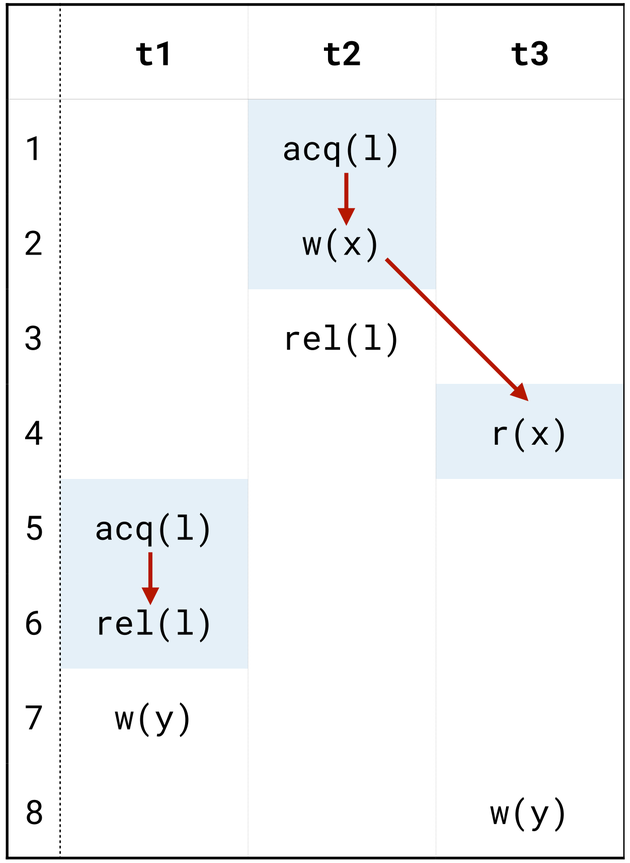

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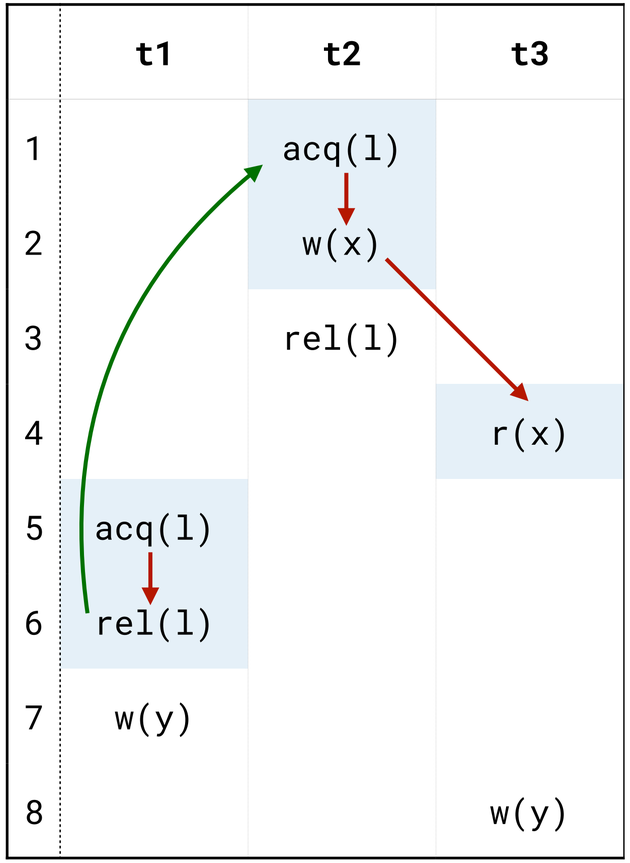

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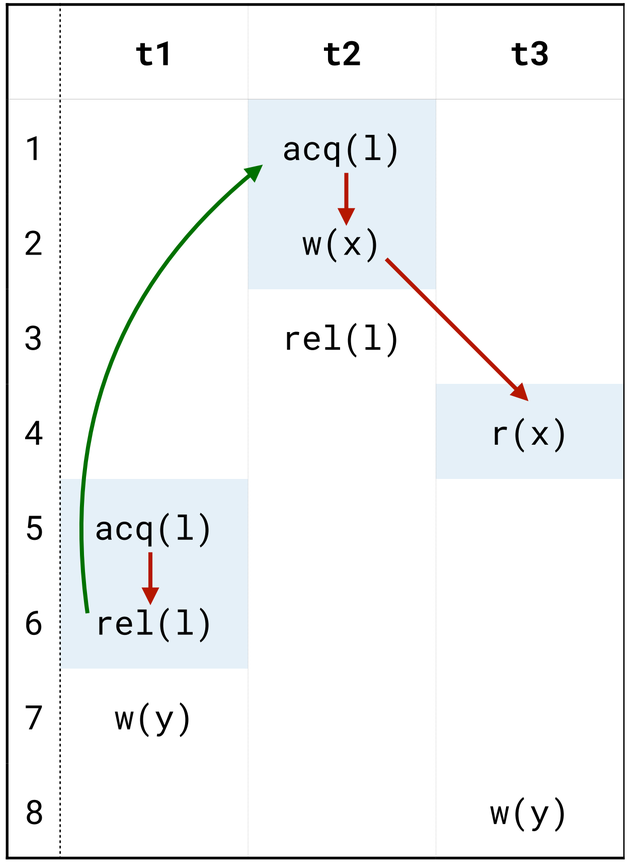

Realizability of Trace Ideals

Given a feasible trace ideal I of σ, check if there is a linearization σ^* of the canonical rf-poset $\mathscr{P}(I)$ such that $\mathrm{RF}_{\sigma^*} \subseteq \mathrm{RF}_\sigma$

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Data Race Prediction as Ideal Realizability

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A pair of conflicting events (e_1, e_2) is a predictable data race of σ

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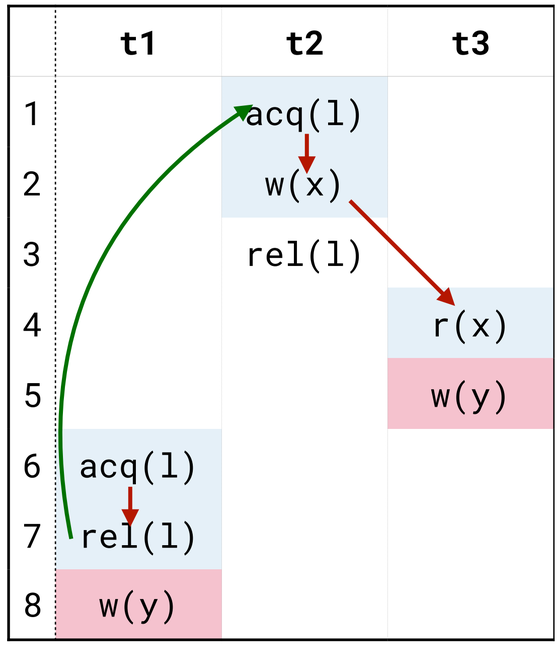

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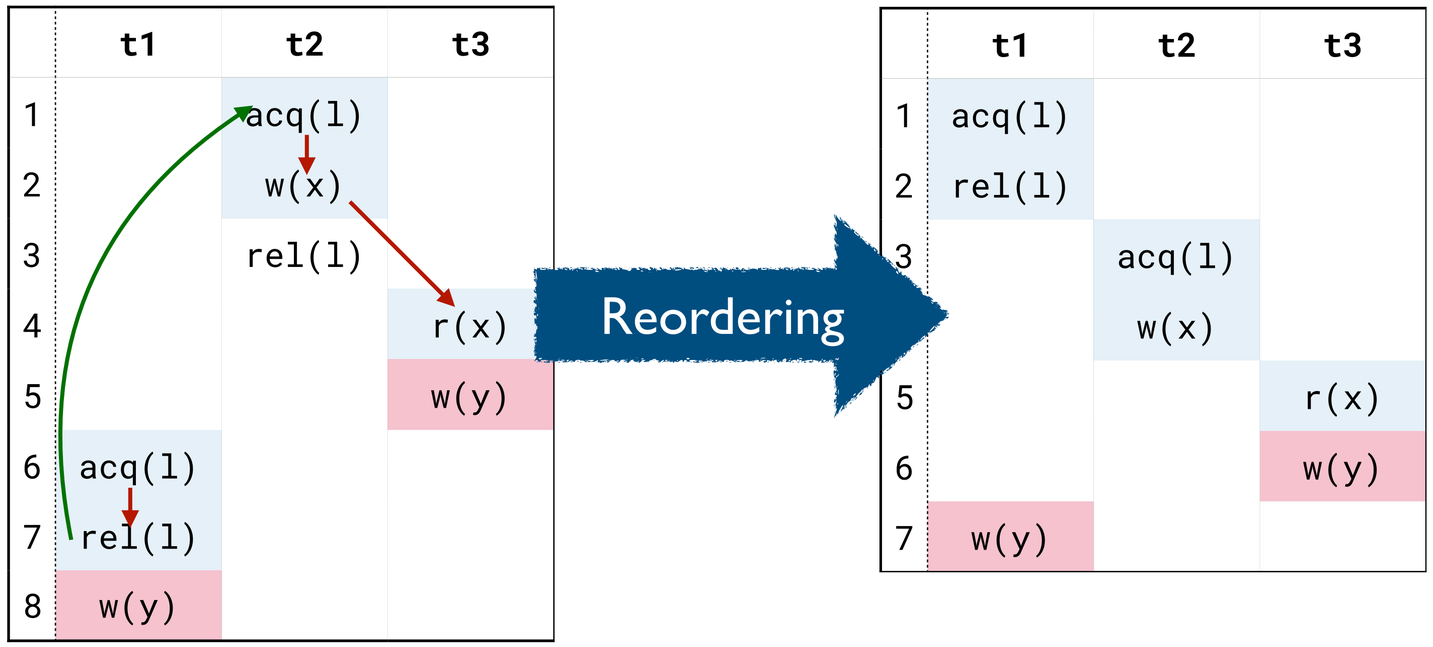

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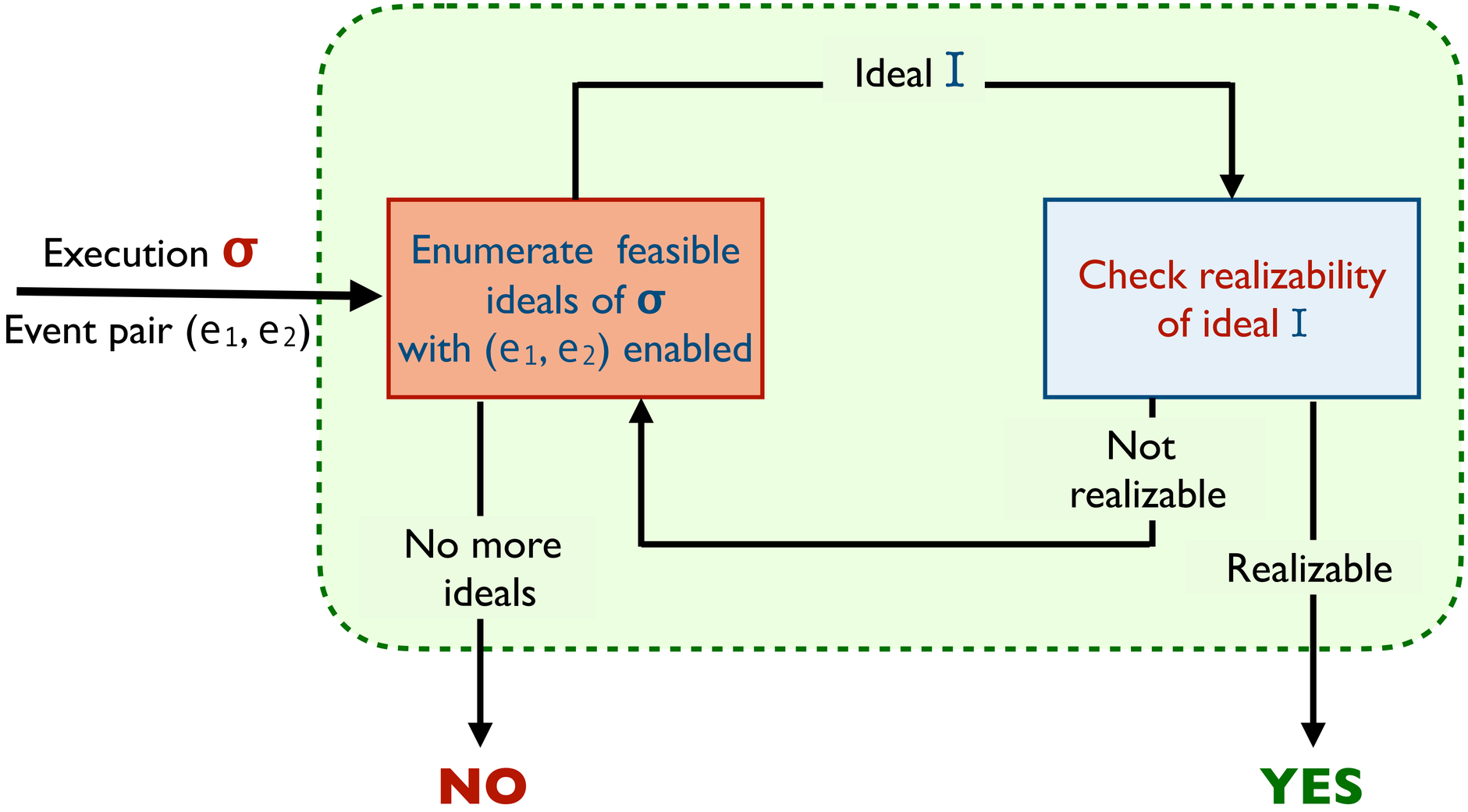

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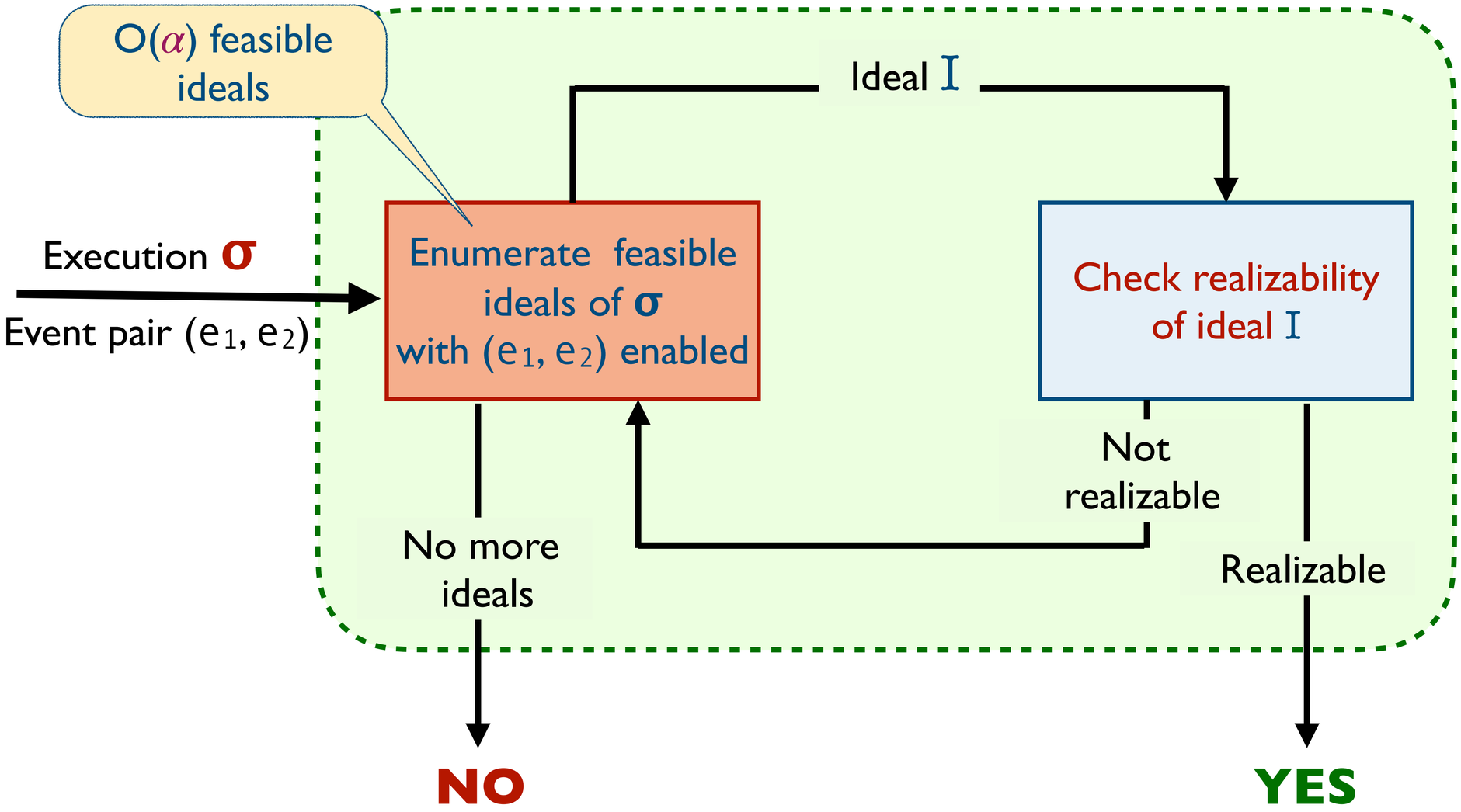

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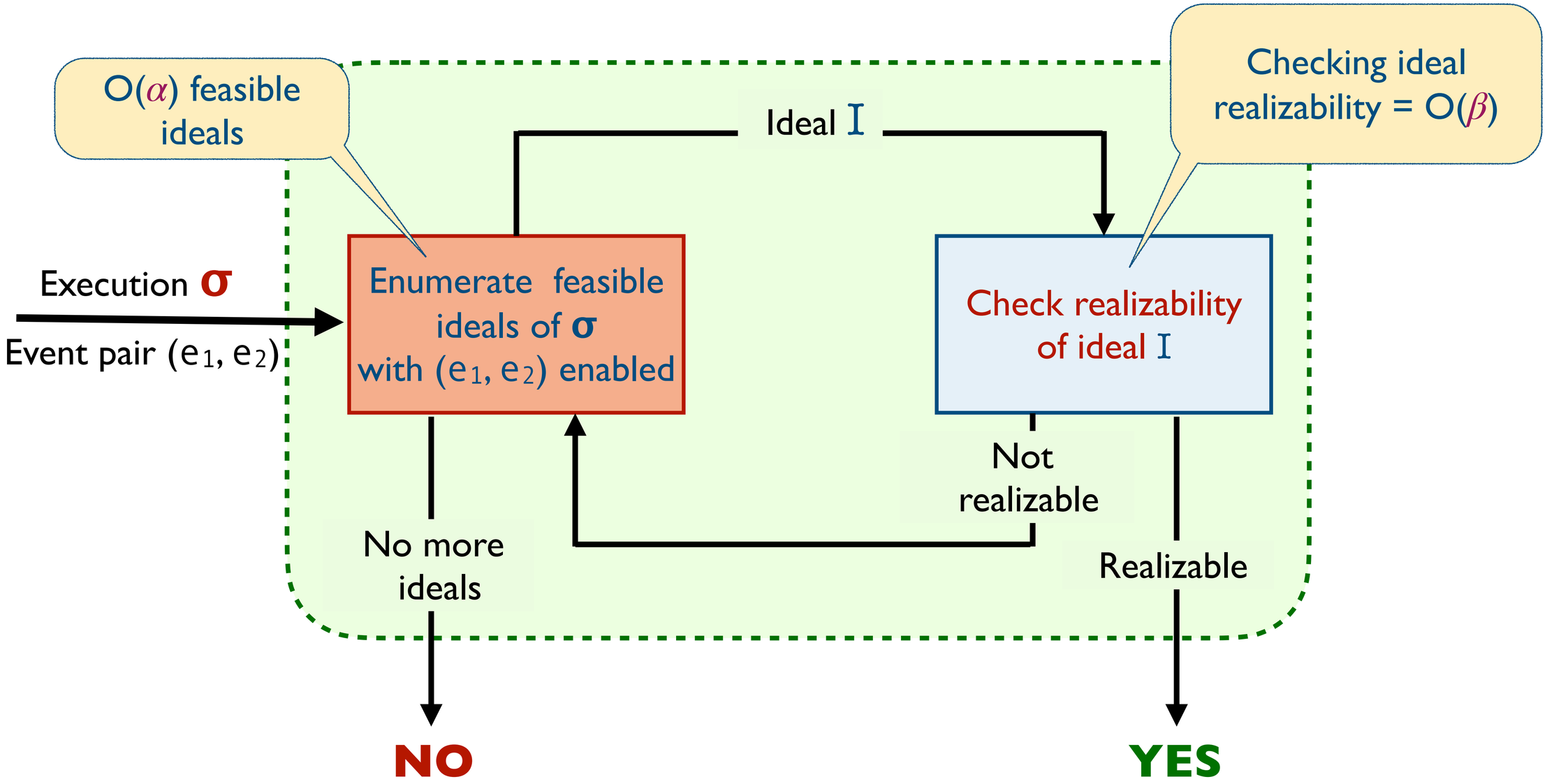

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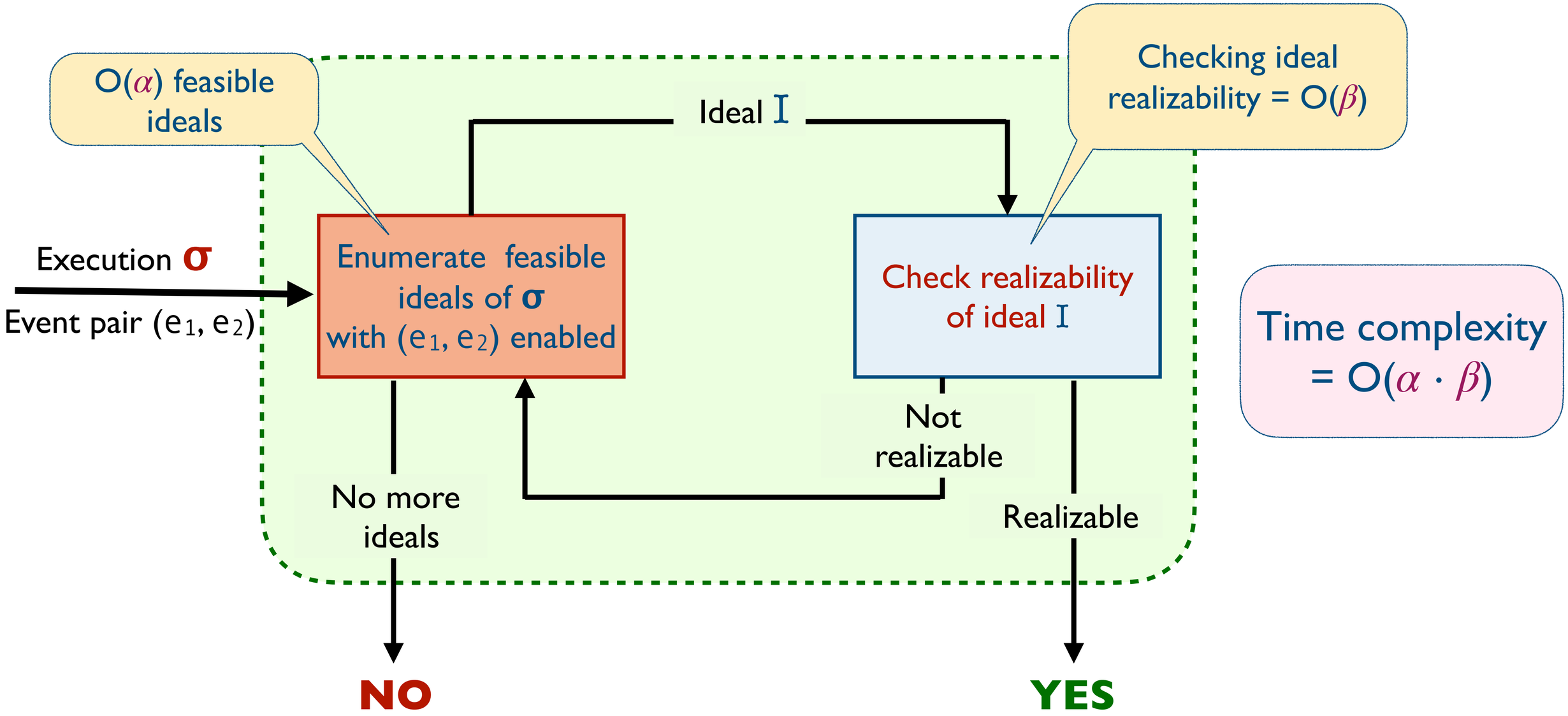

Algorithm for Data Race Prediction

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Lemma

Let σ be an execution with n events,
k threads, lock nesting depth γ and
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Let e_1 and e_2 be events of σ.
There are $O(\min(n, k \cdot \gamma \cdot \zeta)^{k-2})$ feasible ideals
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Let σ be an execution with n events and k threads and let I be a feasible ideal of σ. The realizability of I can be checked in time $O(k \cdot n^k)$.

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Theorem

Let σ be an execution with n events, k threads, lock nesting depth γ and lock-dependence factor ζ. Let (e_1, e_2) be a conflicting pair of events of σ. The dynamic race prediction problem on can be solved in time $O(\min(n, k \cdot \gamma \cdot \zeta)^{k-2} \cdot k \cdot n^k)$.

Outline

1. Trace Ideals and Realizability
2. Algorithm for Data Race Prediction (General case)
3. **Data Race Prediction for Acyclic Communication Topology**
4. Distance Bounded Data Race Prediction
5. Lower Bound (General Case)
6. Lower Bound for 2 threads
7. Conclusions and Future Work

Tree Communication Topology

Tree Communication Topology

The communication topology of σ is an undirected graph $G_\sigma = (V_\sigma, E_\sigma)$:

- V_σ = set of threads in σ
- E_σ = {(t, t') | t ≠ t' are threads that perform conflicting accesses or acquire a common lock}

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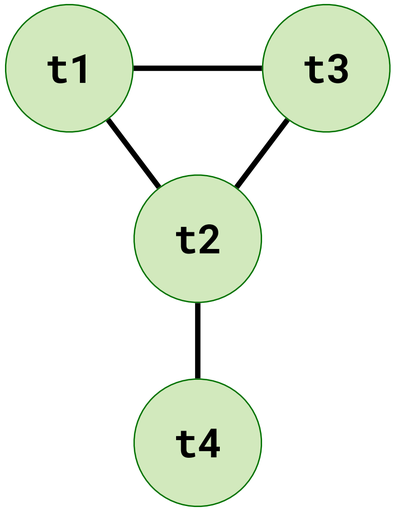
5	acq(l)			
6	rel(l)			
7	w(y)			
8				r(x)
9			w(y)	



Acyclic Communication Topology

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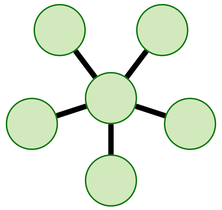

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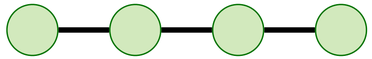
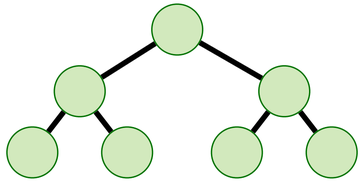
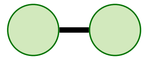


Pipeline

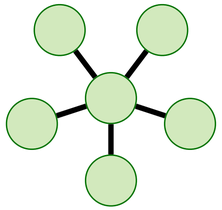


Divide-and-conquer



Two threads

Acyclic Communication Topology



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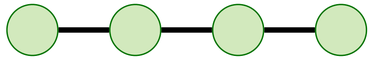

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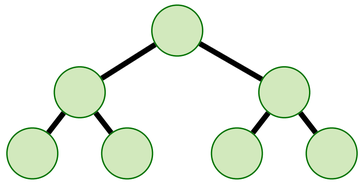

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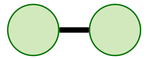
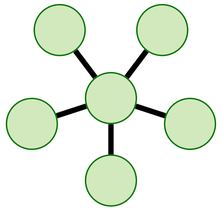


Two threads

Lemma

The realizability of a feasible ideal I of an execution σ (with n events, k threads and d variables) having an acyclic communication topology can be determined in $O(k^2 \cdot d^2 \cdot n^2 \cdot \log n)$ time.

Acyclic Communication Topology



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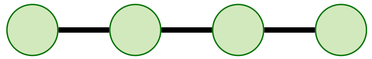

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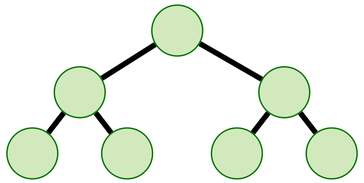

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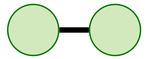

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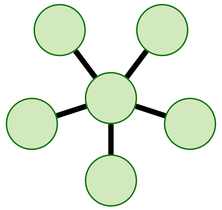
The realizability of a feasible ideal I of an execution σ (with n events, k threads and d variables) having an acyclic communication topology can be determined in $O(k^2 \cdot d^2 \cdot n^2 \cdot \log n)$ time.

Lemma

Let σ be an execution with an acyclic communication topology and let e_1 and e_2 be events of σ. There is a feasible ideal $I_{(e1, e2)}$ such that $I_{(e1, e2)}$ is realizable iff (e_1, e_2) is a predictable data race of σ.

Acyclic Communication Topology



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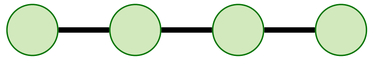

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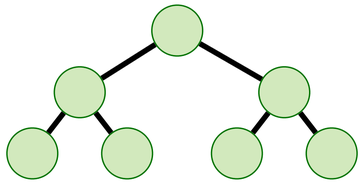

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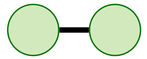

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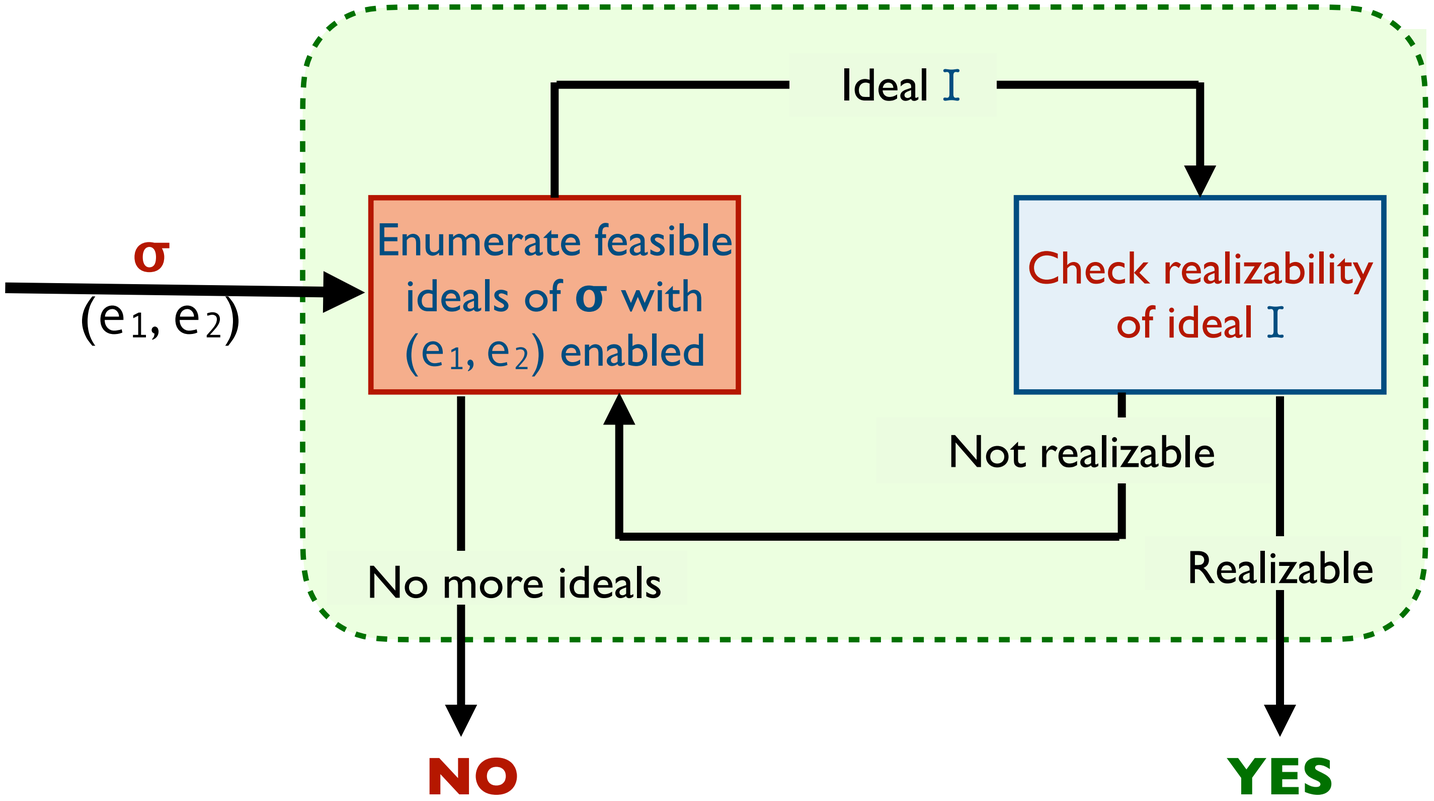

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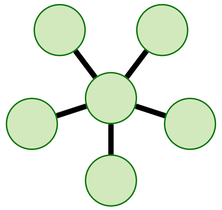

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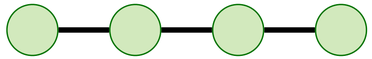

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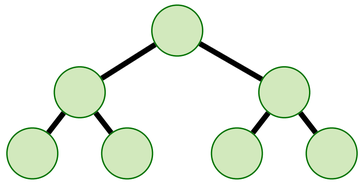

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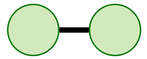

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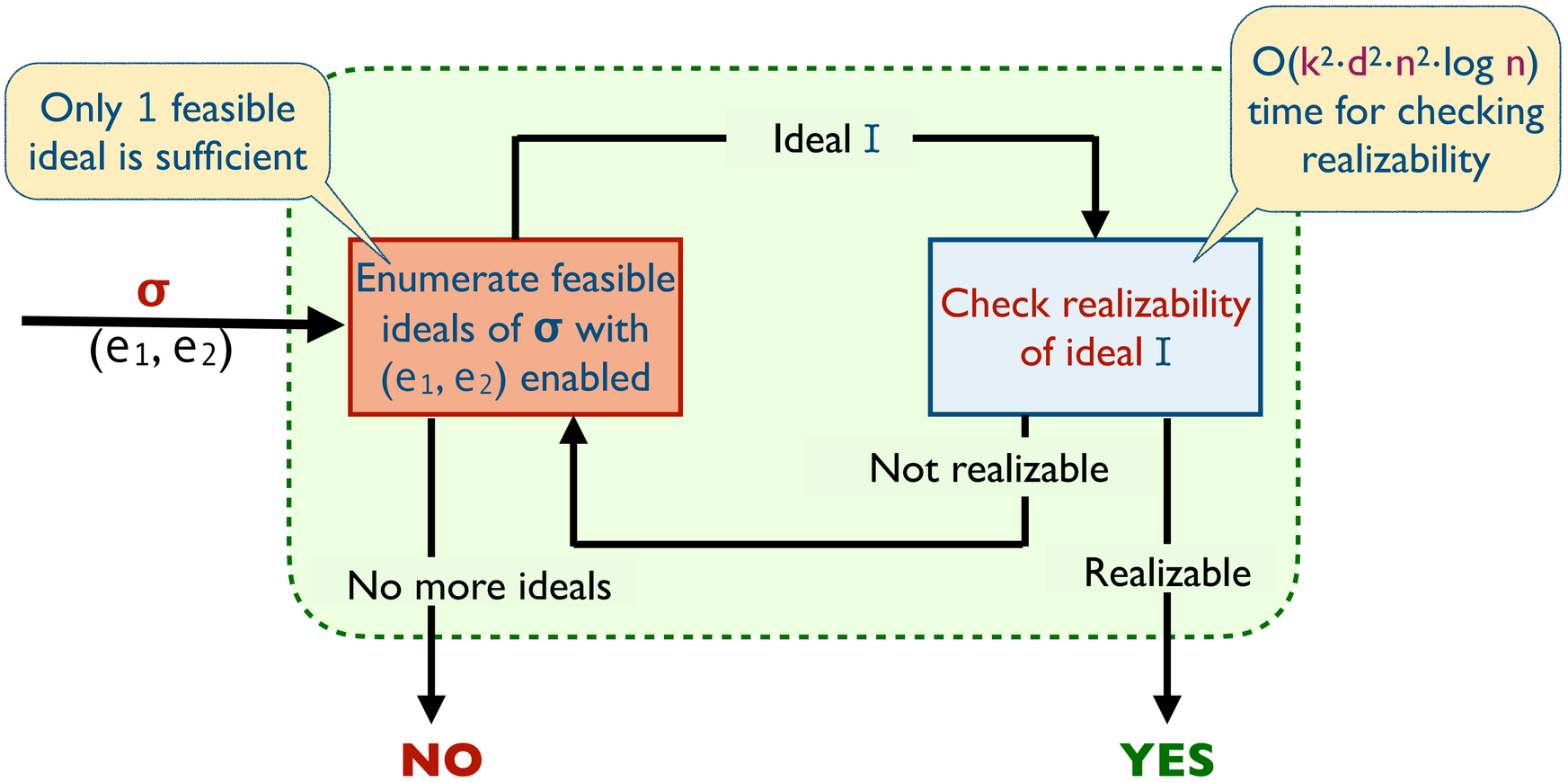

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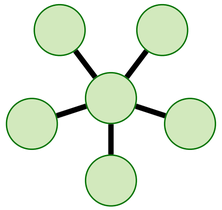

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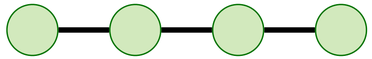

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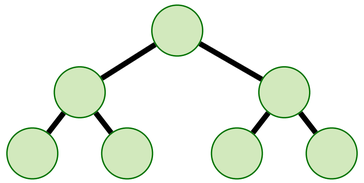

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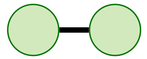

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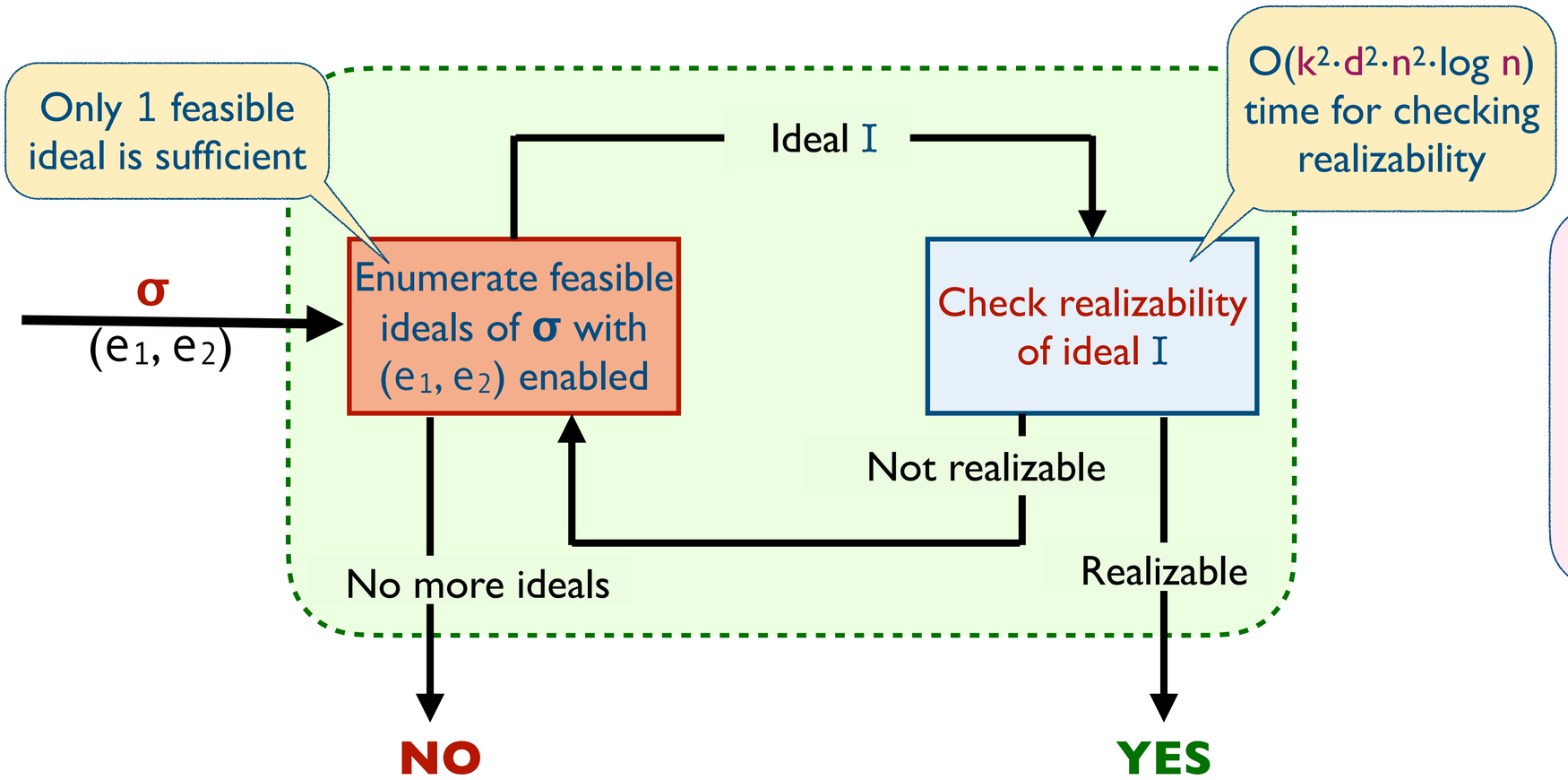

Time complexity when execution has acyclic topology = $O(k^2 \cdot d^2 \cdot n^2 \cdot \log n)$

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Distance-Bounded Data Races

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Set of reversals between σ and its correct reordering ρ:

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$$\mathrm{Rev}(\sigma, \rho) = \{(e_1, e_2) \mid e_1 \text{ and } e_2 \text{ are conflicting writes or acquires of same lock such that } e_1 \leq_\sigma e_2 \text{ and } e_2 \leq_\rho e_1\}$$

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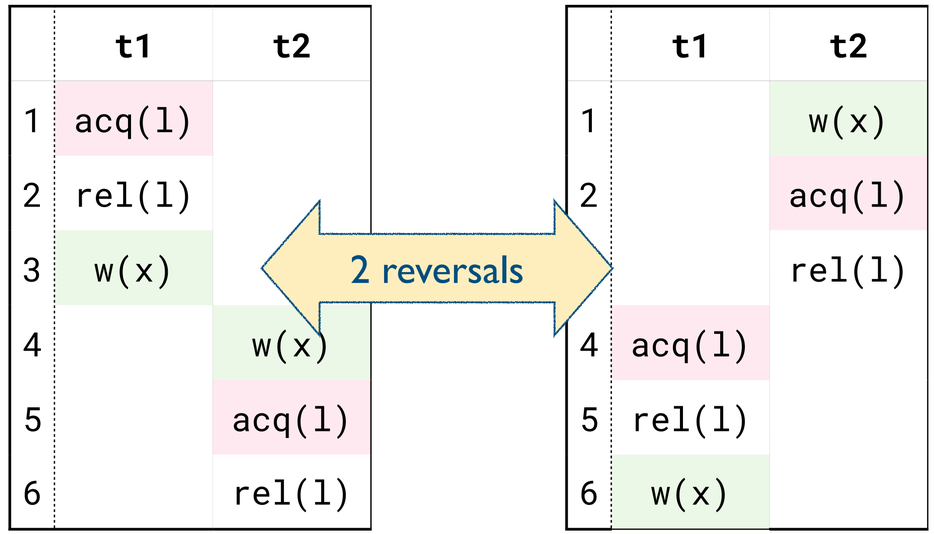

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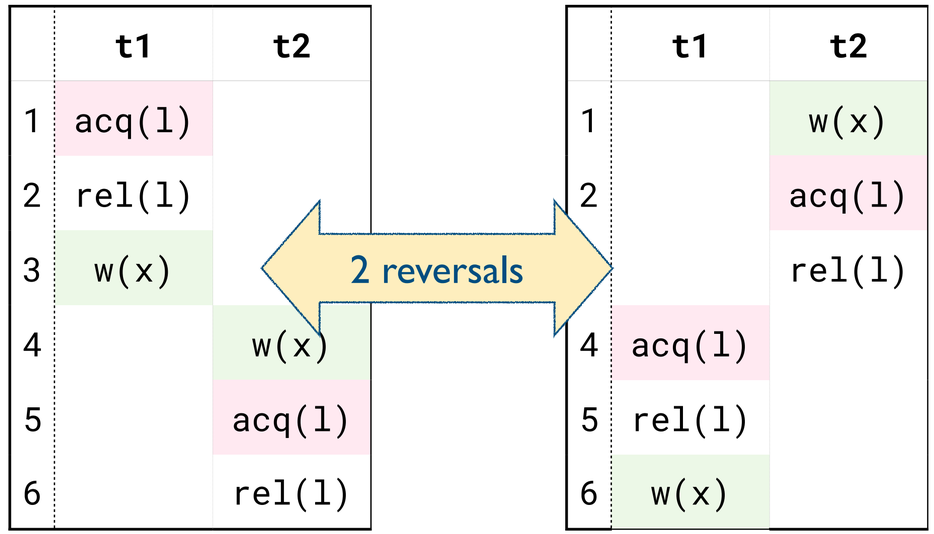

Distance-bounded Race Prediction

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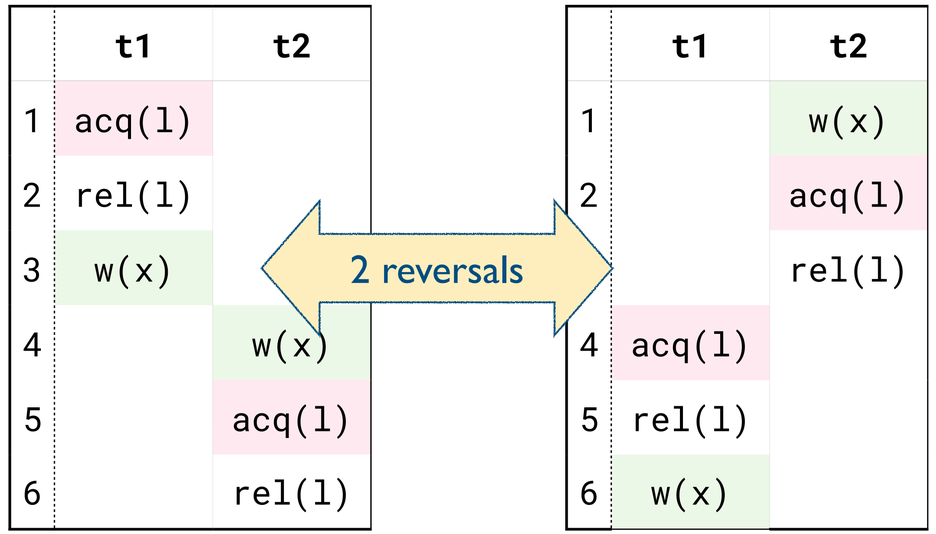

Let $\ell \in \mathbb{N}$ be a constant. Given an execution σ and pair of conflicting events (e_1, e_2) of σ, the answer to the ℓ-bounded data race prediction problem is

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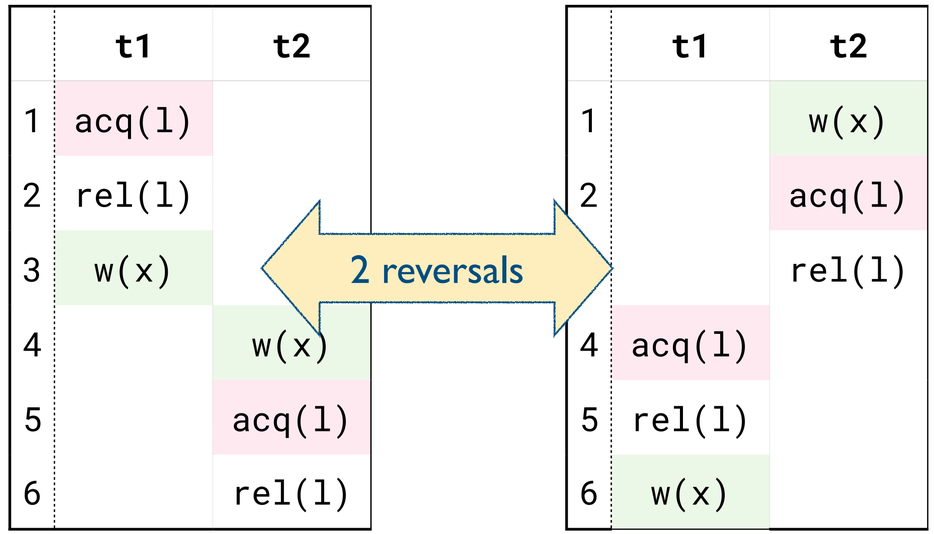

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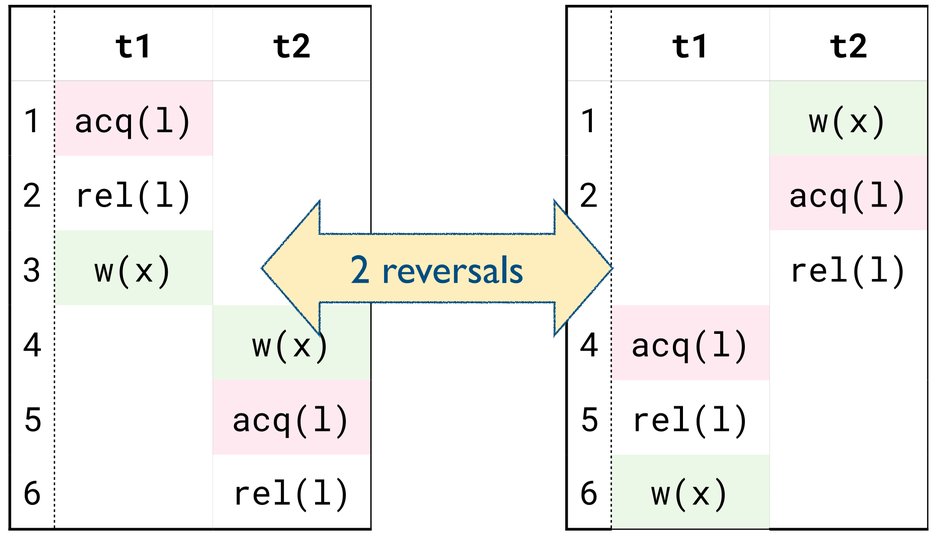

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Let $\ell \in \mathbb{N}$ be a constant. Given an execution **σ** and pair of conflicting events (e_1, e_2) of **σ**, the answer to the ℓ-bounded data race prediction problem is

- NO if (e_1, e_2) is not a predictable race of **σ**
- YES, if there is a correct reordering **σ*** of **σ** witnessing the race (e_1, e_2) such that $\delta(\sigma, \sigma^*) \leq \ell$

Distance-Bounded Data Races

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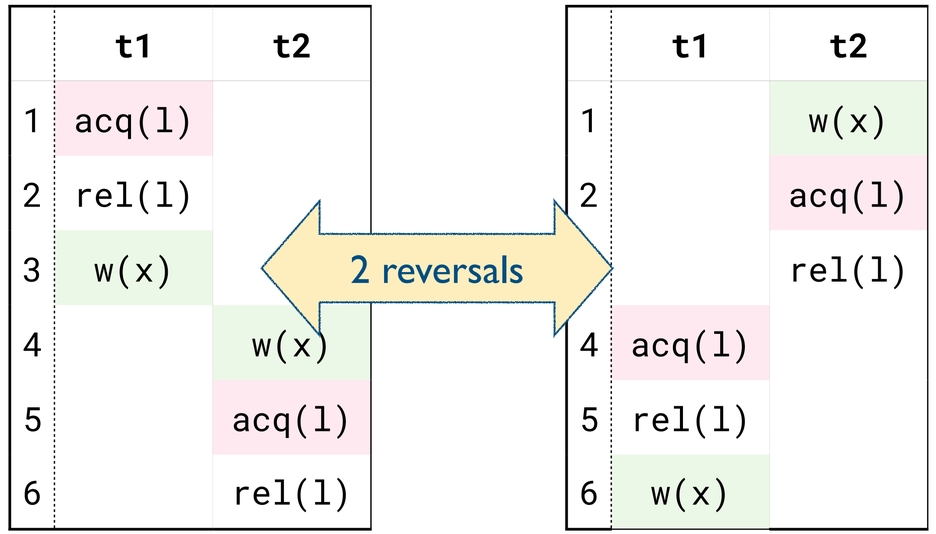

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- YES or NO, otherwise

Distance-Bounded Data Races

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Distance-bounded Ideal Realizability

Let $\ell \in \mathbb{N}$ be a constant. Given an execution σ and a feasible ideal $\mathtt{I}$ of σ, the answer to the ℓ-bounded realizability problem for $\mathtt{I}$ is

- NO if $\mathtt{I}$ is not realizable
- YES, if there is a witness σ^* of realizability of $\mathtt{I}$ such that $\delta(\sigma, \sigma^*) \leq \ell$
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Distance-Bounded Data Races

Distance-bounded Ideal Realizability

Let $\ell \in \mathbb{N}$ be a constant. Given an execution σ and a feasible ideal I of σ, the answer to the ℓ-bounded realizability problem for I is

- NO if I is not realizable
- YES, if there is a witness σ^* of realizability of I such that $\delta(\sigma, \sigma^*) \leq \ell$
- YES or NO, otherwise

Naive Bound

Iterate over all possible subsets of events of I of size $\leq \ell$ of pairs of conflicting writes or lock acquires and invert them.

$O(|I|^{2\ell}) = O(n^{2\ell})$

Distance-Bounded Data Races

Distance-Bounded Data Races

Lemma

Let $\ell \in \mathbb{N}$ be a constant. Let σ be an execution with n events and k threads and let $\mathtt{I}$ be a feasible ideal of σ.
The ℓ-bounded realizability of $\mathtt{I}$ can be checked in time $O(k^{\ell+O(1)} \cdot n)$.

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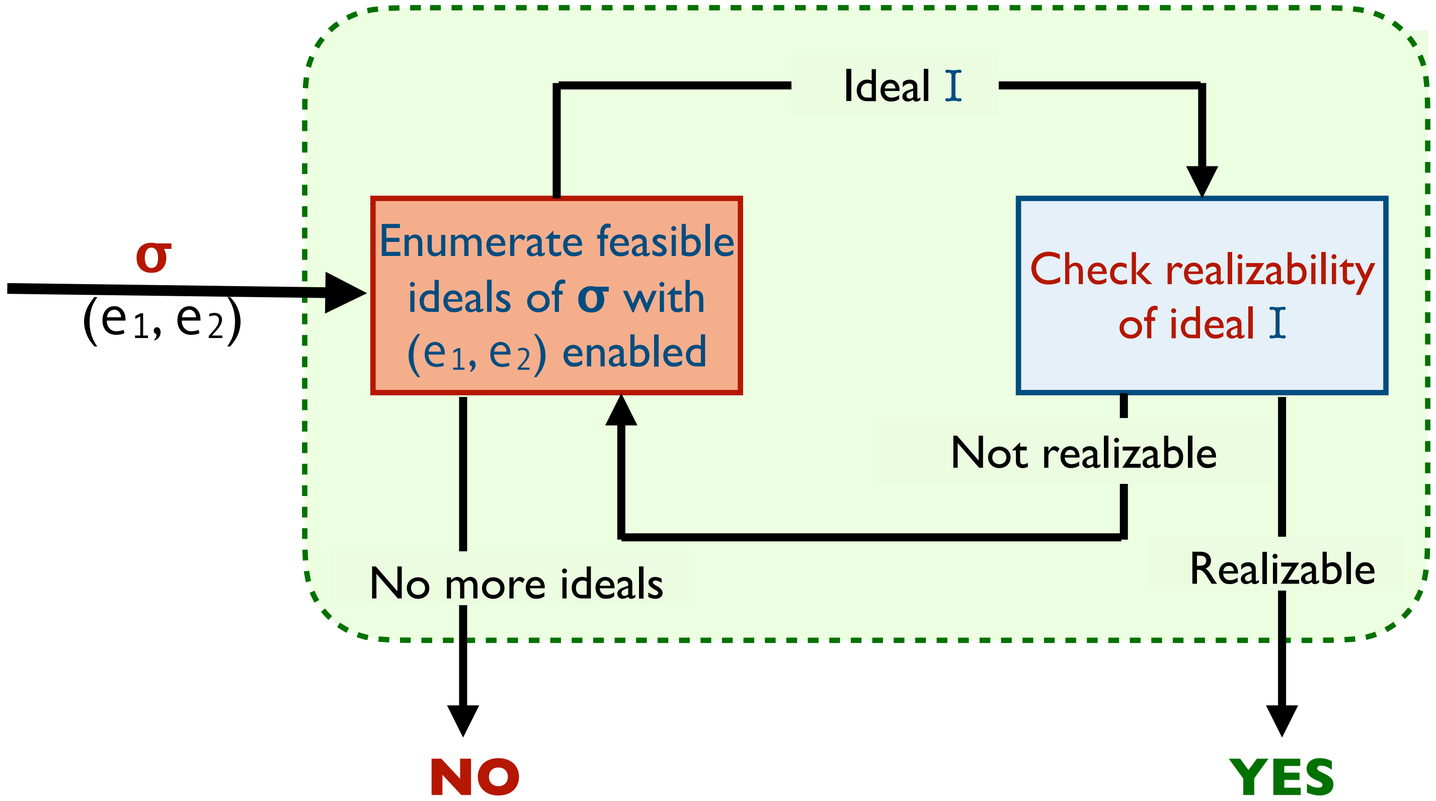

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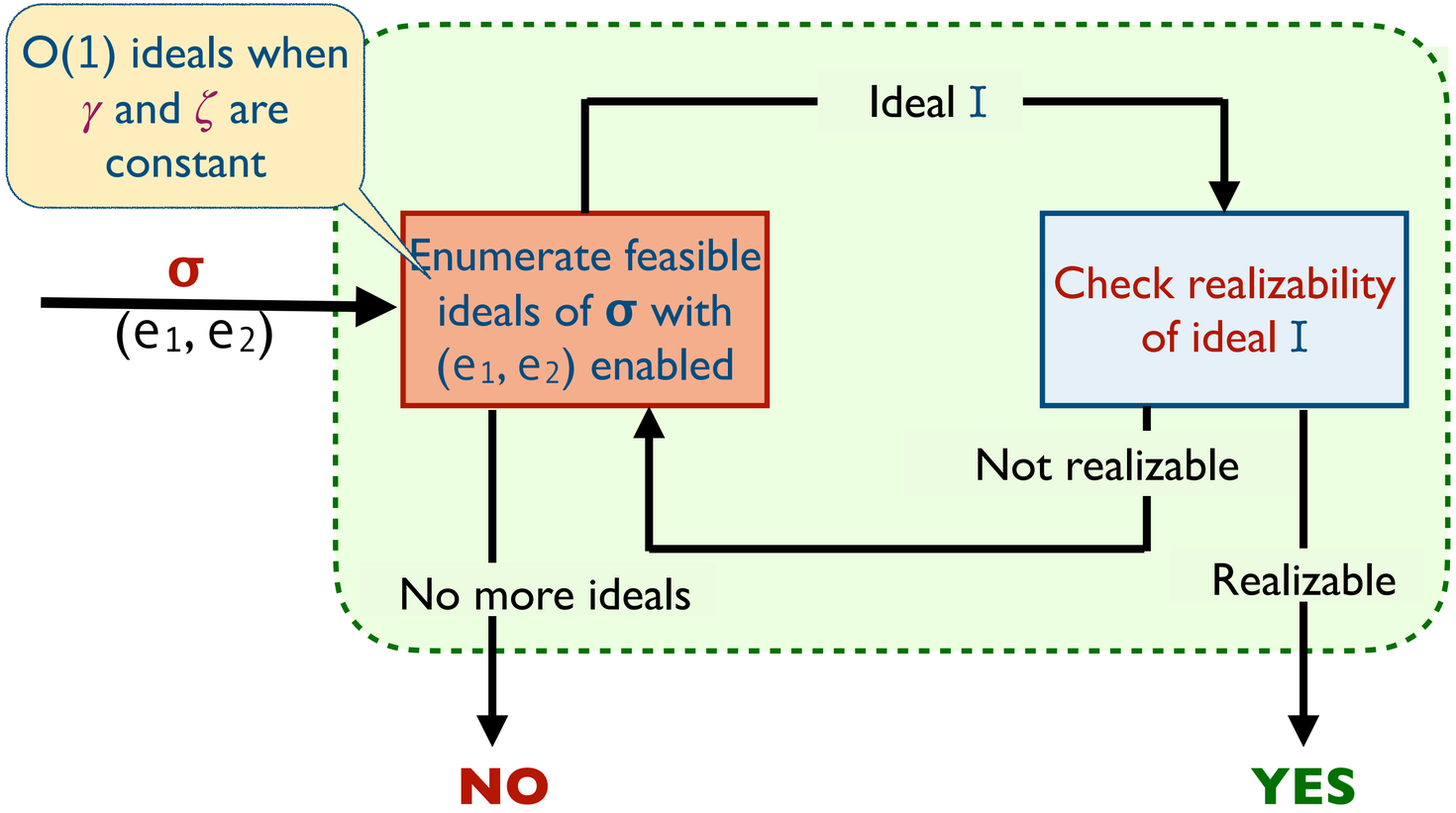

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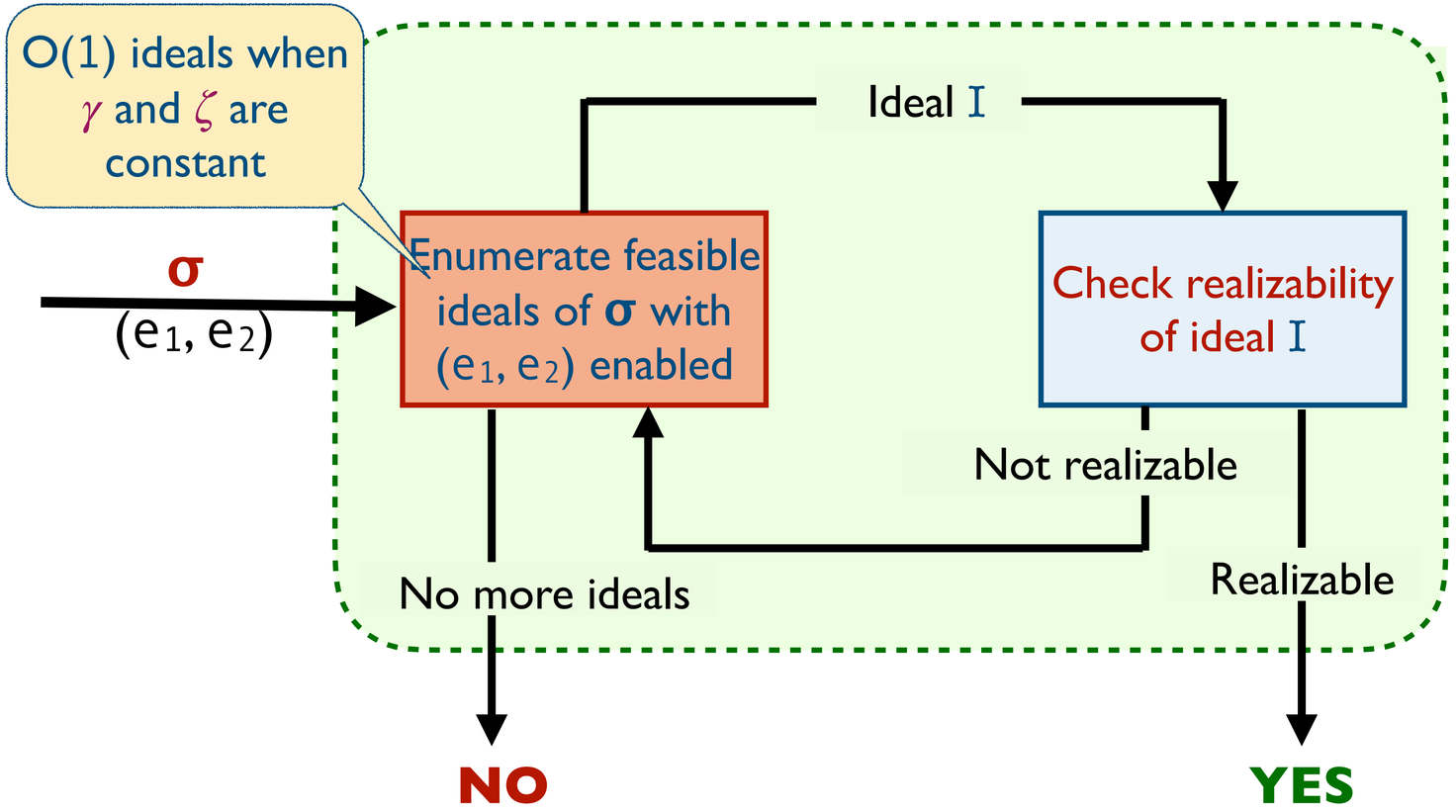

Time complexity of ℓ-bounded data race prediction $= O(k^{\ell+O(1)} \cdot n)$

Outline

1. Trace Ideals and Realizability
2. Algorithm for Data Race Prediction (General case)
3. Data Race Prediction for Acyclic Communication Topology
4. Distance Bounded Data Race Prediction
5. **Lower Bound (General Case)**
6. Lower Bound for 2 threads
7. Conclusions and Future Work

Lower Bound for Data Race Prediction

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Unlikely, unless ETH fails

ETH implies **FPT** ⊊ **W[1]**

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Let σ be an execution with n events, $k \geq 2$ threads and $d \geq 9$ variables (and ≥ 1 lock).

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Applies to acyclic topology

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  - Linear time algorithms
  - (Sub-)linear dependence on d
  - (Sub-)linear space overhead

Thank You!