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# Cooperation in the Minority Game with local information

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## Abstract

The Minority Game was introduced to show basic properties of competitive systems with limited common information resources. M. Paczuski and K. E. Bassler introduced a Minority Game with personal limited information resources, where each agent knows the past actions of randomly chosen neighbours [M. Paczuski, K.E. Bassler, Self-organized Networks of Competing Boolean Agents (1999)]. They asked whether such a system can show cooperation. In this paper we show that agents who are placed in a circle are able to cooperate due to self-organization. Furthermore, we introduce a new evolution method to optimize the cooperation among the agents. © 2000 Elsevier Science B.V. All rights reserved.

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In recent science, in particular in the social sciences and economics, there is a growing interest for systems in which agents with bounded rationality compete for scarce resources. A model that combines general properties of such systems is the Minority Game (MG), which was introduced by Challet and Zhang [1].

In this game, a group of  $N$  (odd) individuals has to decide for one of two possibilities: 0 or 1. The side chosen by less agents is the winner side, so the maximum number of winners is  $(N - 1)/2$ . To make a decision each agent uses a set of  $s$  strategies taken at random from the pool of possible strategies. A strategy contains a set of outputs which refer to all possible inputs. Furthermore, the agents get a bitstring as input that contains the last  $m$  minority sides and each agent uses the most successful strategy in order to react on that input. There are altogether  $2^m$  input possibilities and therefore  $2^{2^m}$  possible strategies. In the simplest case the agents who decide for the minority side get a point. Also all the strategies that predict the winner side correctly get a point no matter if they were actually used or not.

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Now, we suggest another possibility how the agents take up information. We call this model “Minority Game with local information (MGLI)”. The agents are arranged on a circle and everyone gets the previous decisions of his neighbours as input (this is the principle of a cellular automata). For an odd memory  $m$  the decisions of the  $(m-1)/2$  left and right handed neighbours and the own one are known; the asymmetry for even  $m$  is irrelevant. The rest of the procedure is the same: each agent looks at his more successful strategy how to decide for a side in the next timestep. When all have decided the minority side is determined, every agent on this side gets a point, the strategies are valued and the next round begins.

The performance of the system can be described best with the help of the standard deviation of the number of agents deciding for the minority side,  $\sigma$ . The standard deviation indicates how well the agents cooperate. The better the agents use the available resources the smaller it is. To include the number of agents particular emphasis has to be put on  $\sigma^2/N$  [2]. When all agents behave randomly (that occurs when the amount of information is too large)  $\sigma^2/N \rightarrow \frac{1}{4}$ .

At first we analyze the influence of the number of strategies  $s$  on the dynamics of the MGLI. We found that  $\sigma^2/N$  increases just slightly with increasing  $s$  for different values of  $m$  and  $N$  whereas the system behaves randomly for  $s = 1$  since the agents have no possibility for adaptation.

For our further analysis, we set  $s = 2$  to concentrate on the system parameters  $m$  and  $N$ . In Figs. 1 and 2 we plot  $\sigma$  as a function of  $N$  on a log–log scale for different values of  $m$ . For fixed  $m$  nearly all the data fall on a straight line. The slope is approximately 1 for  $m=1$  and 2 and 0.5 for  $m \geq 3$ . Because of this we can generally say that

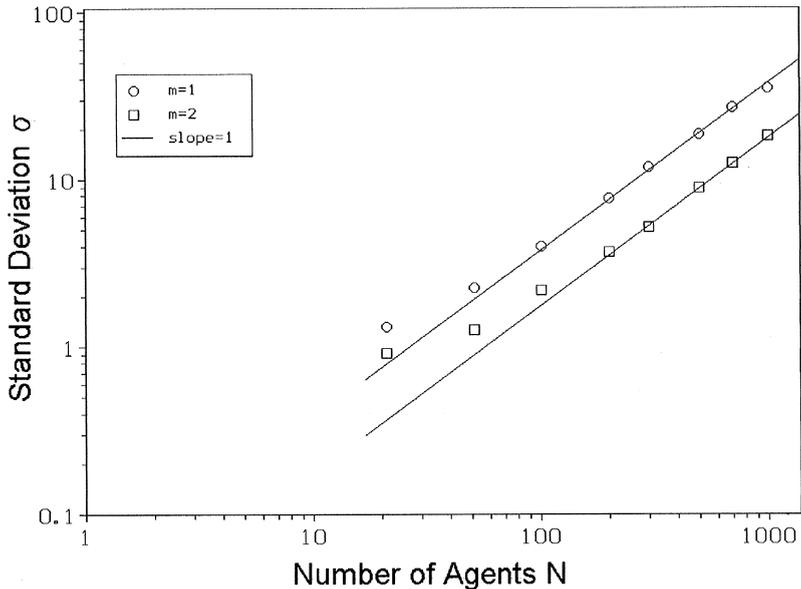


Fig. 1.  $\sigma$  versus  $N$ .

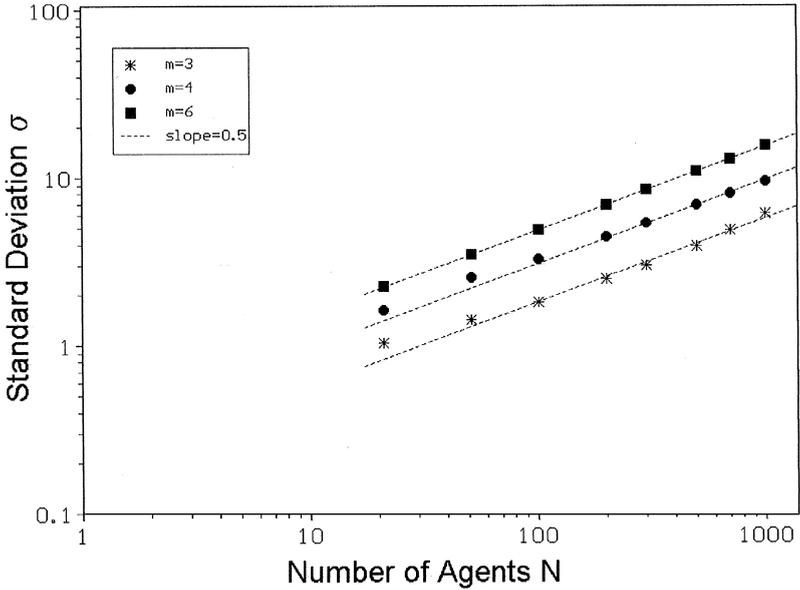


Fig. 2.  $\sigma$  versus  $N$ .

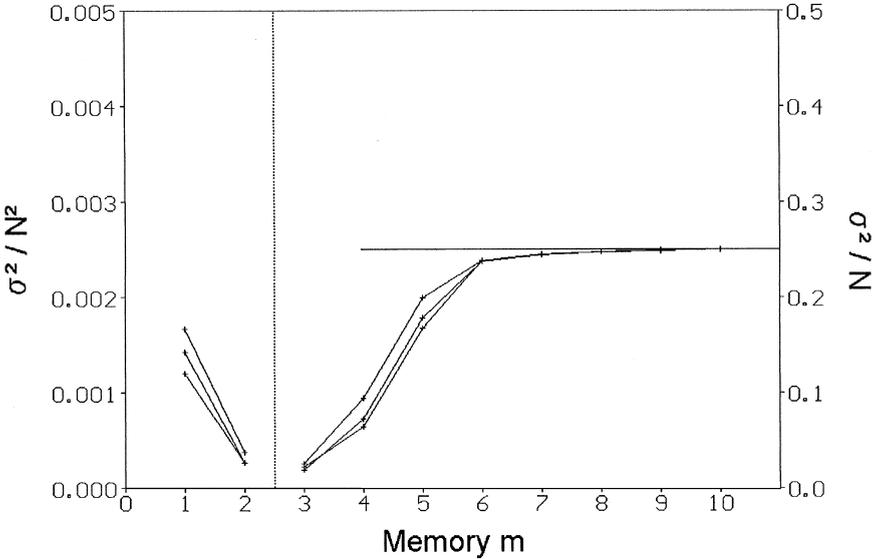


Fig. 3.  $\sigma^2/N^2$  respectively  $\sigma^2/N$  versus  $m$  for  $N = 101, N = 501, N = 1001$  (all points left of the dotted line represent  $\sigma^2/N^2$ , all that are right of it represent  $\sigma^2/N$ ).

$\sigma \sim N$  for  $m < 3$  and  $\sigma \sim \sqrt{N}$  for  $m \geq 3$ . So in the low- $m$ -region  $\sigma^2/N^2$  and in the high- $m$ -region  $\sigma^2/N$  is a function only of  $m$ . To illustrate this we plot  $\sigma^2/N^2$ , respectively  $\sigma^2/N$ , versus  $m$  in Fig. 3 and see that the points fall on a nearly universal curve. The higher  $m$  is the exacter this approximation becomes. What is the cause for

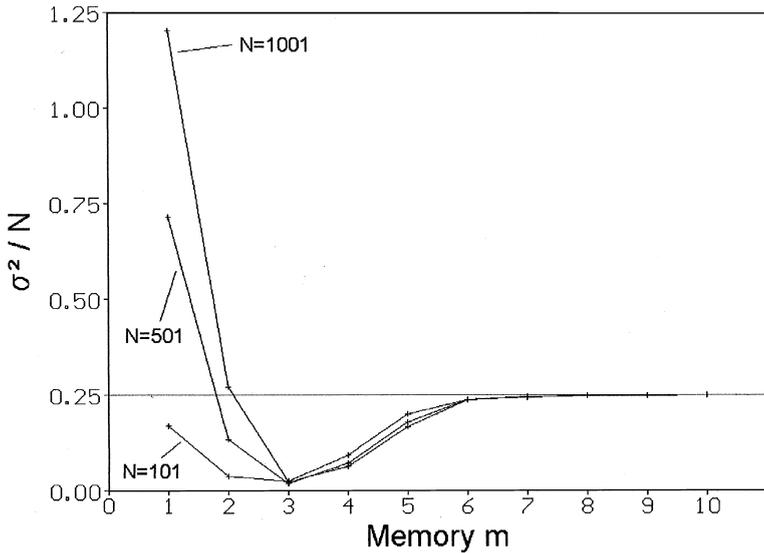


Fig. 4.  $\sigma^2/N$  versus  $m$  for  $N = 101, 501$  and  $1001$ .

the different scaling of  $\sigma$ ? We presume that there is a phase transition between  $m = 2$  and  $3$  since the strategies and therefore the behaviour of the agents is less correlated for  $m \geq 3$  than in the low  $m$  phase in which the agents often make similar choices as a result of the correlation. Compared with the MG this herd effect is much smaller because each agent processes a local different input.

A point we have to put more emphasis on is the  $m = 3$  case. In Fig. 4 we scaled all values with  $\sigma^2/N$  in order to see that for the MGLI the minimum of  $\sigma$  is at  $m = 3$  independent of  $N$  which is different from the original Minority Game. Reason for this is the fact that the agents are less correlated (as mentioned above) so the herd effect is harder to obtain. If we increase  $N$  the correlation among the agents does not increase as rapidly as in the MG since the agents in the MGLI process local information.

How could that strongly developed cooperation among agents with memory  $m = 3$  be explained? For this we assigned every agent a random bitstring as input. As a result, the value of  $\sigma^2/N$  for  $m = 3$  lies very narrow to the random value. If we generate a random bitstring with the length  $N$  each timestep and every agent takes his input from that string the value of  $\sigma^2/N$  for  $m = 3$  is between the original game and that game with totally randomized input. So we can see that the cooperation in the MGLI must base on two factors: firstly there is a connection between the information of the agents and secondly this information must represent the real decisions of the individuals in the last timestep. As a result the agents organize themselves with time which helps them to increase their success significantly. So the self-organization is one of the most important properties of the system.

Furthermore, we want to discuss the question whether the system can be optimized by evolutionary mechanisms. The “genetic code” of an agent consists of two genes:

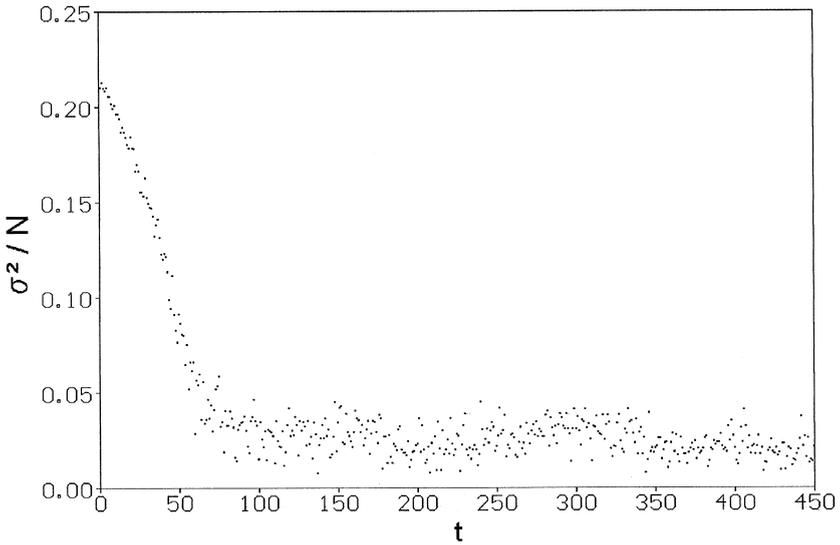


Fig. 5. Result of the local evolution method for 450 evolution steps ( $N = 101$ ).

the intelligence  $m$  and the number of strategies  $s$ . The first evolution method works globally: after  $n$  timesteps the worst agent is replaced by a variant of the best one. He gets the values of  $m$  and  $s$  of the best agent which can be increased or decreased by 1 with a certain probability ( $1 < m < 10$ ,  $1 < s < 10$ ). So populations with mixed  $m$  and  $s$  are allowed.

Now we suggest another (local) evolution possibility. After  $n$  timesteps each agent looks at his direct neighbour to the right and to the left, if the best neighbour has at least 1% more points than the agent, he gets the properties of this neighbour as described above. (To avoid replacement, when the best neighbour has nearly as many points as the agent, the difference must be at least 1%.)

In both methods we set the virtual points of the strategies and the real points of each player to 0 after an evolution step. Furthermore, we take the standard deviation as mutation probability so there are less mutations the more the players cooperate.

For our experiment which was carried out for 101 agents we set  $m = 5$  and  $s = 4$  as initial states. The strategy data were chosen randomly at the beginning and were not changed during evolution. The application of the global evolution method to the MGLI had no effect. The local method had more success (Fig. 5), since it takes into consideration that the success of an agent depends on his local interaction. Reason for this is that the memory of most agents decreases to  $m = 2$  and 3 (Fig. 6); we pointed out that the degree of cooperation is high for these values due to local feedback among the agents and the self-organization as a consequence. Moreover, most agents use  $s = 2$  or 3 strategies (Fig. 7); for these values of  $s$  the degree of cooperation is best. So the system moves towards this state by (local) evolution.

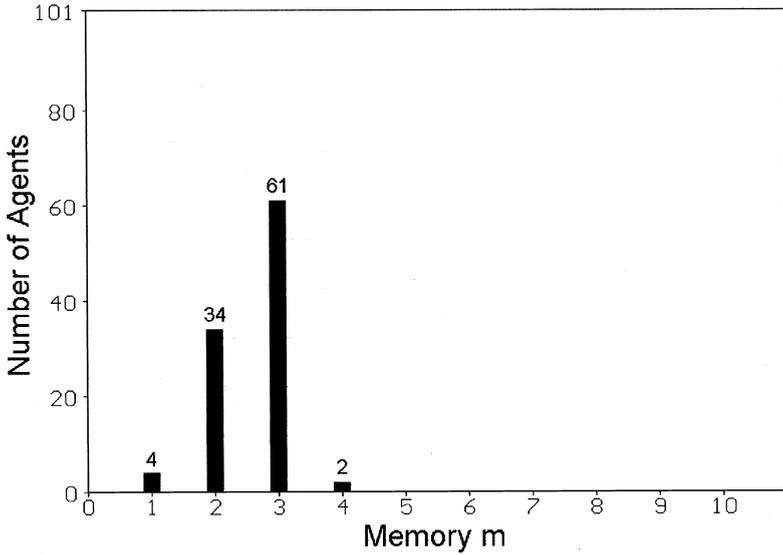


Fig. 6. Distribution of memory  $m$  after 450 evolution steps ( $N = 101$ ).

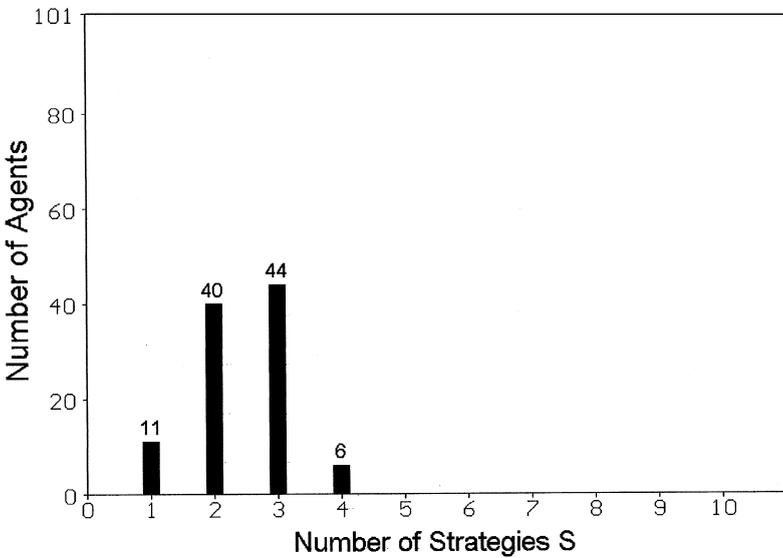


Fig. 7. Distribution of  $s$  after 450 evolution steps ( $N = 101$ ).

In conclusion, we presented an adaptive system that describes competitive situations in which local information is processed. The first result is that the best degree of cooperation is achieved for  $m = 3$  independent of the number of players. Secondly, we have shown that the good system performance bases on self-organization. At last, we optimized the system with the help of a new evolution method.

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### **For further reading**

The following references are also of interest to the reader: [3–7].

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