Formal Reasoning about Programs and Programming Languages

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Introduction

It is important for safety and security that programs are correct, especially in critical applications, *e.g.*, online banking

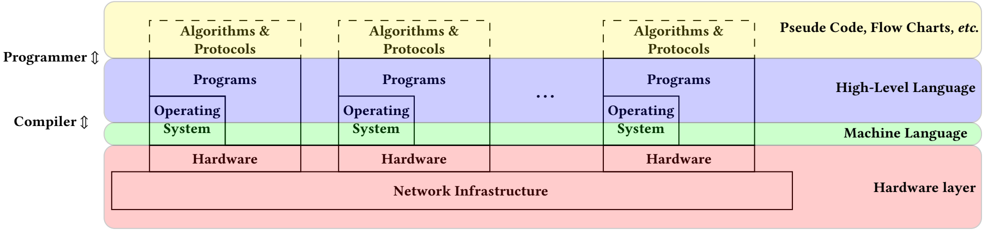
Aim: use formal and mathematical tools to prove correctness of software systems

Methodology:

- Make a mathematical model of the system
- Study the mathematical model using formal logic and mathematical tools

Introduction

There are many levels abstraction consider; all these levels can benefit from formal methods



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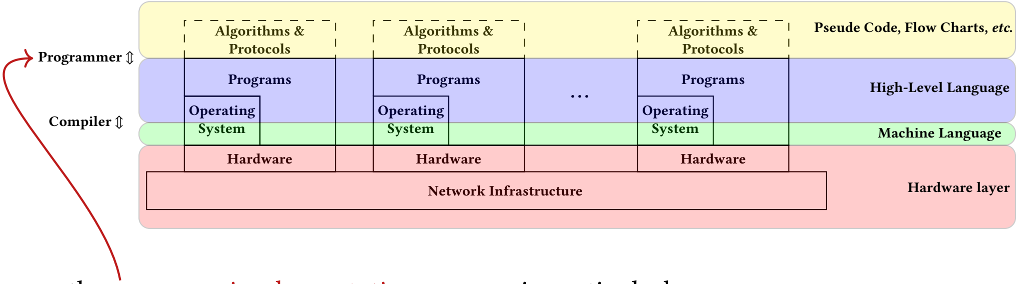

However, the program implementation process is particularly error-prone

- Modern PL features, *e.g.*, concurrency, are challenging to reason about (formally and informally)
- Bugs can introduce serious security vulnerabilities

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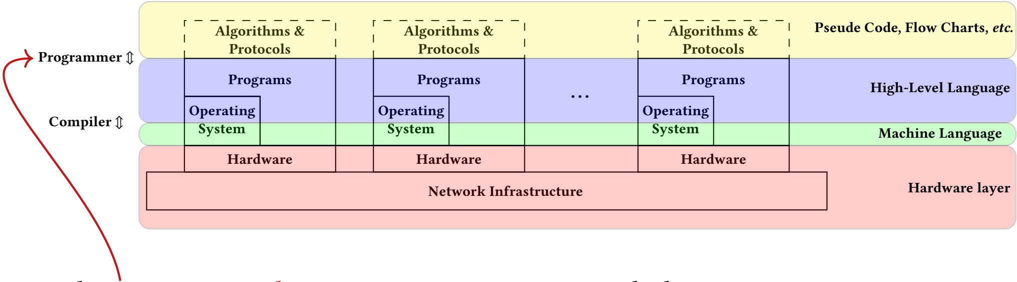

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- Bugs can introduce serious security vulnerabilities

My research focuses on formal verification of programs and programming languages

Example of Security Vulnerability in Implementation: Heartbleed

A bug in OpenSSL's implementation of the heartbeat feature:

- One side sends a heartbeat request message m together with a number l
- The other side sends the first l characters of m back to signal that it is alive

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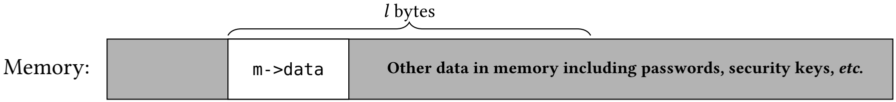
What happens if $l > length(m)$?



l bytes

Memory:

`m->data`

Other data in memory including passwords, security keys, *etc.*

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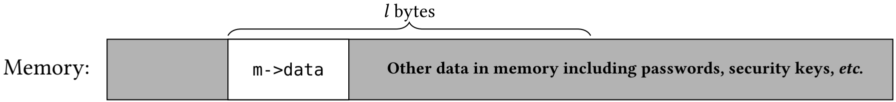

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A simplified version implementation:

```
void answer_heartbeat(SSL *req, unsigned int l){
  if(l > req->length){return;}
  send_reply(l, req->data);
}
```

The fix

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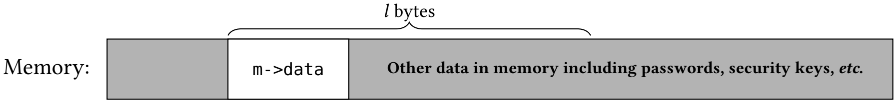
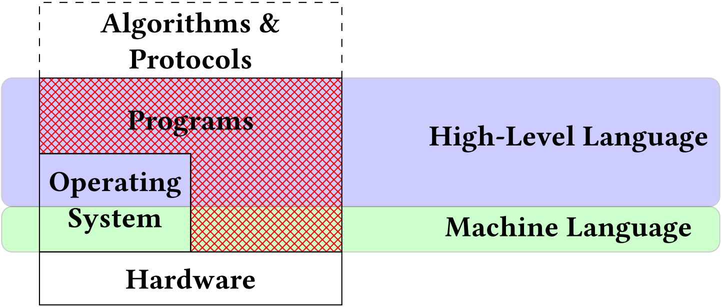


My Research Focus

In my research I focus on reasoning about programs and programming languages



For this purpose I use:

- Formal and mathematical logic: program logics (*the Iris framework*)
- Proof assistants (Coq) to mechanize results (machine-checked mathematical proofs)

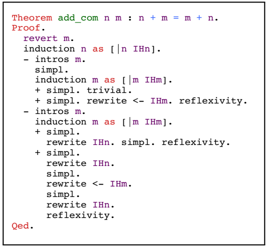
The Proof Assistant Coq

A proof assistant based on the Calculus of Inductive Constructions

- Coq is itself a programming language:
  - Curry-Howard correspondence (types are theorems programs are proofs)
  - It has an interesting meta-theory called *type theory*
- Proofs written and checked against foundational mathematical principles:
  - Coq only understands functions and the concept of induction

An example:

- Commutativity of addition for natural numbers (proven together with Pre-Talent track students)
- Proof automation can help but still this demonstrates the level of formality



```
Theorem add_com n m : n + m = m + n.
Proof.
  revert m.
  induction n as [|n IHn].
  - intros m.
    simpl.
    induction m as [|m IHm].
    + simpl. trivial.
    + simpl. rewrite <- IHm. reflexivity.
  - intros m.
    induction m as [|m IHm].
    + simpl.
      rewrite IHn. simpl. reflexivity.
    + simpl.
      rewrite IHn.
      simpl.
      rewrite <- IHm.
      simpl.
      rewrite IHn.
      reflexivity.
Qed.
```

Proof assistants are the highest standard of rigor for mathematical proofs

The Proof Assistant Coq

We use Coq to reason about state-of-the-art programs and programming languages:

- We define the precise mathematical model of program execution
- The level of details in these models necessitates the use of proof assistants and program logics
- We define program logics (*the Iris framework*) for these programs
- Use these to prove correctness of programs

This is the sate-of-the-art of research in program verification published at the top international conferences, *e.g.*, POPL, ESOP, ICFP

In this talk:

- How we achieve this
- Examples of recent work in this area

How is this feasible?

How can we reason about the state-of-the-art programs at this level of details?

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Modular Proofs!

Modular Proofs and Modular Reasoning about Programs

Curry-Howard correspondence: types are theorems programs are proofs

Software Engineering:

To develop and maintain large programs:

- Divide the program into modules: functions, classes, *etc.*
- Libraries: data structures, networking, GUI, *etc.*

⋮

Proof Engineering:

To develop and maintain large programs:

- Divide the proof into modules: theorems, lemmas, *etc.*
- Libraries: arithmetic, finite sets, *etc.*

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Hence, our program logic supports modular reasoning about realistic effectful programs:

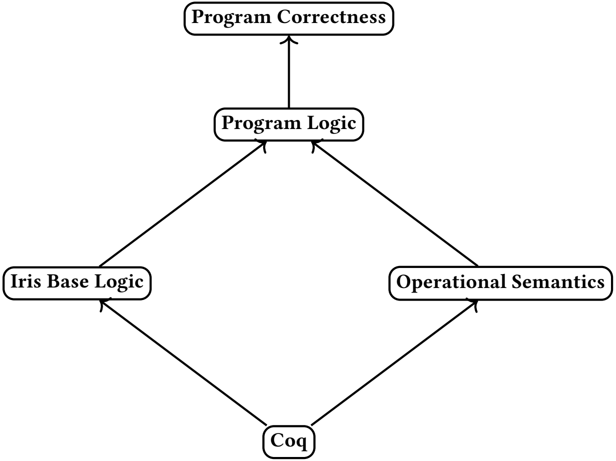
Modular reasoning with respect to program modules

- We reason about each module in isolation and compose those proofs

What is Iris?

A Modular Framework for Constring Program Logics

→ : Built on top of



Program Correctness

Program Logic

Iris Base Logic

Operational Semantics

Coq

What is Iris?

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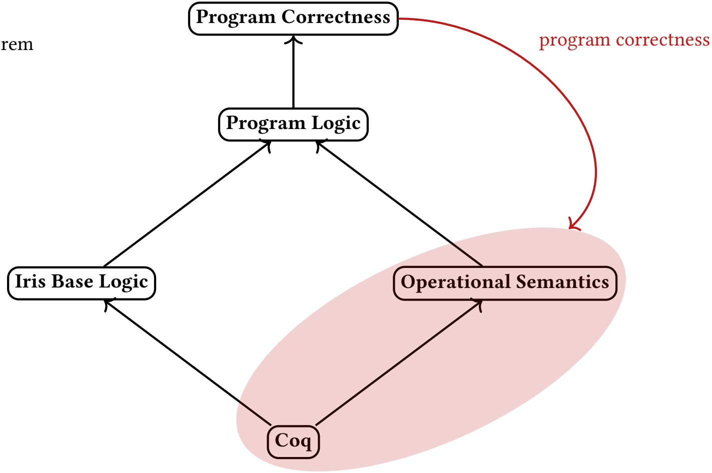
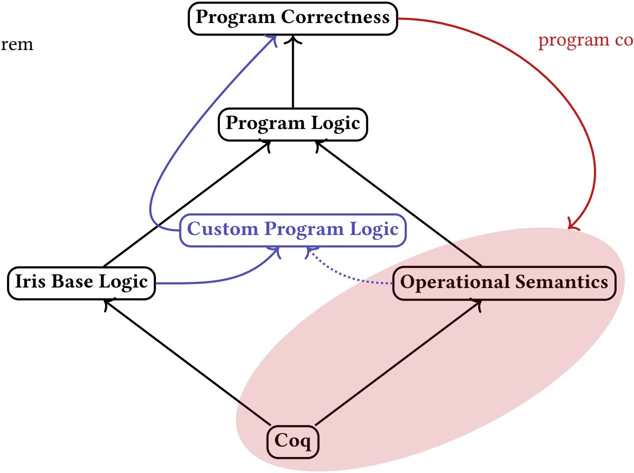
→ : Built on top of
→ : Iris's adequacy theorem



What is Iris?

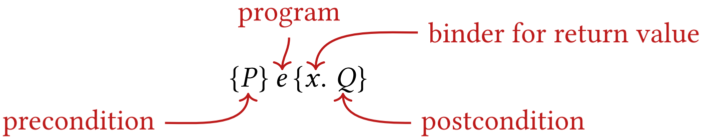
A Modular Framework for Constring Program Logics

→ : Built on top of
→ : Iris's adequacy theorem
□ : User-defined



Program Logic

A Hoare-style logic:



program

binder for return value

$$\{P\}\ e\ \{x.\ Q\}$$

precondition

postcondition

Examples:

$\{\text{True}\}$	$\{\text{isCounter}(c, n)\}$	$\{\text{isCounter}(c, n)\}$
$\quad\text{newCounter}\ ()$	$\quad\text{incr}\ c$	$\quad\text{read}\ c$
$\{x.\ \text{isCounter}(x, 0)\}$	$\{x.\ x = () * \text{isCounter}(c, n + 1)\}$	$\{x.\ x = n * \text{isCounter}(c, n)\}$

Preconditions, postconditions, and invariants[1] allow us to specify conditions for *other* modules.

---

[1] Not presented in this talk

Adequacy Theorem

Theorem (Adequacy)

If we prove

$$\{\mathit{True}\}\; e\; \{x.\ \phi(x)\}$$

in Iris*, then e is safe (e.g., no memory violations) and we have* $\phi(v)$ *for the computed value v.*

Note: this rules out Heartbleed

Example of Modular Reasoning: Function Calls

$\{\text{True}\}$

```
    let c = newCounter () in
    incr c;
    incr c;
    read c
```

$\{x.\ x = 2\}$

Example of Modular Reasoning: Function Calls

$\{\text{True}\}$

$\quad$ `let` $c = \text{newCounter}\ ()$ `in`

$\{\text{isCounter}(c, 0)\}$

$\quad \text{incr}\ c;$

$\{\text{isCounter}(c, 1)\}$

$\quad \text{incr}\ c;$

$\{\text{isCounter}(c, 2)\}$

$\quad \text{read}\ c$

$\{x.\ x = 2 * \text{isCounter}(c, 2)\}$

$\{x.\ x = 2\}$

No need to look at the implementations of newCounter, incr, or read, we just look at the specs.

Example of Modular Reasoning: Concurrency

The parallel composition of two programs e_1 and e_2 (written $e_1||e_2$):

- Runs e_1 and e_2 concurrently in two different threads
- Returns a pair of values (v_1, v_2) corresponding to e_1 and e_2 respectively
- The two programs may work on *shared memory*
- The semantics depends on the order of thread scheduling

The following Hoare-par rule enables modular reasoning about parallel composition:

$$\textsc{Hoare-par}\quad \frac{\{P_1\}\ e_1\ \{x.\ \phi_1(x)\} \qquad \{P_2\}\ e_2\ \{x.\ \phi_2(x)\}}{\{P_1 * P_2\}\ e_1||e_2\ \{x.\ x = (v_1, v_2) * \phi_1(v_1) * \phi_2(v_2)\}}$$

Let’s see a few examples of recent works in this area by my collaborators and I

Reasoning about Distributed Systems (ESOP'20)

Efficient implementations often use advanced features like node-local concurrency and higher-order memory

It is well known that reasoning about distributed programs is difficult

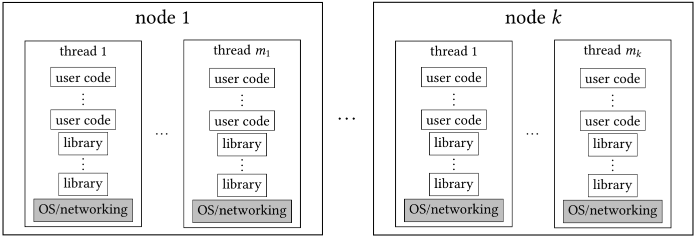
Traditionally, most works focus on verifying a high-level model of the system

We introduced Aneris: a program logic for modular verification of distributed systems

Reasoning about Distributed Systems (ESOP’20)

Modular reasoning about distributed systems:

- Horizontal modularity: nodes, and threads, are verified in isolation and the proofs are composed
- Vertical modularity: library code is verified separately and library clients are verified against the library specs



Reasoning about Causal Consistency (POPL'21)

According to the **CAP** theorem a distributed database cannot satisfy all of the following:

- Consistency: we always read the latest data
- Availability: every request is responded to
- Partition tolerance: system still functions if some of the replicas fail

Causally consistent databases:

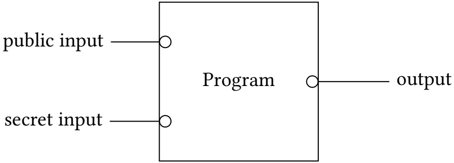
- Sacrifice consistency in favor of availability and partition tolerance
- Even if we don't receive the latest data, we never receive data out of causal order:
    - Example of violation of causal order: receiving response to a message in a group onversation before the message itself

We developed a novel specification (in Aneris) for causally consistent databases, which used to:

- Verify an implementation of a causally consistent database
- Prove correctness of clients of the database (similar to the counter example earlier)

Reasoning about Non-interference (POPL'21)

Non-interference (a security property): output does not leak the secret



A common approach: tracking the level of secrecy of data in the type system

- Each type is annotated with a level, *e.g.*, bool^H, bool^L
- The type system ensures that no data flows from high inputs to low outputs
- Non-interference (termination in-sensitive) in terms of types:

$$\text{TINI:} \qquad \text{for any function } f : \mathsf{bool}^H \to \mathsf{bool}^L \text{ we have } f\ \texttt{true} \approx f\ \texttt{false}$$

Reasoning about Non-interference (POPL'21)

We proved termination-insensitive non-interference:

- For the most advanced type system to date
- Required a novel program logic
- We can reason about both well-typed code and ill-typed code

We do this as follows:

- We define a program logic for termination-insensitive reasoning
- We use it to express non-interference properties of programs of each type such that:

$$\llbracket \mathrm{bool}^H \longrightarrow \mathrm{bool}^L \rrbracket(f) \ \text{ implies } \ f\ \texttt{true} \approx f\ \texttt{false}$$

- We prove that any well-typed program $e : \tau$ we have $\llbracket \tau \rrbracket(e)$
- Hence, the TINI property holds

Other Examples

There are other interesting examples that I did not cover in this talk, *e.g.*,

- Reasoning about machine code (assembly) of so-called capability machines (POPL'21)
- Studying gradual type systems (POPL'21)
- Reasoning about atomicity of advanced concurrent programs (POPL'20)
- Reasoing about continuations (ICFP'19)
- Properties of the ST-monad (POPL'18)
- *etc.*

If you are interested, you can find the full list of my publications at:
`https://cs.au.dk/~timany/publications`

Thanks