manual for the simulation program

DisCo

by Søren Johansen & Bent Nielsen
Institute of Mathematical Statistics
University of Copenhagen

2 June 1993

Manual version 1.0 and DisCo version 1.0

Introduction.

In cointegration analysis of $I(1)$ models the central point is the number of cointegrating relations. Unfortunately the asymptotic distribution of the likelihood ratio test for the cointegration rank is not χ^2, but generalised Dickey-Fuller distributed, with a structure depending on the problem. Sometimes with and sometimes without nuisance parameters. So in almost any situation one needs a new asymptotic table for the test of cointegration. Thus the computer program DisCo was made as a general tool to handle these problems.

With DisCo it is possible to simulate the asymptotic distribution of the likelihood ratio test for cointegration in $I(1)$ models where the statistical analysis is done by a reduced rank regression of variables corrected for the drift. DisCo is limited to consider models with constant and linear drift and the presence of intervention dummies.

1. Statistical Analysis.

1.1. The model and the cointegration hypothesis.

Consider the vector autoregressive (VAR) model for the p-dimensional process written as a reduced form error correction model

$$\Delta X_t = \Pi X_{t-1} + \sum_{i=1}^{k} \Gamma_i \Delta X_{t-i} + \mu_t + \varepsilon_t, \qquad t = 1, \dots, T \tag{1.1.1}$$

with $\varepsilon_1, \dots, \varepsilon_T$ i.i.d. Gaussian. Assume the drift term μ_t is decomposed as

$$\mu_t = \sum_{j=1}^{n} \mu_j k_j(t)$$

where the drift functions $k_j(t)$ are some deterministic functions of t, chosen such that the matrix

$$\begin{pmatrix} k_1(1) & \cdots & k_1(T) \\ \vdots & & \vdots \\ k_n(1) & \cdots & k_n(T) \end{pmatrix} \tag{1.1.2}$$

is of full rank, n.

The hypothesis of at most r cointegrating relations is the hypothesis of reduced rank of the $p \times p$ matrix Π, that is

$$H_n(r): \qquad \Pi = \alpha\beta' \qquad \text{with } \alpha, \beta \in \mathbf{R}^{p\times r}. \tag{1.1.3}$$

The hypothesis $H_n(r)$ is to be tested against the alternative $H_n(p)$.

Example 1. The model with $n = 2$ and the drift functions $k_1(t) = 1$ and $k_2(t) = t$ gives a quadratic trend in the process X_t. It is treated by Johansen (1992b, Section 2). □

1.2. Structure of the drift functions k.

In General the drift functions, k_i, can be anything. The DisCo program, however, is limited to consider functions with a piecewise linear graph.

The asymptotic analysis of the likelihood ratio test is performed by the theory of weak convergence on the space $D[0,1]$. So the process is considered on the interval $[0,1]$ instead of $\{0,1,\ldots,T\}$ by the operation on $t \in [0,1]$ which finds the integer part of tT.

Thus the input to DisCo is limited to functions $k : [0,1] \mapsto [-1,1]$, such that the graph of k is composed of line segments in the space $[0,1] \times [-1,1]$.

An example is $k(t) = t1_{[0,\frac{1}{2})} + \frac{1}{2}1_{[\frac{1}{2},1]}$,

but not $k(t) = t^2 1_{[0,1]}$.

1.3. Restrictions on the model.

Let $A(z)$ denote the characteristic polynomial for the process given by (1.1.1).

$$A(z) = 1 - z - \Pi z - \sum_{i=1}^{k} \Gamma_i z^i (1-z). \tag{1.3.1}$$

Assume that the $I(1)$ condition is fulfilled. That is

$$|A(z)| = 0 \quad \Rightarrow \quad |z| > 1 \quad \text{or} \quad z = 1. \tag{1.3.2}$$

When $rank(\Pi) < p$ assume further that the matrix

$$\alpha'_{\perp}\Psi\beta_{\perp}, \tag{1.3.3}$$

where

$$\Psi = -\frac{\partial}{\partial z}A(z)\bigg|_{z=1} = I + \Pi - \sum_{i=1}^{k}\Gamma_i \tag{1.3.4}$$

is of full rank.

Application of Granger's Theorem, see e.g. Johansen (1991, Th. 4.1) shows that the process (1.1.1) under $H_n(r)$ can be represented as

$$X_t = X_0 + C\sum_{j=1}^{t}\varepsilon_j + \sum_{i=1}^{n}C\mu_i\sum_{j=1}^{t}k_i(j) + Y_t - Y_0. \tag{1.3.5}$$

Here

$$Y_t = C^*(L)\left(\varepsilon_t + \sum_{i=1}^{n}k_i(t)\right) \tag{1.3.6}$$

is some trend stationary process, $C = \beta_{\perp}(\alpha'_{\perp}\Psi\beta_{\perp})^{-1}\alpha'_{\perp}$ and $C^*(z)$ is a power series found by defining

$$\tilde{A}(z) = (\alpha, \alpha_{\perp})'A(z)(\beta, \beta_{\perp}(1-z)^{-1}) \tag{1.3.7}$$

and let $C^*(z) = (\tilde{A}(z)^{-1} - \tilde{A}(1\diamond)^{-1})/(1-z)$.

This result implies that the asymptotic analysis has to distinguish between the situations where $^{1)}$ μ_i is zero or not, $^{2)}$ $C\mu_i$ or rather $\alpha'_{\perp}\mu_i$ is zero or not, $^{3)}$ $\alpha'_{\perp}\mu_i$ equals $\alpha'_{\perp}\mu_j$ for $i \neq j$ or not and where $^{4)}$ μ_i equals μ_j for $i \neq j$ or not.

Introduce the notation $H_{m,n-m}(r)$ such that the model has $m + n - m = n$ drift functions. The m first of these correspond to unrestricted parameters, while the other $n - m$ of these has $\alpha_{\perp}$-restricted parameters

Now, assume for simplicity that the number of drift functions in the model is $n = 1$. Then the drift has the structure

$$\mu_t = \mu_1 k_1(t) = (\alpha\beta_1' + \alpha_{\perp}\gamma_1')k_1(t). \tag{1.3.8}$$

Here it is of interest to study the hypotheses

$$H_{1,0}(r): \qquad \beta_1 \text{ unrestricted}, \qquad \gamma_1 \text{ unrestricted} \tag{1.3.9}$$

and

$$H_{0,1}(r): \qquad \beta_1 \text{ unrestricted}, \qquad \gamma_1 = 0. \tag{1.3.10}$$

Of course both β_1 and γ_1 could be zero, that is

$$H_{0,0}(r): \qquad \beta_1 = 0, \quad \gamma_1 = 0. \tag{1.3.11}$$

But in this case $n = 0$!

When $n = 2$ the drift has the structure

$$\mu_t = (\alpha\beta_1' + \alpha_\perp\gamma_1')k_1(t) + (\alpha\beta_2' + \alpha_\perp\gamma_2')k_2(t), \tag{1.3.12}$$

Then the following hypotheses are of interest.

$$H_{2,0}(r): \qquad \beta_1, \beta_2 \text{ unrestricted}, \qquad \gamma_1, \gamma_2 \text{ unrestricted}\ . \tag{1.3.13}$$

$$H_{1,1}(r): \qquad \beta_1, \beta_2 \text{ unrestricted}, \qquad \gamma_1 \text{ unrestricted}, \gamma_2 = 0. \tag{1.3.14}$$

$$H_{0,2}(r): \qquad \beta_1, \beta_2 \text{ unrestricted}, \qquad \gamma_1 = \gamma_2 = 0. \tag{1.3.15}$$

If both $\gamma_2 = 0$ and $\beta_2 = 0$ then the model reduces to (1.3.8).

1.4. The statistical analysis and the likelihood ratio test.

The statistical analysis of the model will depend on the structure of the deterministic drift μ_t. For all the models reduced rank regression is applied, see Anderson (1951), Johansen (1988) or Johansen & Juselius (1990). As will be shown below the models can all reformulated as

$$\Delta X_t = \alpha\beta^{*\prime}X_t^* + \Gamma U_t + \varepsilon_t \tag{1.4.1}$$

for suitable definitions of β^*, X_t^*, Γ and U_t.

First ΔX_t and X_t^* are corrected U_t, which gives the residuals $R_0(t)$ and $R_1(t)$. This regression leads to a new equation

$$R_0(t) = \alpha\beta^{*\prime}R_1(t) + \varepsilon_t \tag{1.4.2}$$

which is analysed by a reduced rank regression of $R_0(t)$ on $R_1(t)$. First, for fixed β^*, α is estimated by simple regression. Further β^* is estimated by defining $S_{ij} = \frac{1}{T}\sum_{i=1}^{T} R_i(t)R_j'(t)$ and considering the eigenvalue problem

$$\left|\lambda S_{11} - S_{10}S_{00}^{-1}S_{01}\right| = 0 \tag{1.4.3}$$

which has the solutions $1 > \lambda_1 > \cdots > \lambda_p > 0$. The parameter β^* is estimated from the space spanned by the eigenvalues corresponding to the r largest eigenvalues. Further the likelihood ratio test of $H(r)$ against $H(p)$ has the form

$$-2\log Q(H(r)|H(p)) = -T \sum_{i=r+1}^{p} \log(1-\lambda_i). \tag{1.4.4}$$

As illustration consider the hypotheses above, where $n = 2$.

The hypothesis $H_{2,0}(r)$: $\mu_t = \mu_1 k_1(t) + \mu_2 k_2(t)$.
Here $\beta^* = \beta$ and $X_t^* = X_{t-1}$. Further $U_t = (\Delta X_{t-1}', \ldots, \Delta X_{t-k}', k_1(t), k_2(t))'$.

The hypothesis $H_{1,1}(r)$: $\mu_t = \mu_1 k_1(t) + \alpha\beta_2' k_2(t)$.
In this case the model is rewritten as

$$\begin{aligned}\Delta X_t &= \alpha\beta' X_{t-1} + \sum_{i=1}^{k} \Gamma_i \Delta X_{t-i} + \mu_1 k_1(t) + \alpha\beta_2' k_2(t) + \varepsilon_t \\ &= \alpha(\beta', \beta_2')'(X_{t-1}', k_2(t))' + \sum_{i=1}^{k} \Gamma_i \Delta X_{t-i} + \mu_1 k_1(t) + \varepsilon_t.\end{aligned}$$

Thus the residuals $R_0(t)$ and $R_1(t)$ are found by regressing ΔX_t and $X_{t-1}^* = (X_{t-1}', k_2(t))'$ on $U_t =$
$(\Delta X_{t-1}', \ldots, \Delta X_{t-k}', k_1(t))'$ and $\beta^* = (\beta, \beta_2)$ is estimated instead of β.

The hypothesis $H_{0,2}(r)$: $\mu_t = \alpha\beta_1' k_1(t) + \alpha\beta_2' k_2(t)$.
As above the model is rewritten by letting $X_{t-1}^* = (X_{t-1}', k_1(t), k_2(t))$. and $\beta^* = (\beta, \beta_1, \beta_2)$ Thus $U_t = (\Delta X_{t-1}', \ldots, \Delta X_{t-k}')'$.

1.5. Asymptotic analysis.

Assume again that $n = 0, 1$ or 2. Consider drift functions $k_1, \ldots, k_n$ such each function $k_i(t)$ has a graph which is piecewise linear, cf. Section 1.2. Then the following asymptotic result is obtained.

Theorem. *Assume that the drift functions $k_i[Tt]$ converges to $g_i(t)$ in $D[0,1]$ and further that the trend term $K_i([Tt]) = \sum_{s=1}^{[tT]} \kappa_i(s)$ should be normalized by $T^{-\eta_i}$ so that*

$$T^{-\eta_i} K_i([Tt]) \to \int_0^t g_i(s)\, ds \overset{\text{def}}{=} G_i(t). \tag{1.5.1}$$

Assume further that $\eta_1, \eta_2 > \frac{1}{2}$. *Then*

$$-2\log Q(H(r)|H(p)) \xrightarrow{w} tr\left\{\int_0^1 dB\,F'\left(\int_0^1 F\,F'\,du\right)^{-1}\int_0^1 F\,(dB)'\right\}, \quad (1.5.2)$$

where F *depends on the structure of the hypothesis which is tested.*

The hypothesis $H_{0,0}(r)$**:** $\mu_t = 0$.

$$F = B. \quad (1.5.3)$$

The hypothesis $H_{1,0}(r)$**:** $\mu_t = \mu_1 k_1(t)$.
Assume that $\alpha'_\perp\mu_1 \neq 0$ *corresponding to* $(H_{1,0}(r)\setminus H_{0,1}(r))$. *With this assumption* F *is equal to*

$$(G_1(u), B_2(u), \ldots, B_{p-r}(u))' \text{ corrected for } (g_1(u)). \quad (1.5.4)$$

The hypothesis $H_{0,1}(r)$**:** $\mu_t = \alpha\beta_1' k_1(t)$
Assume $rank(\alpha'\mu_1) = 1$ $(H_{0,1}(r)\setminus H_{0,0}(r))$. F *is equal to*

$$(B_1(u), \ldots, B_{p-r}(u), k_1(u))'. \quad (1.5.5)$$

The hypothesis $H_{2,0}(r)$**:** $\mu_t = \mu_1 k_1(t) + \mu_2 k_2(t)$
There are two situations.

1) $G_1(u) = \int_0^u g_1(s)\,ds$ *is proportional to* $g_2(u)$. *Assume that the rank of* $(\alpha'_\perp\mu_1, \alpha'_\perp\mu_2)$ *is larger than 0. Then* F *is*

$$(G_2(u), B_2(u), \ldots, B_{p-r}(u))' \text{ corrected for } (g_1(u), g_2(u)). \quad (1.5.6)$$

2) $G_1(u) = \int_0^u g_1(s)\,ds$ *is not proportional to* $g_2(u)$. *Assume that the rank of* $(\alpha'_\perp\mu_1, \alpha'_\perp\mu_2)$ *is 2. This assumption require that* $p - r \geq 2$. *Then* F *is*

$$(G_1(u), G_2(u), B_3(u), \ldots, B_{p-r}(u))' \text{ corrected for } (g_1(u), g_2(u)). \quad (1.5.7)$$

The hypothesis $H_{1,1}(r)$**:** $\mu_t = \mu_1 k_1(t) + \alpha\beta_2' k_2(t)$.
Assume that $\alpha'_\perp\mu_1 \neq 0$ *corresponding to* $(H_{1,1}(r)\setminus H_{1,0}(r))$. *With this assumption* F *is equal to*

$$(G_1(u), B_2(u), \ldots, B_{p-r}(u), g_2(u))' \text{ corrected for } (g_1(u)). \quad (1.5.8)$$

The hypothesis $H_{0,2}(r)$**:** $\mu_t = \alpha\beta_1' k_1(t) + \alpha\beta_2' k_2(t)$.
Then F *is equal to*

$$(B_1(u), \ldots, B_{p-r}(u), g_1(u), g_2(u))'. \tag{1.5.9}$$

Sketch of the proof. The proof is obtained by a generalisation of the proof of Johansen (1991, Th. 2.1). The first step is to notice that the process by Granger's Theorem could be rewritten as a process on $[0,1]$, see (1.3.5)–(1.3.6).

$$X_{[Tu]} = X_0 + C\sum_{i=1}^{[Tu]} \varepsilon_i + \sum_{j=1}^{n} C\mu_j K_j([Tu]) + Y_{[Tu]} - Y_0 \tag{1.5.10}$$

where

$$Y_{[Tu]} = C^*(L)\left(\varepsilon_{[Tt]} + \sum_{i=1}^{n} k_i([Tt])\right) \tag{1.5.11}$$

and $C = \beta_\perp(\alpha_\perp'\Psi\beta_\perp)^{-1}\alpha_\perp$. By the Lemma below it is seen that the process $Y_{[Tu]}$ is trend stationary and of order $O_p(1)$.

The next step is to study the asymptotic behaviour of the process $X_{[Tu]}$ in different directions. The first distinction is the orthogonal decomposition

$$(\beta, \beta_\perp).$$

Here $\beta' X_{[Tu]} = \beta'(Y_{[Tu]} - Y_0 + X_0)$ are the trend stationary directions, cf. (1.5.10) above, and $\beta_\perp' X_t$ are the non stationary directions of the process. Since the random walk has to be normalised by $\sqrt{T}$ and $K_i(t)$ by T^{η_i} it is convenient to decompose $\beta_\perp$ in the directions, τ, where the deterministic trend dominates and the orthogonal directions, ξ, without deterministic trend.

Note that τ span a subspace of $\mathbf{R}^p$ with rank two for the hypothesis $H_{2,0}$ with $G_1(u)$ nonproportional to $g_2(u)$ (1.5.7). A a subspace of rank one for the hypotheses $H_{1,0}$ (1.5.4), $H_{2,0}$ with $G_1(u)$ proportional to $g_2(u)$ (1.5.6) and $H_{1,1}$ (1.5.8). And a subspace of rank zero for the hypotheses $H_{0,0}$ (1.5.3), $H_{0,1}$ (1.5.5) and $H_{0,2}$ (1.5.9).

Thus the process is studied in the normalised directions

$$\left(\beta, \frac{\tau_1}{T^{\eta_1}}, \frac{\tau_2}{T^{\eta_2}}, \frac{\xi}{\sqrt{T}}\right). \tag{1.5.12}$$

The first step of the statistical analysis is a regression analysis. Thus the residuals from this, R_t, should be studied rather than X_t. In some situations the residuals of the cumulated deterministic trend is zero in the direction τ_2, say; see e.g. Example 1 below. Then the τ_2 direction will give (the residuals of) a Brownian motion by a normalisation by $\sqrt{T}$ rather than T^{η_2}. In this case the decomposition (1.5.2) should be reformulated.

Extended with these considerations the proof of Johansen (1991, Th. 2.1) could be followed. $\square$

Lemma. *Assume the* $I(1)$ *conditions (1.1.4) – (1.1.5) are fulfilled. Let the drift functions*
$k_1([Tu]),\ldots,k_n([Tu])$ *be functions defined on* $[0,1]$ *with a piecewise linear graph. Define the residuals* $R(Y_{[Tu]})$ *of*

$$Y_{[Tu]} = C^*(L)\left(\varepsilon_t + \sum_{i=1}^{n} k_i([Tu])\right) \tag{1.5.13}$$

regressed on $k_1,\ldots,k_n$*. Then*

$$R(Y_{[Tu]}) = O_p(1). \tag{1.5.14}$$

Proof. Since the characteristic polynomial of the process fulfil $|A(z)| = 0 \Rightarrow |z| > 1$ or $z = 1$, (1.3.2), the polynomial $\tilde{A}(z) = (\alpha,\alpha_\perp)'A(z)(\beta,\beta_\perp(1-z)^{-1})$ (1.3.7) satisfy $|\tilde{A}(z)| = 0 \Rightarrow |z| > 1$. This implies the existence of a $\delta > 0$ and a power series $C^*(z) = (\tilde{A}(z)^{-1} - \tilde{A}(1)^{-1})/(1-z)$ which is convergent for $|z| < 1+\delta$. Further $C^*(z)$ has the representation

$$C^*(z) = \sum_{j=0}^{\infty} c_j z^j \qquad \text{for} \qquad |z| < 1+\delta. \tag{1.5.15}$$

Let ε_t be a stationary process. Then $C^*(L)\varepsilon_t$ is stationary and $O_p(1)$.

Let $k(t) = (at+b)1_{[T_1,T_2]}$, where $T_1, T_2 \to \infty$ as $T \to \infty$. Then $C^*(L)k(t)$ is a linear function since

$$\begin{aligned} C^*(L)k(t) &= \sum_{j=1}^{\infty} c_j(a(t-j)+b)1_{[0,t-T_1]}(j) \\ &= (at+b)\sum_{j=0}^{\infty} c_j 1_{[T_1,T_2]} - a\sum_{j=0}^{\infty} j c_j 1_{[T_1,T_2]} + O_p(t) \sum_{j=t-(T_1-1)}^{\infty} j c_j. \end{aligned} \tag{1.5.16}$$

Regression of this term on $k(t)$ gives the residuals

$$0 + O_p(\frac{1}{T}) + O_p(1) \sum_{j=t+1}^{\infty} j c_j. \tag{1.5.17}$$

Generalisation of this argument to functions defined on $[0,1]$ and with a piecewise linear graph finish the proof. $\square$

Remark that the error terms $O_p(1)\sum_{j=t+1}^{\infty}$ implies that the intervals k is defined on must include a certain number of observations to avoid more small sample problems than is strictly necessary.

1.6. Examples.

Example 1.

Again the model permitting quadratic trend is studied. Then $n = 2$ and the drift functions are $k_1(t) = 1$ and $k_2(t) = t$. The model is

$$\Delta X_t = \Pi X_{t-1} + \sum_{i=1}^{k} \Gamma_i \Delta X_{t-i} \mu_1 + \mu_2 t + \varepsilon_t. \tag{1.6.1}$$

Note, that $\int_0^u g_1(u)\,ds \propto g_2(u)$ and that the drift functions fulfil the assumption (1.5.6) of the Theorem.

$***$ $\quad H_{2,0}(r) : \mu_t = \mu_1 k_1(t) + \mu_2 k_2(t)$.
Assume that $rank(\alpha'_\perp \mu_1, \alpha'_\perp \mu_2) \geq 1$. Then the structure of F is

$$F = \begin{pmatrix} u^2 - u - \frac{1}{6} \\ B_2(u) - \bar{B}_2 - (u - \frac{1}{2})\tilde{A}_2 \\ \vdots \\ B_{p-r}(u) - \bar{B}_{p-r} - (u - \frac{1}{2})\tilde{A}_{p-r} \end{pmatrix}$$

where $\tilde{A}_i = \int_0^1 (s - \frac{1}{2}) B_i(s)\,ds / \int_0^1 (s - \frac{1}{2})^2\,ds$ and $\bar{B}_i = \int_0^1 B_i(s)\,ds$, since the deterministic part of F is found by regressing

$$\begin{pmatrix} \frac{1}{T^2}\sum_1^{[Tu]} i \\ \frac{1}{T}\sum_1^{[Tu]} 1 \\ 0 \\ \vdots \\ 0 \end{pmatrix} \text{ on } (1,u) \text{ which gives the residuals } \begin{pmatrix} \text{const } (u^2 - u - \frac{1}{6}) \\ 0 \\ 0 \\ \vdots \\ 0 \end{pmatrix}.$$

See also Johansen (1992b). $\square$

Example 2.

Here a model with linear trend and an intervention dummy in the middle of the time interval is studied. Then $n=2$, $k_1(t)=1_{[0,\frac{T}{2})}(t)$ and $k_2(t)=1_{[\frac{T}{2},T]}(t)$. The model is

$$\Delta X_t = \Pi X_{t-1} + \sum_{i=1}^{k} \Gamma_i \Delta X_{t-i} + \mu_1 1_{[0,\frac{T}{2})}(t) + \mu_2 1_{[\frac{T}{2},T]}(t) + \varepsilon_t. \qquad (1.6.2)$$

Note, that the drift functions fulfil the assumption (1.5.7) of the Theorem.

$***$ $\quad H_{2,0}(r): \mu_t = \mu_1 k_1(t) + \mu_2 k_2(t).$
Assume that $rank(\alpha'_\perp \mu_1, \alpha'_\perp \mu_2) = 2$. In this case F is found to be

$$F = \begin{pmatrix} (u-\frac{1}{4})1_{[0,\frac{1}{2})}(u) \\ (u-\frac{3}{4})1_{[\frac{1}{2},1]}(u) \\ B_3(u) - \tilde{A}_3(u) \\ \vdots \\ B_{p-r}(u) - \tilde{A}_{p-r}(u) \end{pmatrix}$$

where $\tilde{A}_i(u) = 2\int_0^{\frac{1}{2}} B_i\, ds 1_{[0,\frac{1}{2})}(u) + 2\int_{\frac{1}{2}}^1 B_i\, dt 1_{[\frac{1}{2},1]}(u)$.

When the hypothesis $H_{2,0}(r)$ and $rank(\alpha'_\perp \mu_1, \alpha'_\perp \mu_2) = 1$ the asymptotic distribution of the likelihood ratio test is not given by this F. Thus if the rank seems to be 1 by an inspection of the estimates the following hypothesis should be investigated.

$***$ $\quad H^{\delta}_{2,0}(r): \mu_t = \mu_1 k_1(t) + \mu_2 k_2(t)$ and $\gamma_2 = \delta\gamma_1$, $\delta \neq 0$.
In this model $rank(\alpha'_\perp \mu_1, \alpha'_\perp \mu_2) = 1$. The model involve two parameter matrices of reduced rank which can be estimated by a switching algorithm, cf. Johansen & Juselius (1992, Section 5.3). Since the statistical analysis involve two reduced rank problem the asymptotic distribution of the likelihood ratio test can not be simulated by DisCo.

$***$ $\quad H_{1,1}(r): \mu_t = \mu_1 k_1(t) + \alpha\beta'_2 k_2(t).$
Assume $\alpha'_\perp \mu_1 \neq 0$. Then F is found to be

$$F = \begin{pmatrix} (u-\frac{1}{4})1_{[0,\frac{1}{2})}(u) \\ B_2(u) - \bar{B}_2(u) \\ \vdots \\ B_{p-r}(u) - \bar{B}_{p-r}(u) \\ 1_{[\frac{1}{2},1]}(u) \end{pmatrix}$$

where $\bar{B}_i(u) = 2\int_0^{\frac{1}{2}} B_i\, dt 1_{[0,\frac{1}{2})}(u)$.

$\square$

2. Hypothesis hierarchy.

In Section 1 a number of hypotheses were suggested and discussed. One has to be very careful and not test hypothesis without any interpretation and meaning. Further some of the test can be avoided by inspecting the estimates.

Example 1. An hypothesis hierarchy of the model permitting quadratic trend is suggested by Johansen (1992b). □

Example 2. The model is

$$\Delta X_t = \Pi X_{t-1} + \sum_{i=1}^{k} \Gamma_i \Delta X_{t-i} \mu_1 1_{[0,\frac{T}{2})}(t) + \mu_2 1_{[\frac{T}{2},T]}(t) + \varepsilon_t. \tag{2.1}$$

A number of interesting hypotheses could be suggested.

$$\begin{aligned} H_{2,0}(r): &\quad \beta_1, \beta_2 \text{ unrestricted}, &\quad \gamma_1, \gamma_2 \text{ unrestricted} . \\ H_{2,0}^{\gamma_1=\gamma_2}(r): &\quad \beta_1, \beta_2 \text{ unrestricted}, &\quad \gamma_1 = \gamma_2. \end{aligned}$$

Under $H_{2,0}^{\delta_0}(r)$ the model can be reformulated such that

$$\mu_t = (\alpha\tilde{\beta}_1' + \alpha_\perp\tilde{\gamma}_1')(k_1(t) + k_2) + \alpha\tilde{\beta}_2' k_2(t).$$

By this hypothesis $H_{2,0}^{\gamma_1=\gamma_2}(r)$ is equal to

$$\tilde{H}_{1,1}(r): \quad \tilde{\beta}_1, \tilde{\gamma}_1 \text{ unrestricted}, \qquad \tilde{\beta}_2 \text{ unrestricted}.$$

with the subhypotheses

$$\begin{aligned} \tilde{H}_{1,0}(r): &\quad \tilde{\beta}_1, \tilde{\gamma}_1 \text{ unrestricted}, &\quad \tilde{\beta}_2 = 0. \\ \tilde{H}_{0,1}(r): &\quad \tilde{\beta}_1 \text{ unrestricted}, \tilde{\gamma}_1 = 0, &\quad \tilde{\beta}_2 = 0. \\ H_{0,0}(r) = \tilde{H}_{0,0}(r): &\quad \tilde{\beta}_1 = \tilde{\gamma}_1 = 0, &\quad \tilde{\beta}_2 = 0. \end{aligned}$$

By these hypotheses the following hierarchy is etablished

$$H_{2,0}(r) \quad \supset \quad \tilde{H}_{1,1}(r) \quad \supset \quad \tilde{H}_{1,0}(r) \quad \supset \quad \tilde{H}_{0,1}(r) \quad \supset \quad H_{0,0}(r)$$

Notice, that the asymptotic distribution of the test of $H_{2,0}(r)$ against $H_{2,0}(p)$ needs the asumption that $rank(\alpha'_\perp\mu_1, \alpha'_\perp\mu_2) = 2$. Thus if the rank seems to be 1 by an inspection of the estimates this assumption should be investigated, cf. Example 2 of Subsection 1.6. □

Anyway when a hierarchy system has been etablished it is recommended to follow the testing ideas of Pantula (1989). Pantula described the idea in connection with univariate analysis, while Johansen (1992a) introduced the idea in the cointegration analysis.

3. Simulation principles.

The limit distribution of $-2\log Q(H_{m,n-m}(r)|H_{m,n-m}(p))$ does not depend on the parameters $\Gamma_1,\ldots,\Gamma_k$ and $\gamma_1,\ldots,\gamma_n$ Further it does only depend of the parameters $\alpha,\beta,\ \beta_1,\ldots,\beta_m$ through the rank and of the dimensions p and r only through the difference $p-r$. So the same limit distribution would be obtained by the $p-r$ dimensional model

$$H_{m,n-n}(p-r): \qquad \Delta Y_t = \Pi Y_{t-1} + \sum_{i=1}^{m} \mathbf{e}_i k_i(t) + \sum_{i=m+1}^{n} \Pi k_i(t) + \varepsilon_t.$$

and test the hypothesis

$$H_{m,n-n}(0): \qquad \Pi = 0.$$

Here $\mathbf{e}_1,\ldots \mathrm{e}_p$ are the coloumns of the unit matrix. This model corresponds to restrict the parameters of the drift functions $k_{m+1},\ldots,k_n$ to be in the α-space and the parameters of $k_1,\ldots,k_m$ to be unrestricted.

Under the null hypothesis the process Y_t fulfil the equation

$$\Delta Y_t = \begin{pmatrix} \varepsilon_1(t) & + & k_1(t) \\ \vdots & & \vdots \\ \varepsilon_m(t) & + & k_m(t) \\ \varepsilon_{m+1}(t) & & \\ \vdots & & \\ \varepsilon_{p-r}(t) & & \end{pmatrix}. \tag{3.1}$$

The process Y_t is simulated by generating data from this equation. Together with the drift functions with $\alpha_\perp$-restricted parameters the vector process

$$Y_t^* = \begin{pmatrix} \sum_{i=1}^t \Delta Y_i \\ k_{m+1}(t) \\ \vdots \\ k_n(t) \end{pmatrix} = \begin{pmatrix} \sum_{i=1}^t \varepsilon_1(i) & + & \sum_{i=1}^t k_1(i) \\ \vdots & & \vdots \\ \sum_{i=1}^t \varepsilon_m(i) & + & \sum_{i=1}^t k_m(i) \\ \sum_{i=1}^t \varepsilon_{m+1}(i) & & \\ \vdots & & \\ \sum_{i=1}^t \varepsilon_{p-r}(i) & & \\ & k_{m+1}(t) & \\ & \vdots & \\ & k_n(t) & \end{pmatrix}$$

is obtained. The vectors ΔY_t and Y_t^* are then regressed on the variables $k_1(t), \ldots,$ $k_m(t)$ which is done by some orthogonal projection with matrix representation P. This gives the residuals

$$R_0(t) = P\Delta Y_t = \varepsilon_t \qquad \text{and} \qquad R_1(t) = PY_t^*. \tag{3.2}$$

From this the matrices $TS_{01} = \sum_{i=1}^T R_0(t)R_1'(t)$ and $TS_{11} = \sum_{i=1}^T R_1(t)R_1'(t)$ is calculated. Further the trace distribution (1.5.2) is asymptotically equivalent to and therefore calculated by

$$\operatorname{tr}\left\{S_{01}\left(\frac{S_{11}}{T}\right)^{-1}S_{10}\right\} = \operatorname{tr}\left\{\sum R_0R_1'\left(\sum R_1R_1'\right)^{-1}\sum R_1R_0'\right\}. \tag{3.3}$$

4. An interesting simulation result.

Consider a model with a intervention dummy at the time ζT where $\zeta \in [0,1)$, e.g. the model

$$\Delta X_t = \Pi X_{t-1} + \mu 1_{[\zeta T, T]} + \varepsilon_t. \tag{4.1}$$

It is plausible that the distribution of the likelihood ratio test of $H(r)$ against $H(p)$ is between the distributions found from the models

$$\Delta X_t = \Pi X_{t-1} + \mu 1_{[0,1]} + \varepsilon_t. \qquad (\zeta = 0) \tag{4.2}$$

and

$$\Delta X_t = \Pi X_{t-1} + \varepsilon_t. \qquad (\zeta = 1) \tag{4.3}$$

This is not the case, however. When $d \to 0$ the asymptotic distribution of the test statistic corresponding to the model (4.1) will converge to that of (4.2), whereas it does not converge to that of (4.3) for $d \to 1$.

Actually when $p - r = 1$ the distribution found from model (4.1) is χ^2_1 for all $d \in [0,1)$, while it is Dickey-Fuller distributed in model (4.2).

This phenomenon will be the topic of a forth coming paper.

5. Handling DisCo.

5.1. Installation.

The installation on the hard disc is done by copying the files 'DisCo.EXE' and 'DisCo.HLP' from the installation diskette.

The DisCo files needs 60.000 free bytes on the harddisc. During the simulations DisCo creates a temporary file 'ERASEME' with the sample of simulated data. The size of this file depends on the number of iterations. Each iteration needs 14 bytes from the disc. The size of the temporary file is seen in the following table.

N	Bytes
1,000	14,000
10,000	140,000
100,000	1,400,000
1,000,000	14,000,000

Table 5.1.1.

The program must be installed on a PC with the DOS operative system. A 80486 processor is recommended but a 80387 or a 80386 processor will be sufficient. Further each simulation require 1200 bytes. A Colour monitor is recommended too but is not necessary.

5.2 How to get started.

Disco has an editor section and a simulation section. The program starts in the editor section. Here the simulation jobs are designed. It is possible to design more than one model before switching the program into simulation mode. Then the jobs are simulated.

The editor section is based on a menu system. Each page gives some action possibilities, numbered '0' - '6'. These are chosen by pressing the number.

When the chosen action is to input a variable a green box will come up on the screen. Here the required value is printed. The value is accepted by pressing the 'RETURN' key. Then the box is left.

In the program different background colours are used. If a monochrome screen is used the colours are represented by a shadow.

GREEN: Some input is expected after the sign '>'. In green boxes the ARROW keys, the keys BACKSPACE, DELETE, HOME, END work as intented. A green box is sometimes left automatically, sometimes by the RETURN key.

RED: Gives an error message and the just printed sequence is not accepted. See also appendix E.

BLUE: Gives some general messages.

The program has two HOTKEYS. They are ALT-X in the editor mode and CTRL-BREAK in the simulation mode. When these keys are pressed the execution of the program is stopped and the computer returns to DOS-mode.

The jobs and the simulation results are stored as ASCII-files, which can be read e.g. using the DOS command 'Type'.

5.3. Description of the drift functions k.

DisCo is limited to study drift functions $k : [0,1] \mapsto [-1,1]$, with a graph composed of line segments in the space $[0,1] \times [-1,1]$, cf. Section 1.2. An example is $k(t) = t1_{[0,\frac{1}{2}]} + \frac{1}{2}1_{[\frac{1}{2},1]}$, but not $k(t) = t^2$.

The function is coded by its breakpoints. As an example $k(t) = t1_{[0,\frac{1}{2}]} + \frac{1}{2}1_{[\frac{1}{2},1]}$ has the breakpoints

$$(0,0) \to \left(\frac{1}{2}, \frac{1}{2}\right) \qquad \text{and} \qquad \left(\frac{1}{2}, \frac{1}{2}\right) \to \left(1, \frac{1}{2}\right).$$

A more detailed description of this procedure is placed in Example 4 in the next Section.

Remark that DisCo is limited to study functions composed of at most 6 line segments. Further DisCo allows at most 3 drift function with unrestricted parameters and at most 3 drift function with $\alpha_\perp$-restricted parameters

Warning. When more than one drift function is used make sure that the matrix (1.1.2) is of full rank. DisCo does not check this property. If two equal drift functions are put in as having unrestricted parameters the simulated table would be asymptotically equal to the case with only one of the drift functions. In other situations DisCo will break down.

5.4. How many repetitions ?

The fractiles are estimated by the empirical fractile of the simulated data. That is, the p-fractile is estimated by the ordered observation with the found as the integer part of p times the number of observations N.

A rough estimate of the necessary number of repetitions to get a satisfactory estimate is found from the following considerations. For a sample $X_1, \ldots, X_n$ of i.i.d. observations the p-value of x is defined as $p = P\{X_i > x\}$. This p is estimated by

$$\hat{p} = \frac{1}{N} \sum_{i=1}^{N} 1_{\{X_i > x\}}.$$

Using the binomial distribution the standard deviation of $\hat{p}$ is found to be

$$\sqrt{\frac{p(1-p)}{N}}.$$

Then the estimate of p relative to p is

$$\frac{\hat{p}}{p} = \left(1 \pm 2\sqrt{\frac{1-p}{pN}}\right).$$

The following table gives relative errors for various p and N values

p	$N = 1,000$	$10,000$	$100,000$
0.025	0.39	0.12	0.039
0.05	0.28	0.087	0.028
0.1	0.095	0.03	0.0095
0.5	0.0032	0.01	0.0032

Table 5.4.1.

Thus for estimating the 95 % fractile with two determining digits $N = 10,000$ is a reasonable choice, and DisCo has this default value.

5.5. Examples.

Example 3.

Consider the model

$$H_{1,0}(r) : \Delta X_t = \alpha\beta' X_{t-1} + \varepsilon_t \qquad t = 1, \ldots, T.$$

Here α and β are $p \times r$-matrices. The hypothesis should be tested against $H_0(p)$. DisCo can simulate the asymptotic distribution.

Assume $p - r = 2$. Set $T = 400$, which is a reasonable discretisation of the Brownian Motion. Further the number of repetitions, N, is set to $N = 100$. This is a small choice to keep the running time down.

Start DisCo from DOS by writing 'DISCO' + 'any key'

Now PAGE 0 will come up. Press '2' to start editing the example. A green box come up on the screen and ask for a name of the job, with default 'sim'. Write 'Ex3'+RETURN.

Now PAGE 2 will come up. The default values are OK except the N, T and p-r values. Press '2' to adjust the N-value. A green box will come up. Write '100'+'DEL' +'RETURN'. The T value is adjusted similarly by '3'+ '400'+'RETURN'. The number of nonstationary directions $(p-r)$ are adjusted by '4' + '2' + 'RETURN'. Now the variables on the page is OK. Accept by '0'.

Now PAGE 3 will come up. Page 3 is not of interest in this example. So just accept by '0'. By this command the job is put into the ASCII file with the name 'EX3.IN', which will be stored at the current path.

Now PAGE 0 will come up again. To start simulation press '4'.

Now PAGE 4 will come up. At the right hand side of this page the simulation queue is shown. It has one member which is 'Ex3'. Press '1' to start the simulations.

Now the simulation page will come up. At the right bottom corner is shown the number of repetition which is simulated right now for the current simulation job. The simulations will soon be finished and the results will be put on the screen. They will also be stored in the ASCII file 'EX3.OUT' at the current path. A blue box will be put on the screen too. This indicates that the simulation job is done and the queue is empty. Go back to page 0 by pressing 'any key'.

Now PAGE 0 will come up again. Leave DISCO by '0'+'any key'. $\square$

Example 4.

Consider the extended model

$$H_{1,1}(r): \Delta X_t = \alpha\beta' X_{t-1} + \mu_0 \mathbf{1} + \alpha\beta_1' \left(t - \frac{T}{2}\right) * 1_{[T/2,T]} + \varepsilon_t \qquad t = 1, \ldots, T.$$

In some application it could be reasonable to extend this model with a constant drift in the interval $[T/2, T]$. But this is omitted here. Here α and β are $p \times r$-matrices. The hypothesis should be tested against $H_2^*(p)$. DisCo can simulate the asymptotic distribution of the likelihood ratio test.

Assume again that $p - r = 2$, $T = 400$, $N = 100$.

Start DisCo again and arrive at PAGE 0.

The program has some usable default values but to get life easier the values from Example 3 could replace these. This is done by pressing '3' and write the name 'Ex3'+'RETURN'. Now the values of this example have to be put in. Start the editor by pressing '2' and write 'Ex4'+'RETURN'.

Now PAGE 2 will come up. The values N, T, $p - r$ are now OK. In this example the drift has two components. One with an unrestricted parameter corresponding to $\mu_0 \mathbf{1}$ and one with an $\alpha_\perp$-restricted parameter corresponding to $\alpha\beta_1'(t - T/2)1_{[T/2,T]}$.

So press '5'+ '1'+'RETURN' to put the number of drift functions with unrestricted parameters equal to 1 and press '6'+ '1'+'RETURN' to put the number of drift functions with $\alpha_\perp$ restricted parameters equal to 1. Press '0' to accept and leave this page.

Now PAGE 3 will come up. Here the structures of the drifts are declared. Start with the one with unrestricted parameter. Press '1'. A green box will now come up on the screen. Write 'C'+'RETURN' for constant. Now take the other drift function. Press '4' and get the green box again. If the indicator function $1_{[T/2,T]}$ is omitted the drift is linear, so in this case press 'L'+'RETURN' for linear. On the interval $[0, 1]$ the indicator function corresponds to $1_{[\frac{1}{2},1]}$. This drift could be composed by two line segments so press 'M'+'RETURN' for mixed.

Now PAGE 3a will come up. The following line is showed on the screen.

```
1.  (0.00;0.00) -> (1.00;0.00)
```

Now, choose '1'. A green box gives possibility of choosing the desired coordinates. x1 = 0 is trivial and omitted. Thus the y1 coordinate is showed in the green box. It is OK, so write 'RETURN'. Now the x2 coordinate is comming up. Write '05'+'RETURN'. The y2 coordinate is OK so press 'RETURN'. Then the green box is left. A new screen is set up with the lines

```
1.  (0.00;0.00) -> (0.50;0.00)
2.  (0.50;0.00) -> (1.00;0.00)
```

Now, choose '2' and print the coordinates y1=0, x2=1, y2=1 by '0' + 'RETURN' + 'RETURN' + '1' + 'RETURN' Then the green box is left. A new screen is shown with the lines

```
1.  (0.00;0.00) -> (0.50;0.00)
2.  (0.50;0.00) -> (1.00;1.00)
```

Accept by '0'.

Now DISCO returns to PAGE 3. Accept and save by '0'.

Now PAGE 0 will come up again. Start simulation by '4'.

Now PAGE 4 will come up. The job queue is now 'Ex4'. Example 3 could be simulated right after 'Ex4' by extending the job queue. Press '2' and write

'Ex3'+'RETURN'. It could be removed from the queue again by writing '3' and then '2'+'RETURN'. Start the simulations by pressing '1'. □

Appendix A: The code.

A.1. Code.

The program is written in Turbo Pascal 5 (TP) created by Borland International. The original TP source text is included on the installation diskette. It is split up in the main program 'DISCO.PAS' and the units 'EDIT_UN.PAS', 'SIM_UNIT.PAS', 'SIM_IO.PAS', 'MATRIX.PAS' and 'SORT.PAS'.

PLEASE, delete the comment at the begining of the program and the units if the original source text is modified.

A.2. Random number generator.

The program uses the random number generator from TP. We have not found any documentation of this random number generator. But it is probably very good. Further it works faster than a self made generator created in TP would do. When one is interested in improving the generator it should be done by linking an assempler code to the source code.

The random number generator gives 'independent' random numbers between 0 and 1. Two of these are transformed into two independent Gaussian variables by an algorithm using polar coordinates of $\mathbf{R}^2$, see Box & Müller (1958) or Rubinstein (1981).

Appendix B: The input and output files.

Corresponding to Example 3, testing $H(r)$ against $H(p)$ in the model

$$H_{1,0}(r) : \Delta X_t = \alpha\beta' X_{t-1} + \varepsilon_t \qquad t = 1, \ldots, T$$

DisCo gives the input and output files:

EX3.IN:

```
93.4.16 is the input file version.  Be careful with the format of this file.
1          = Fixed Random Start, 1=True, 0=False
100        = N
400        = T
2          = # non stationary directions
0          = # drift functions with UnRestricted parameters
0          = # drift functions with Restricted parameters
The first  drift function with UnRestricted parameter:
0   0   0   0   0   0   0   0   0   0   0   0   0   0   0   0   0   0
The second drift function with UnRestricted parameter:
```

```
0  0  0  0  0  0  0  0  0  0  0  0  0  0  0  0  0  0
The third  drift function with UnRestricted parameter:
0  0  0  0  0  0  0  0  0  0  0  0  0  0  0  0  0  0
The first  drift function with Restricted parameter:
0  0  0  0  0  0  0  0  0  0  0  0  0  0  0  0  0  0
The second drift function with Restricted parameter:
0  0  0  0  0  0  0  0  0  0  0  0  0  0  0  0  0  0
The third  drift function with Restricted parameter:
0  0  0  0  0  0  0  0  0  0  0  0  0  0  0  0  0  0
```

and EX3.OUT:

```
* * * Result of Simulation * * *
16:53:30  (1993.  5.31)  to  16:54:  6  (1993.  5.31)
Input Data:
N=    100   T=  400
p-r =  2  # UnRes drift =  0  # Res drift =  0
This is the file "ex3.OUT"
Mean     =    6.022   (   0.640)
Variance =   10.231
0.0 %    =    0.507
1.0 %    =    0.507
2.5 %    =    0.912
5.0 %    =    2.156
10.0 %   =    2.414
25.0 %   =    3.658
50.0 %   =    5.136
80.0 %   =    8.838
90.0 %   =   10.635
95.0 %   =   11.920
97.5 %   =   13.006
99.0 %   =   13.956
100.0 %  =   14.630
```

In this table the mean and the variance are usual empirical moments. The number between the parentheses after the mean value is the estimated 2 times the standard deviation of the empirical mean.

Further the 0 % value and the 100 % value are the smallest and the largest value of the simulated sample.

Corresponding to Example 4, testing $H_{1,1}(r)$ against $H_{1,1}(p)$ in the model

$$H_2^*(r) : \Delta X_t = \alpha\beta' X_{t-1} + \mu_0 \mathbf{1} + \alpha\beta_1'(t - \frac{T}{2})1_{[\frac{T}{2},T]} + \varepsilon_t \qquad t = 1, \ldots, T$$

DisCo gives the input and output files:

EX4.IN:

```
93.4.16 is the input file version.  Be careful with the format of this file.
1          = Fixed Random Start, 1=True, 0=False
100        = N
400        = T
2          = # non stationary directions
1          = # drift functions with UnRestricted parameters
1          = # drift functions with Restricted parameters
The first  drift function with UnRestricted parameter:
100 100 100   0   0   0   0   0   0   0   0   0   0   0   0   0   0   0
The second drift function with UnRestricted parameter:
0   0   0   0   0   0   0   0   0   0   0   0   0   0   0   0   0   0
The third  drift function with UnRestricted parameter:
0   0   0   0   0   0   0   0   0   0   0   0   0   0   0   0   0   0
The first  drift function with Restricted parameter:
0  50   0   0 100 100   0   0   0   0   0   0   0   0   0   0   0   0
The second drift function with Restricted parameter:
0   0   0   0   0   0   0   0   0   0   0   0   0   0   0   0   0   0
The third  drift function with Restricted parameter:
0   0   0   0   0   0   0   0   0   0   0   0   0   0   0   0   0   0
```

and EX4.OUT:

```
* * * Result of Simulation * * *
16:55:43  (1993.  5.31)  to  16:56:58  (1993.  5.31)
Input Data:
N=    100   T=  400
p-r =  2  # UnRes drift =  1  # Res drift =  1
This is the file "ex4.OUT"
Mean      =   12.984   (   0.907)
Variance =   20.583
0.0 %     =    2.248
1.0 %     =    2.248
2.5 %     =    4.975
5.0 %     =    6.528
10.0 %    =    7.945
25.0 %    =    9.625
50.0 %    =   12.155
80.0 %    =   16.871
90.0 %    =   18.396
95.0 %    =   20.607
97.5 %    =   21.310
99.0 %    =   25.729
100.0 %  =   28.203
```

Appendix C: Running times.

The simulations take often long time. The running time depends linearly on the number of repetitions and exponentially on T and the dimensions of the model.

Example 5. The model

$$H_{1,1}(p): \qquad \Delta X_t = \Pi X_{t-1} + \mu_1 + \alpha\beta_1' t + \varepsilon_t$$

was studied and the asymptotic distribution of the likelihood ratio test of $H_{1,1}(r)$ against $H_{1,1}(p)$ was simulated on a 486 PC with 33 MHz processor. With $T = 400$ and $N = 6000$ the following running times was obtained.

Table C.1.

$p - r$	1	3	5	7
hour:min.sec	0:09.54	0:27.05	0:49.32	1:17.21

Appendix D: The menu pages.

PAGE 0:
has the possibilities:

'`0. Quit`' Exit from DisCo.

'`1. Help`'

'`2. Edit a new Job`' A green box ask for a name of the job. This should be of up to 8 characters ('a'-'z') or digits. The job will automatically get the surname '.IN'. DisCo will go to page 2.

'`3. Initialise Default Values`' A green box ask for a name of a job file. This file is then readed. The file could be edited by chosing '2'.

'`4. Simulation`' DisCo will go to page 4.

PAGE 2:
has the possibilities:

'`0. Accept`' DisCo will go to page 3.

'`1. The starting values of the random generator should be ...`'
Range: 'Fixed' → 'Randomized'. Default 'F'
When fixed is choosed a new simulation will give the same result.

'`2. N = ...`'
Range: 0 → 1,000,000. Default 10,000.

'`3. T = ...`'
Range: 2 → 800. Default 400.
Remark T should be strictly larger than the sum of the number of non-stationary directions and the number of drift functions with $\alpha_\perp$-restricted parameter.

'`4. # of non stationary directions = ...`'
Range: 1 → 17. Default 1.

`‘5.  # of drift functions with UnRestricted parameters = ...’`

Range: $0 \rightarrow 3$. Default 0.

`‘6.  # of drift functions with Restricted parameters = ...’`

Range: $0 \rightarrow 3$. Default 0.

PAGE 3:

has the possibilities:

`‘0.  Accept and Save ...’`

The job is saved in a file on the current path. Further the job is put into the job queue and DisCo will go to page 0.

```
‘1.  The first  drift function with UnRestricted parameter is ...’
‘2.    -  second -     -        -    -            -         -      ’
‘3.    -  third  -     -        -    -            -         -      ’
‘4.  The first  drift function with Restricted parameter is ...’
‘5.    -  second -     -        -    -          -         -      ’
‘6.    -  third  -     -        -    -          -         -      ’
```

Range: Constant - Linear - Mixed.

If Mixed is choosed DisCo will go to page 3a.

PAGE 3a:

is used when a mixed drift function is wanted. That is a regressor of up to 6 linepieces. For handling this page see example 4. See also the Sections 1.2 and 5.3. Remember, that the scale of the ‘y’-values has no importance. That is why DisCo wants ‘y’ to be in the range $-1 \rightarrow 1$.

Page 4:

The edited jobs are automatically put into the job queue. The full queue is showed and has a capacity of 30 jobs.

Page 4 gives the possibilities

```
‘0.  To page 1’
‘1.  Simulate the Queue’
‘2.  Add a Job’
‘3.  Delete a Job’
```

When the program works the current repetion number of the actual simulation is printed on the screen. When the program is finished a blue box is put on the screen.

Appendix E: The error codes.

`ERROR 1:  These characters must be letters !`

The pressed key is not a letter (’A’→’Z’,’a’→’z’) and is not accepted. Just continue, but write letters !

`ERROR 2: These characters must be digits !`

The pressed key is not a digit ('0'→'9') and is not accepted. Just continue, but write digits !

`ERROR 3: These characters must be letters or digits !`

The pressed key is not a letter ('A'→'Z','a'→'z') or a digit ('0'→'9') and is not accepted. Just continue, but write letters or digits !

`ERROR 4: This character must be a F or a R !`

The pressed key is not a 'F' or a 'R' and is not accepted. Just continue, but write a 'F' or a 'R' ! Remark, DisCo would also accept a 'f' or a 'r' respectively or a '1' or a '0' respectively.

`ERROR 5: This character must be a C, a L or a M !`

The pressed key is not a 'C', 'L' or 'M' and is not accepted. Just continue, but write a 'C' or 'L' or 'M' ! Remark, DisCo would also accept a 'c', 'l' or 'm' respectively or a '1', '2' or '3' respectively.

`ERROR 6: Out of range !`

The pressed value is out of range. The range is printed in the green box. Write a new value and continue.

`ERROR 7: T <= # NonStat + # Res Drift.   Input a new T !`

The wanted values of T, $p-r$ and the number of drift functions with $\alpha_\perp$-restricted parameters will give singularities during the simulations. The page can not be left before proper values of T, $p-r$ and the number of drifts has been put in.

`ERROR 8: p-r < # UnRes drift.   Input a new p-r !`

The system is overidentified.

`WARNING 9: Zero Function !   Dim:= Dim-1  -  unless You press "Esc" !`

The typed drift function is zero. To avoid singularities DisCo omit this drift function and let the number of the drift functions one down. By pressing 'ESCAPE' this reaction is avoided and it is possible to put in another drift function.

`WARNING 10: N must be <= 10,000 or a multiplum of 10,000 to get fractiles !`

The sorting procedure can only sort less than 10,000 numbers or blocks of 10,000 numbers. So if N is larger than 10,000 the data are devided into blocks of 10,000 numbers. Each of these is sorted and fractiles are found for each block. The result and output is a mean of the fractiles of the blocks. These are not the fractiles of the simulated numbers, but in comparison with the true fractiles they have the same variance.

`ERROR 11: N is to large !   Not sufficient free bytes at the current drive !`

Choose a smaller N or leave DisCo and clean up the harddisc. See also Section 5.1, Installation.

`ERROR 12: File does not exist.   Press Any Key !`

The file does not exist at the current path. Maybe it does exist on another path. If this file must be used then exit to DOS and move the file.

`ERROR 13: The file does not have the proper format.   Press Any Key !`

Examples of the proper format of an 'IN' file is shown in Appendix B. Be very careful !

`ERROR 14:  Max length of the queue is ....    Press Any Key !`

The queue is filled up and it can not be extended. If this job should be inclosed in the queue then delete another job.

`ERROR 15:  The simulation queue is empty.    Press Any Key !`

It is not possible to delete any job, because the queue is empty. If the last key pressed had been '0' this message had been avoided.

`RUN TIME ERROR 20:  Inversion of singular matrix !`

The reason for the singularity could be the existence of a linear dependence between the drift functions with $\alpha_\perp$ restricted parameters. Or a combination of these with some drift functions with unrestricted parameters.

Since the T value is choosed strictly greater than $p - r$ plus the number of drift with restricted parameter, the singularity should not be caused by reduced rank of the matrix of generated data. At least this would occure with probability zero, see also formula (1.1.2) and Section 5.3.

References.

Anderson, T. W. (1951), *Estimating linear restrictions on regression coefficients for multivariate normal distributions*, Annals of Mathematical Statistics **22**, 327–351.

Box & Müller (1958), *A note on the generation of random normal deviates*, Annals of Mathematical Statistics **29**, 610–611.

Johansen, S. (1988), *Statistical Analysis of Cointegration Vectors*, Journal of Economic Dynamics and Control **12**, 231–254.

Johansen, S. (1990), *Maximum likelihood estimation and inference on cointegration - with applications to the demand for money*, Oxford Bulletin of Economics and Statistics **52**, 169–210.

Johansen, S. (1991), *Estimation and hypothesis testing of cointegration vectors in gaussian autoregressive models*, Econemetrica **59**, 1551–1580.

Johansen, S. (1992a), *Determination of cointegration rank in the presence of a linear trend*, Oxford Bulletin of Economics and Statistics **54**, 383–397.

Johansen, S. (1992b), *The role of the constant term in cointegration analysis of nonstationary variables*, Preprint, KUIMS, 1992, 1.

Johansen, S. & Juselius, K. (1990), *Maximum likelihood estimation and inference on cointegration - with applications to the demand for money*, Oxford Bulletin of Economics and Statistics **52**, 169–210.

Johansen, S. & Juselius, K. (1992), *Testing structural hypotheses in a multivariate cointegration analysis of the PPP and the UIP for UK*, Journal of Econometrics **53**, 211–244.

Pantula, S.G. (1989), *Testing for unit roots in time series data*, Economic Theory **5**, 256–271.

Rubinstein, R.Y. (1981), *Simulation and the Monte Carlo method*, Wiley.