Difference between the version 1.3 and 1.4 of DisCo[1]

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DisCo 1.4 can be found on the web-site

A bug has been found in the version 1.3, which is now corrected. Fortunately, in many cases the old version 1.3 would not give misleading results.

The purpose of DisCo is to simulate limit distributions of test for cointegration rank in the presence of structual breaks in the deterministic terms of a vector autoregressive model. Thus, let $U(t)$, $R(t)$ denote vectors of unrestricted and restricted drift components respectively. These are of dimension $p_U \geq 0$, $p_R \geq 0$, say. The model for a p-dimensional time series, X_t, is then formulated as

$$\Delta X_t = \Pi X_{t-1} + \mu U(t) + \gamma R(t) + \varepsilon_t$$

for $t = 1, \ldots, T$. Here the innovations are independently Gaussian with variance Ω, the initial value of the time series is fixed and the parameters vary freely. The hypothesis of cointegration is given by the hypothesis that the rank of Π does not exceed $r < p$ and that the matrices Π, γ have the same column space. Thus there exist $(p \times r)$-matrices α, β and a $(p_R \times r)$-matrix β_R such that

$$\Pi = \alpha\beta' \text{ and } \gamma = \alpha\beta_R.$$

Alternatively the model can be formulated as

$$\Delta X_t = \Pi^* \begin{pmatrix} X_{t-1} \\ R(t) \end{pmatrix} + \mu U(t) + \varepsilon_t \qquad (1)$$

where Π^* is a matrix of dimension $p \times (p + p_R)$. The hypothesis of interest is then that the rank of Π^* is at most r.

The mistake in the old version is that the code is based on the model

$$\Delta X_t = \Pi^* \begin{pmatrix} X_{t-1} \\ R(t-1) \end{pmatrix} + \mu U(t) + \varepsilon_t \qquad (2)$$

[1] The distribed bug in DisCo version 1.3 has been discovered by Rocco Mosconi, Maria Cristina Leali and Graziano Vigano.

rather than on (1). If no breaks are included in the drift terms, U, R, that is when the models of Johansen (1995) are analysed, then (2) is just a reparametrisation of (1). However, in the presence of breaks the limit distribution of the rank tests in the two models may be different. To illustrate this consider models with one break, at time T_B, so that

$$\Delta X_t = \alpha\beta' X_{t-1} + \mu_t + \varepsilon_t.$$

The following five models are of particular interest:

$$\begin{aligned}
H_0 &: \mu_t = 0, \\
H_1^* &: \mu_t = \begin{cases} \alpha\beta_1' & \text{for } t < T_B, \\ \alpha\beta_2' & \text{for } t \geq T_B, \end{cases} \\
H_1 &: \mu_t = \begin{cases} \mu_1 & \text{for } t < T_B, \\ \mu_2 & \text{for } t \geq T_B, \end{cases} \\
H_2^* &: \mu_t = \begin{cases} \mu_1 + \alpha\beta_1' t & \text{for } t < T_B, \\ \mu_2 + \alpha\beta_2' t & \text{for } t \geq T_B, \end{cases} \\
H_2 &: \mu_t = \begin{cases} \mu_1 + \gamma_2 t & \text{for } t < T_B, \\ \mu_2 + \gamma_2 t & \text{for } t \geq T_B. \end{cases}
\end{aligned}$$

In cases H_0, H_1, H_2 the restricted component, R, is absent so that the models (1) and (2) are identical.

In case H_1^* the restricted component is

$$R(t) = \begin{pmatrix} 1_{[0,T_1[}(t) \\ 1_{[T_1,T]}(t) \end{pmatrix},$$

so that the models (1) and (2) are slightly different. However, the limit distributions of the rank tests are identical. The reason is that the limit distribution appears to be a continuous function of the break point. Since DisCo only approximates the limit distribution there will be a small discrepancy of the outcome of the two versions of the program.

In case H_2^* the restricted component is

$$R(t) = \begin{pmatrix} 1_{[0,T_1[}(t) \\ 1_{[T_1,T]}(t) \\ t1_{[0,T_1[}(t) \\ t1_{[T_1,T]}(t) \end{pmatrix}.$$

It can be proved that the limit distributions of the two models are entirely different. I believe the model (1) is appropriate. A more detailed account on this issue will appear on this web-site soon.

References

Johansen, S. (1995): Likelihood-based inference in cointegrated vector-autoregressive models; Oxford University Press.