Simplicity in Computational Geometry

Sven Skyum's Algorithm for Computing the Smallest Enclosing Circle

Gerth Stølting Brodal

Sven Skyum - farewell celebration, Department of Computer Science, Aarhus University, September 5, 2014

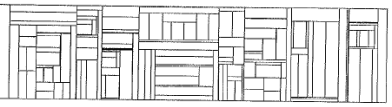
ISSN 0105-8517

A simple algorithm for computing the smallest enclosing circle

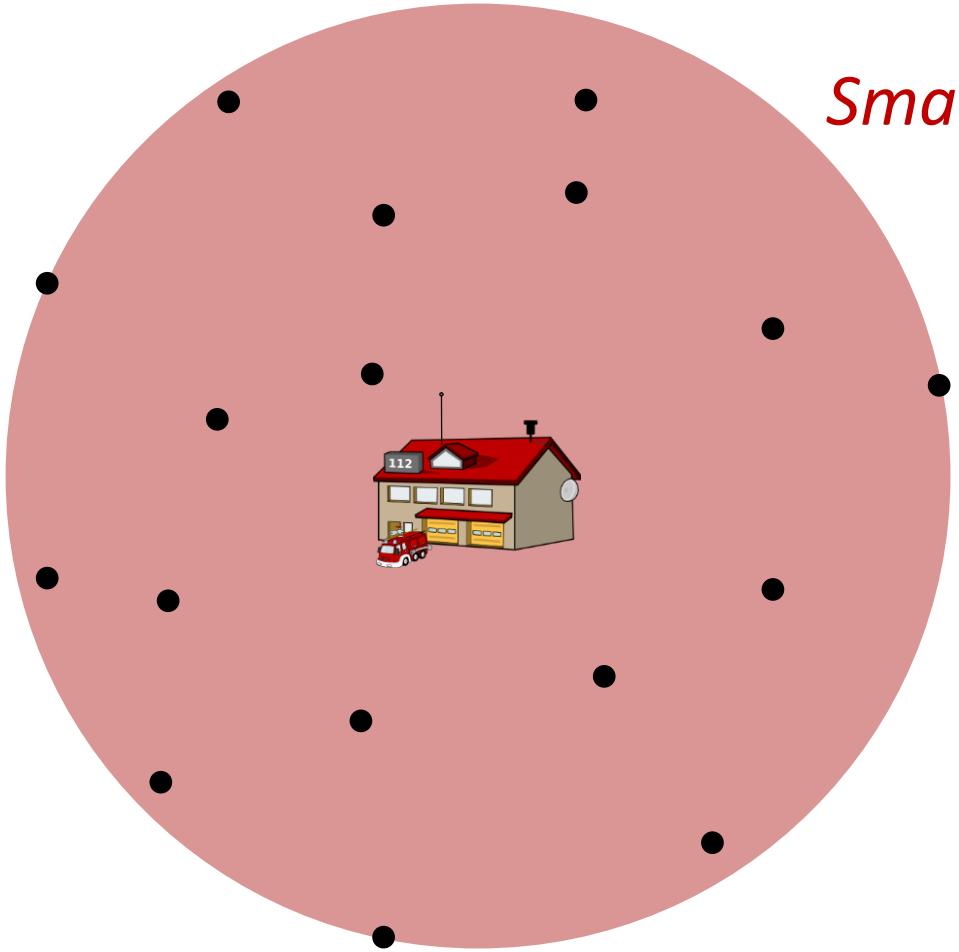
Sven Skyum

DAIMI PB – 314
June 1990

COMPUTER SCIENCE DEPARTMENT
AARHUS UNIVERSITY
Ny Munkegade, Building 540
DK-8000 Aarhus C, Denmark



Sven Skyum, *A Simple Algorithm for Computing the Smallest Enclosing Circle*. Information Processing Letters, Volume 37, Issue 3, 18 February 1991, Pages 121–125



Smallest Enclosing Circle

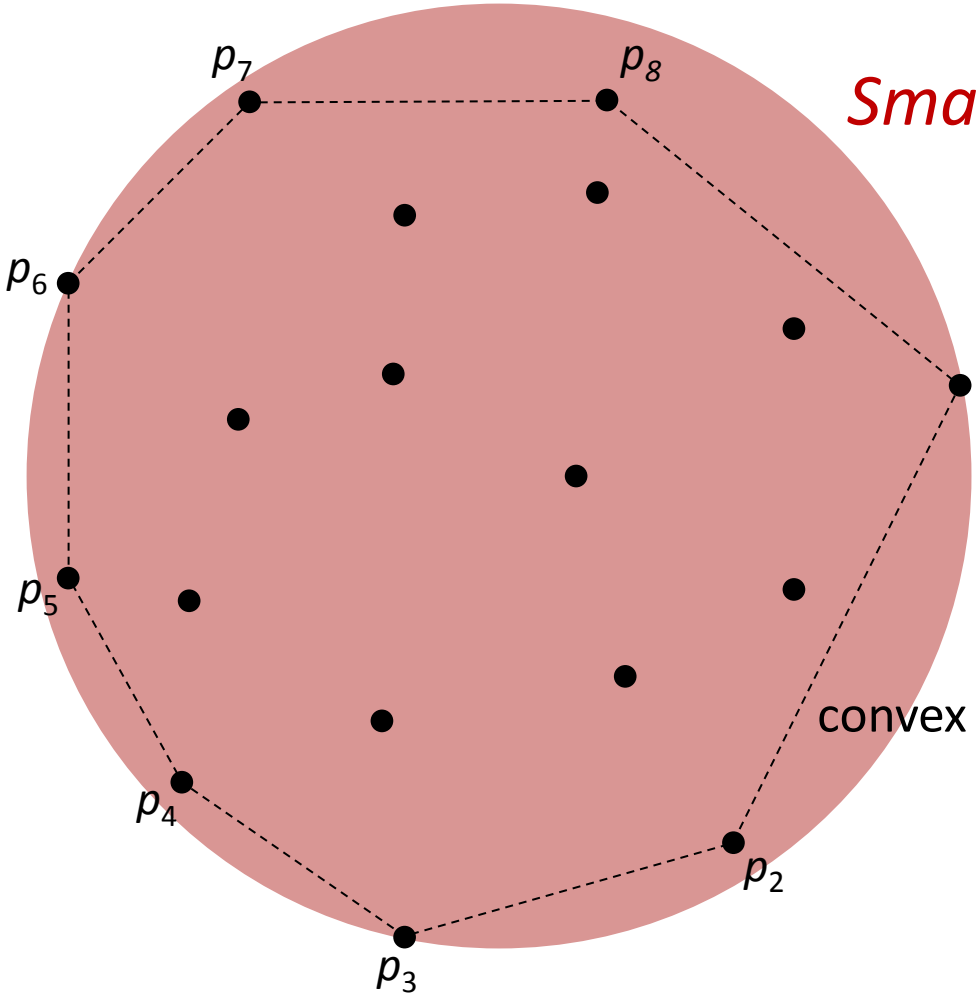
History

Year	Result	Authors
1857	problem posed	Sylvester
1860	”graphical solution procedure”	Pierce
1965	quadratic programming $\min_{p_0 \in \mathbb{R}^2} \max_i (x_i - x_0)^2 + (y_i - y_0)^2$	Lawson
1966		Zhukhovitsky, Avdeyeva
	$O(n^4)$	”The obvious”
1972	$O(n^3)$, $O(h^3 \cdot n)$, $O(n^2)$	Elzinga, Hearn
1975	$O(n \cdot \log n)$	Shamos, Hoey
1977	$O(n \cdot \log n)$	Preparata
1981	$O(n \cdot h)$	Chakraborty, Chaudhuri
1983	$O(n)$	Megiddo
1991	$O(n \cdot \log n)$	Skyum
1991	$O(n)$, expected	Welzl

Just because a problem $\mathcal{A}$ can be formulated as a special case of $\mathcal{B}$ is no reason for believing that a general method for solving $\mathcal{B}$ is an efficient way of solving $\mathcal{A}$
- Preparata & Shamos, 1985

…the involved constants hidden in O(n) are large.
- Skyum, 1991

However his method is not nearly as easy to describe and to implement, and the dependence of the constant in d falls far behind the one achieved by our method.
- Welzl, 1991

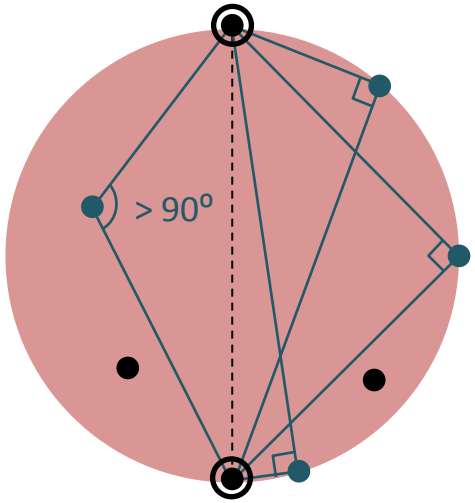


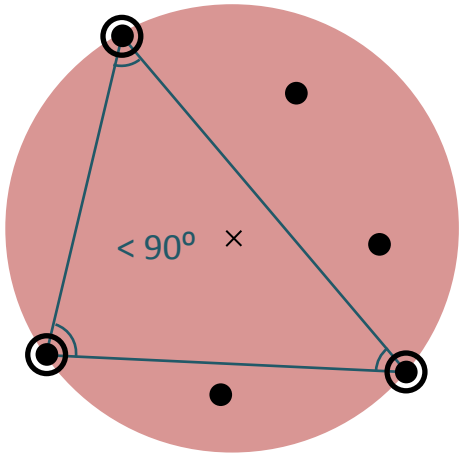
Smallest Enclosing Circle

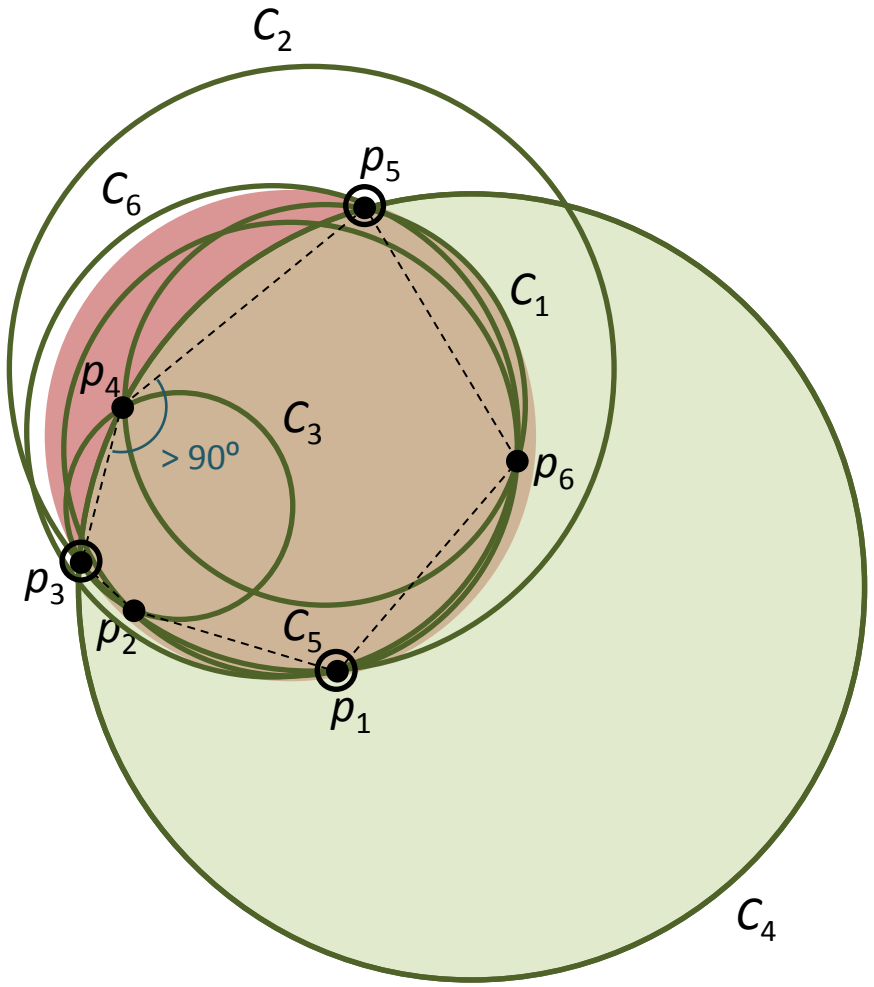
convex hull – O(n·log n) time

Convex polygon S = (p_1, p_2, p_3, … , p_n)

Observations



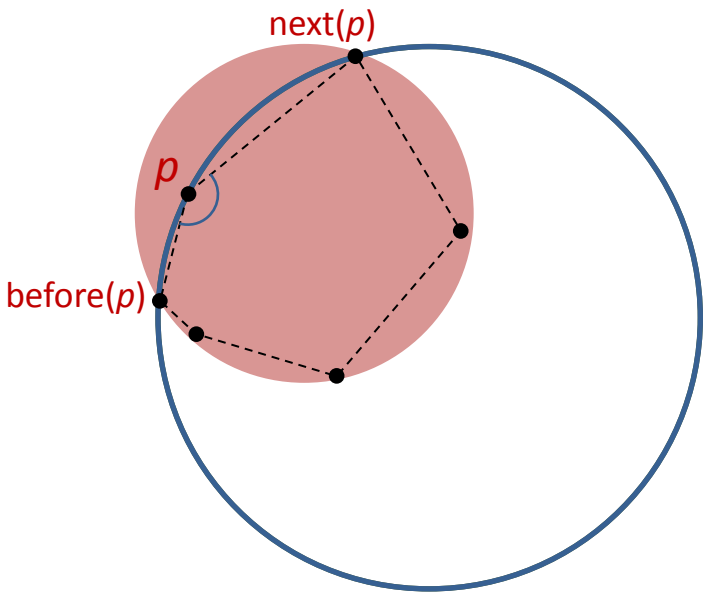




Rademacher, Toeplitz 1957

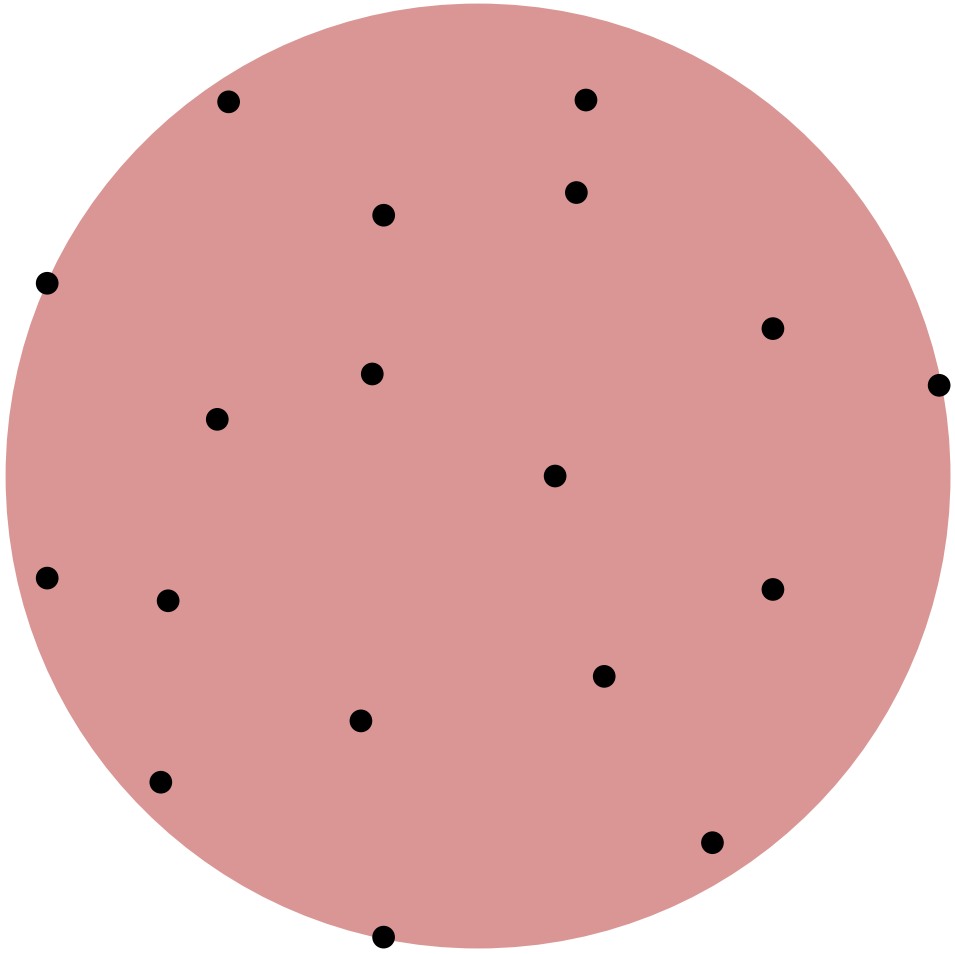
Algorithm 1.

if $|S| \neq 1$ **then**
  finish := false;
  repeat
  (1) find p in S
      (radius(before(p), p, next(p)), angle(before(p), p,next(p))
    in the lexicographic order;
  (2) **if** angle(before(p), p, next(p)) $\leq \pi/2$ **then**
      finish := true
    else
      remove p from S
    fi
  until finish
fi;
{ answer is SEC(before(p), p, next(p)) }



Top 20 citing Skyum's algorithm

1. Movement-assisted sensor deployment
2. Distributed control of robotic networks: a mathematical approach to motion coordination algorithms
3. Smallest enclosing disks (balls and ellipsoids)
4. Coordination and geometric optimization via distributed dynamical systems
5. Design Techniques and Analysis
6. Circle formation for oblivious anonymous mobile robots with no common sense of orientation
7. Reactive data structures for geographic information systems
8. Distributed circle formation for anonymous oblivious robots
9. Imaging knee position using MRI, RSA/CT and 3D digitisation
10. The organization of mature Rous sarcoma virus as studied by cryoelectron microscopy
11. Hyperbolic Voronoi diagrams made easy
12. Collaborative area monitoring using wireless sensor networks with stationary and mobile nodes
13. Approximating smallest enclosing balls with applications to machine learning
14. The deployment algorithms in wireless sensor net works: A survey
15. Adaptive and distributed coordination algorithms for mobile sensing networks
16. ISOGRID: An efficient algorithm for coverage enhancement in mobile sensor networks
17. A novel hybrid approach to ray tracing acceleration based on pre-processing & bounding volumes
18. Fast neighborhood search for the nesting problem
19. Local strategies for connecting stations by small robotic networks
20. Algorithmic problems on proximity and location under metric constraints



Thank You

Sven