Fully Persistent B-Trees

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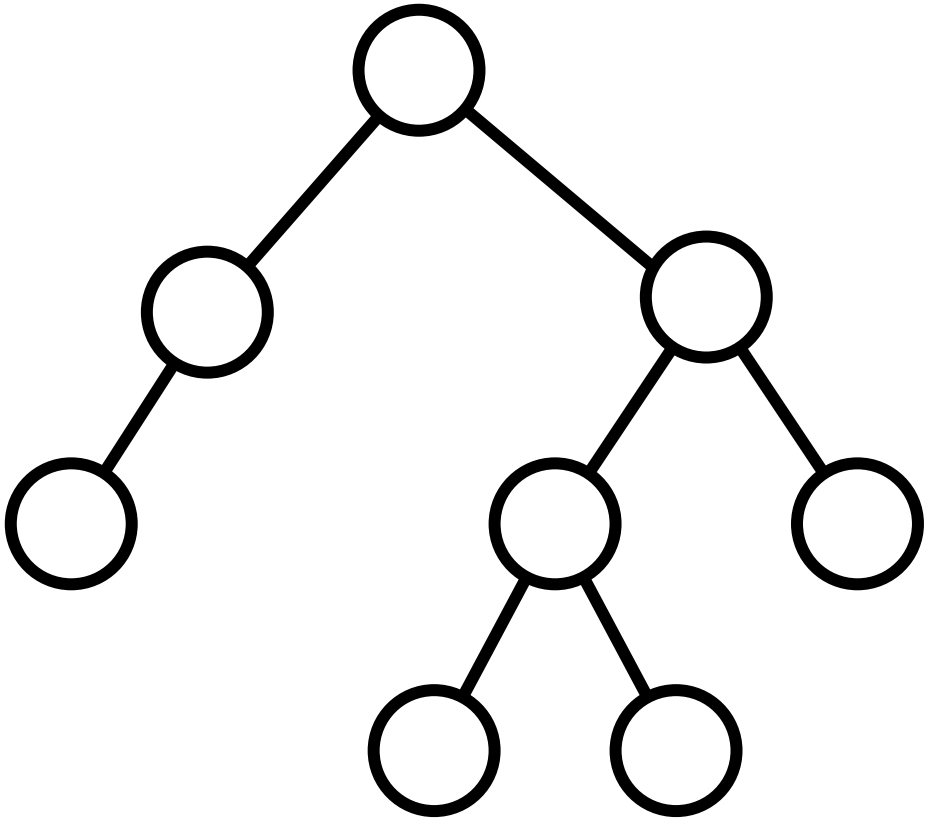
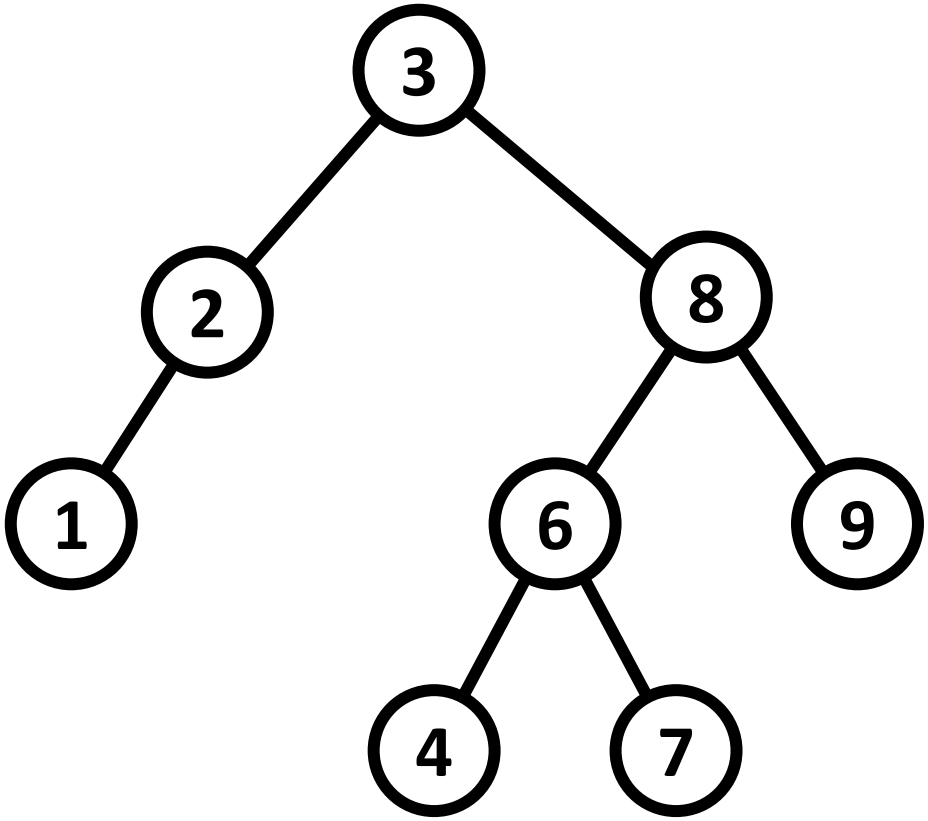
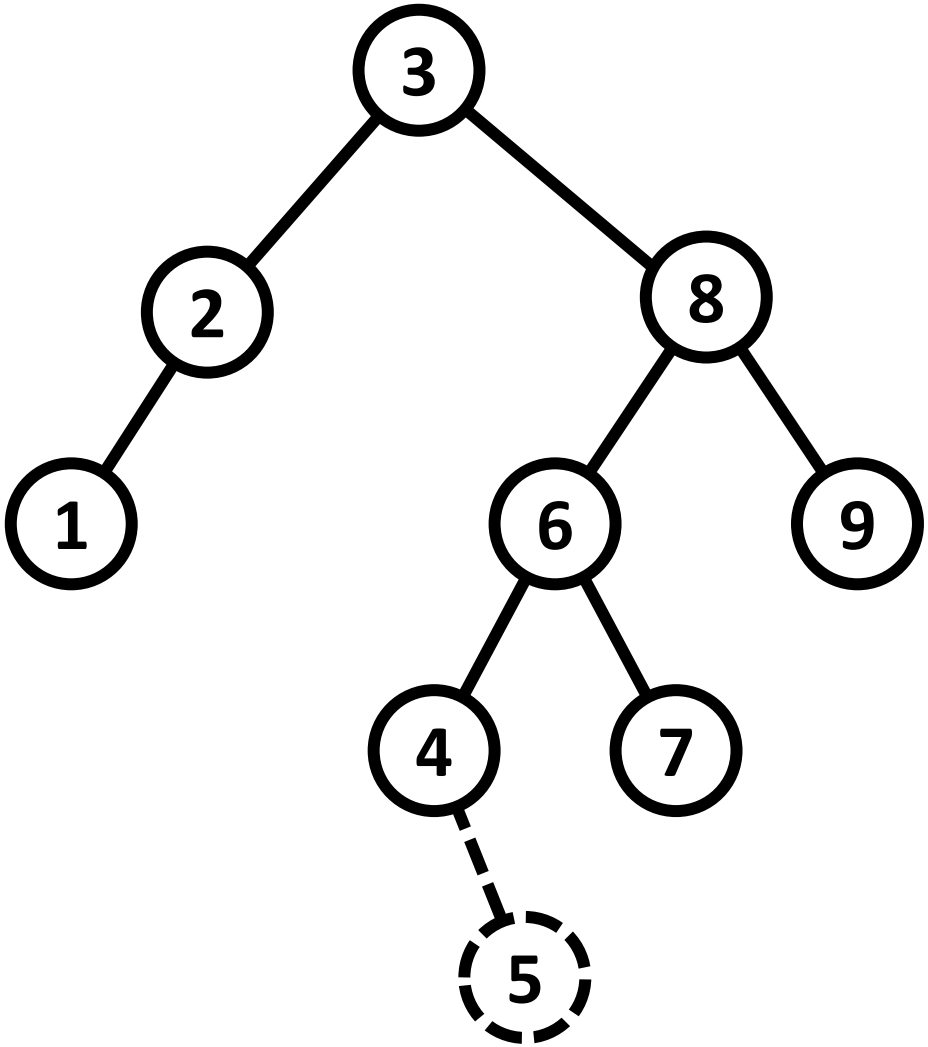
Binary Tree



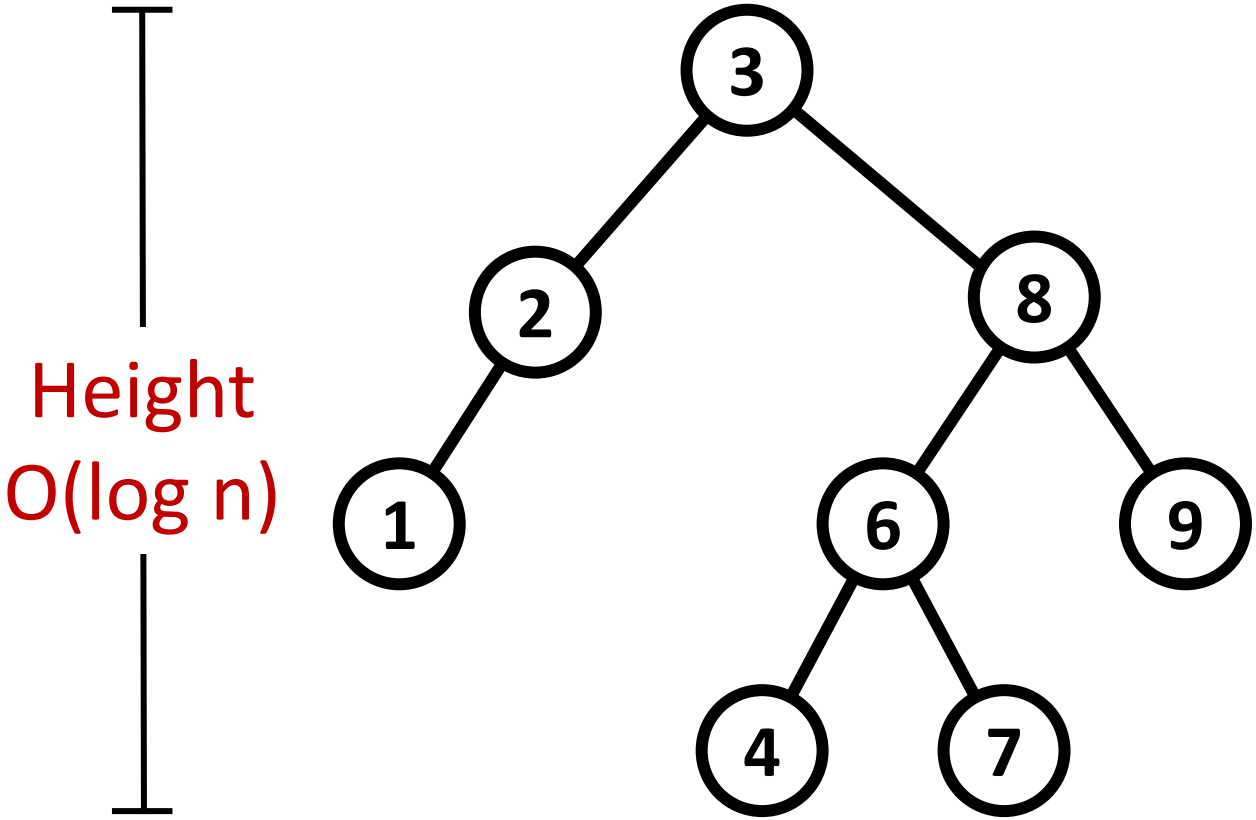
Binary Search Tree



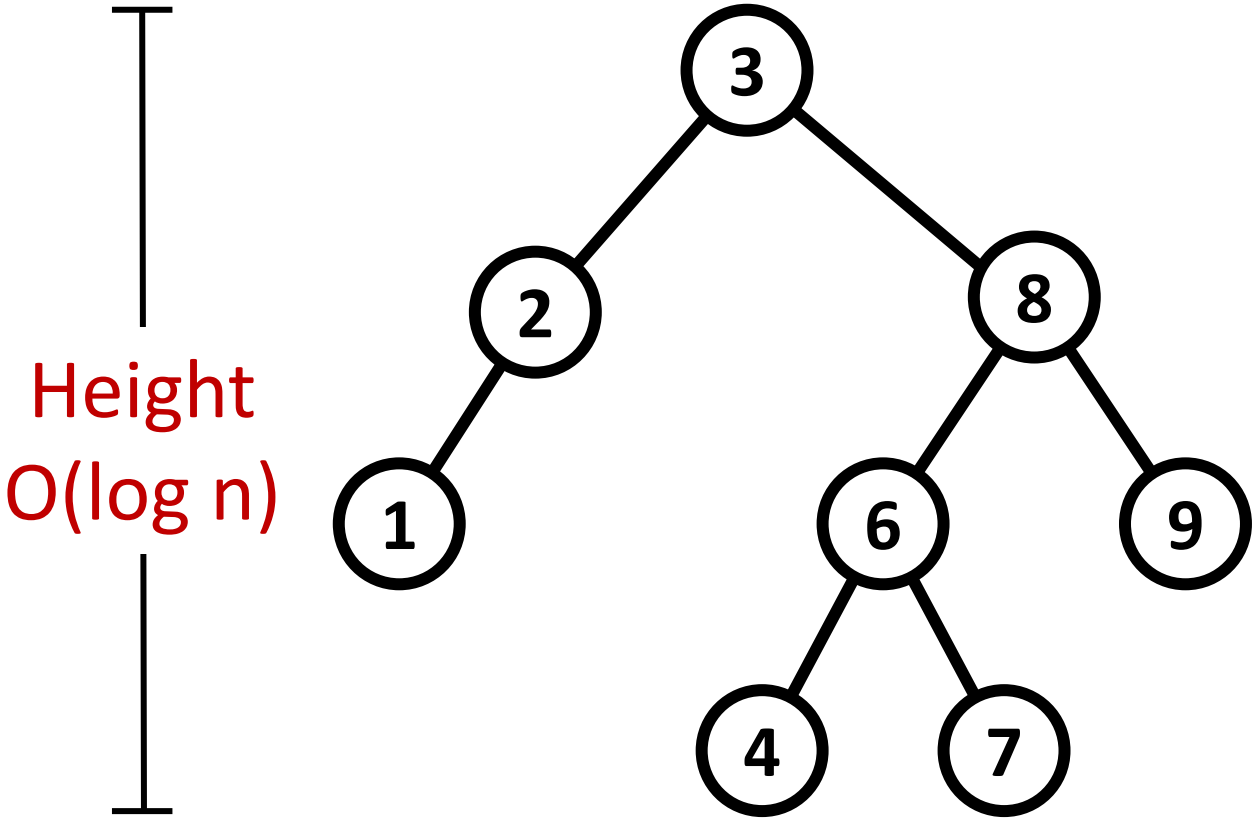
Insert(5)



Balanced Binary Search Tree

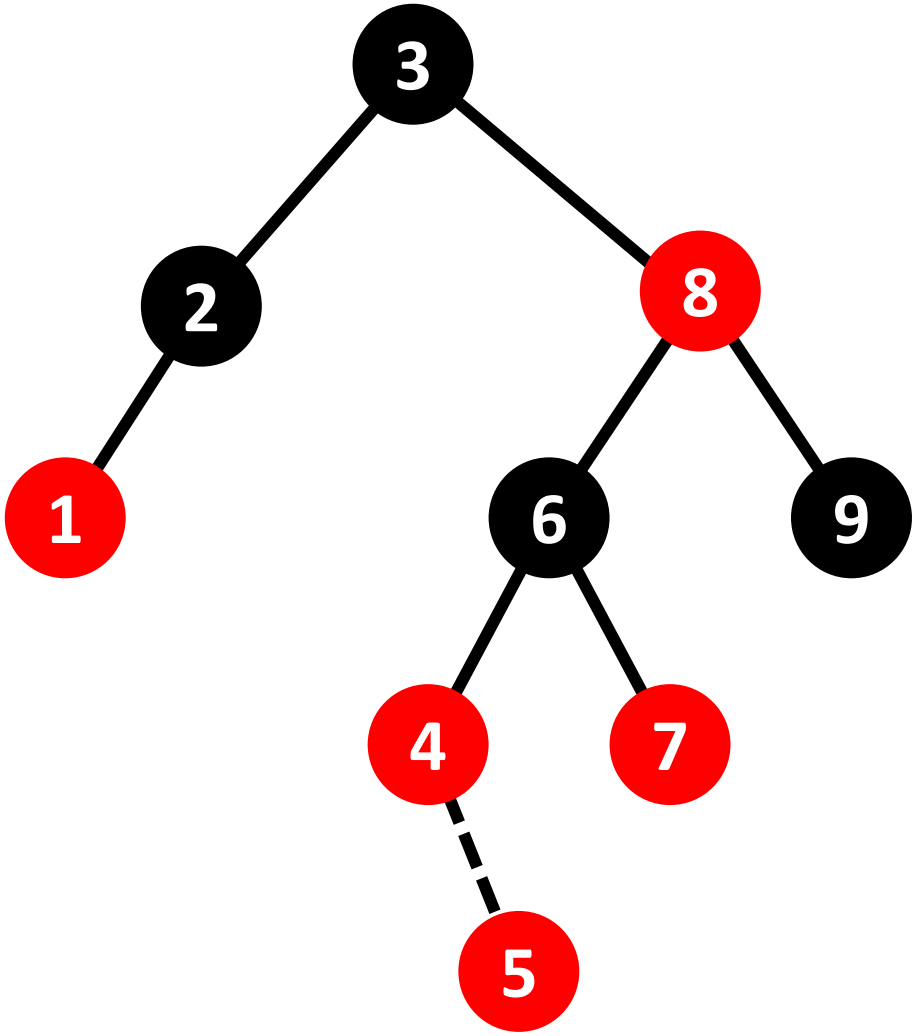


Report(2,7)



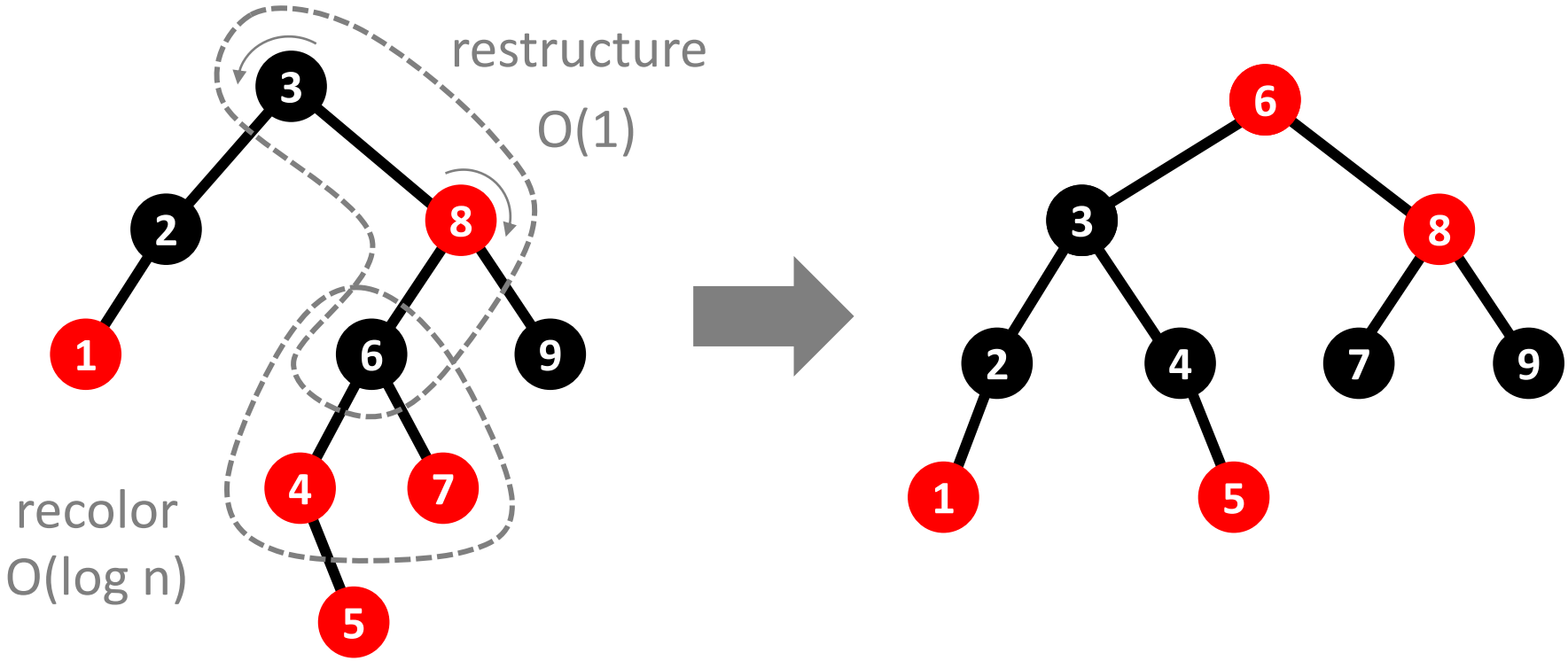
Time O(log n + t)

Red-Black Tree



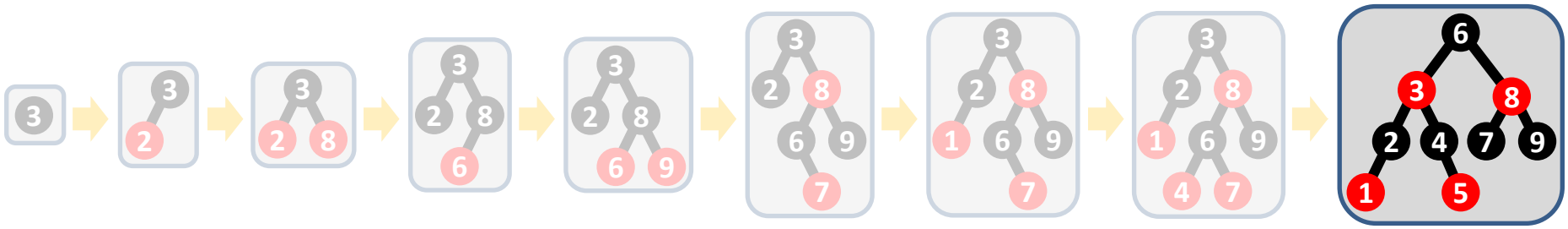
Bayer 1972

Red-Black Tree - rebalancing



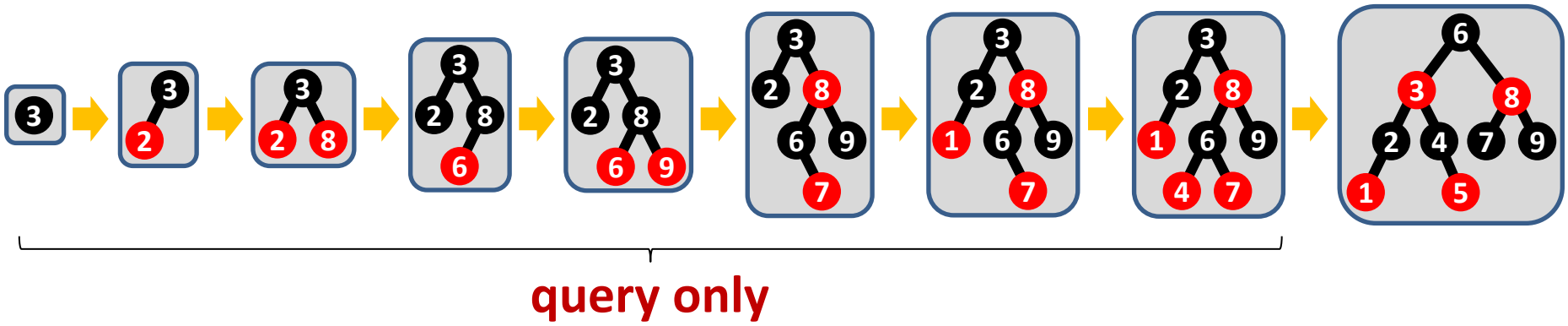
Bayer 1972

Ephemeral Red-Black Tree



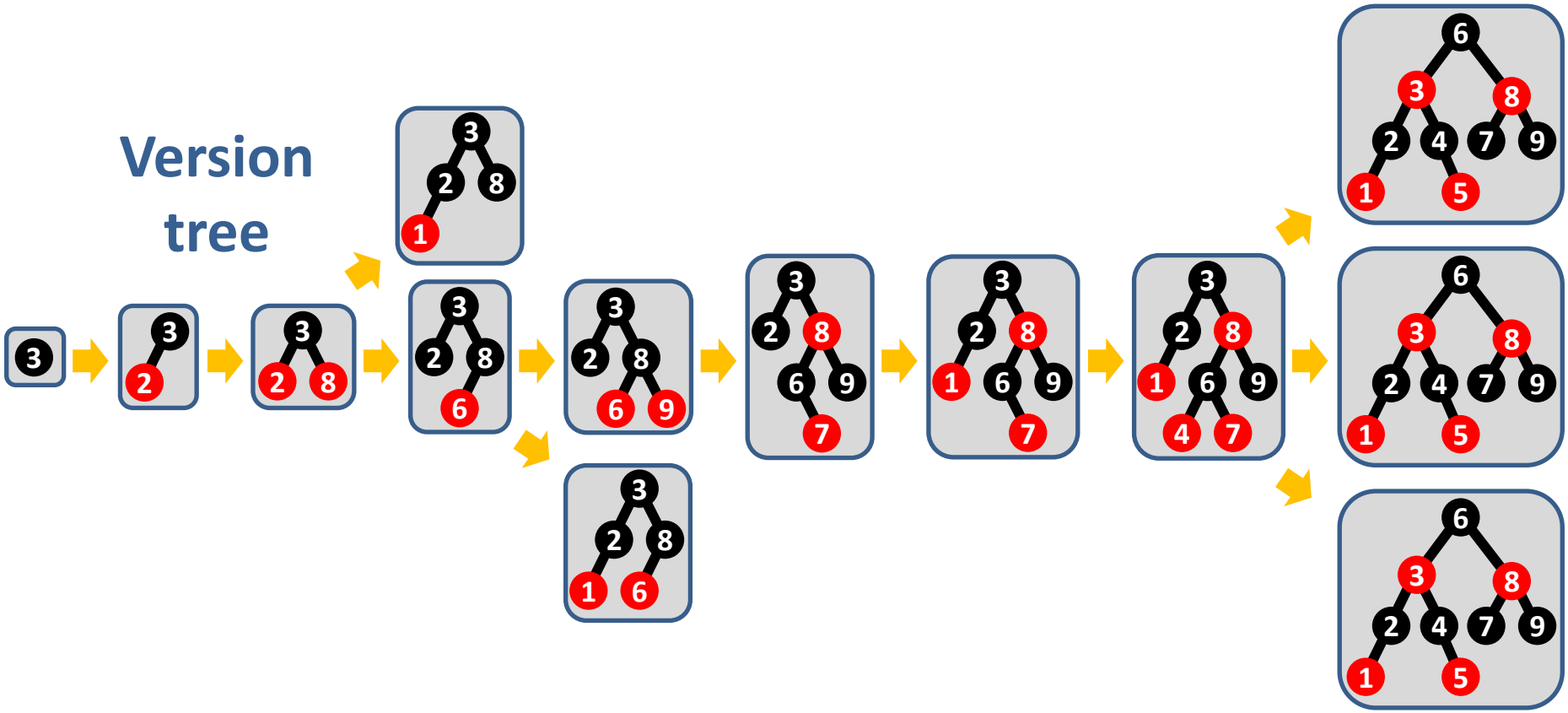
Driscoll, Sarnak, Sleator, Tarjan 1986

Partially Persistent Red-Black Tree



Driscoll, Sarnak, Sleator, Tarjan 1986

Fully Persistent Red-Black Tree



Driscoll, Sarnak, Sleator, Tarjan 1986

Fully Persistent Red-Black Tree

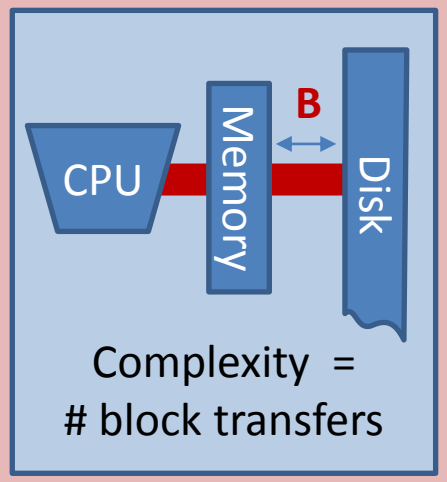
Contributions of Driscoll et al.

1. **Red**-**black** trees with O(1) modifications per update (displacement paths)
2. General technique to make pointer based data structures fully persistent

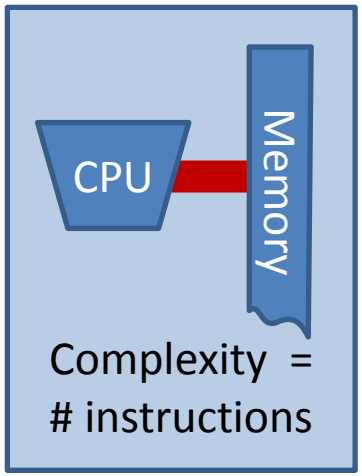
Expensive to record updates performing **many modifications**

Driscoll, Sarnak, Sleator, Tarjan 1986

IO model



RAM model



Binary Search Trees

Red-Black Trees
Bayer 1972

B-Trees
Bayer 1972

Maintaining Order in a List
Dietz, Sleator 1987

Red-Black Trees w. Displacement Paths
Driscoll et al. 1986

Incremental B-Trees

IO Efficient Node Splitting

Full Persistence: Node splitting
Driscoll et al. 1986

Fully Persistent B-Trees

Fully Persistent Balanced Binary Search Trees
Driscoll et al. 1986

Results

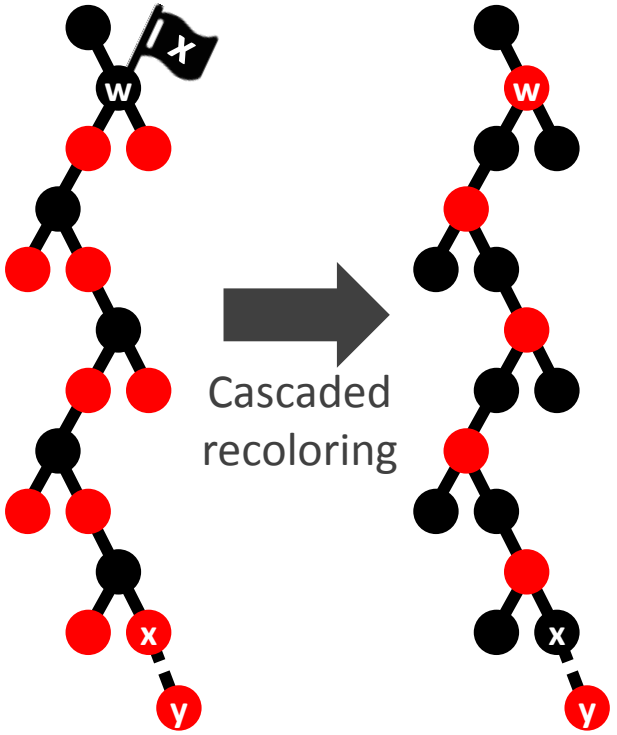
Incremental B-tree with range searches in $O(\log_B n + t/B)$ IOs, updates in $O(\log_B n)$ IOs with O(1) key and pointer modifications, O(n/B) space

Any emphemeral pointer based IO data structure with O(1) indegree can be made fully persistent with O(1) overhead per block access and $O(\log_2 B)$ IOs amortized overhead per field modification

Fully persistent B-tree with updates in $O(\log_B n + \log_2 B)$ IOs amortized, range searches in $O(\log_B n + t/B)$ IOs, O(m/B) space

	Persistent B-tree result	Query	Update
Partial	Arge, Danner, Teh 2003	$\log_B n + t/B$	$\log_B n$
Full	Lanka, Mays 1991	$(\log_B n + t/B) \cdot \log_B m$	$\log_B n \cdot \log_B m$
	This talk	$\log_B n + t/B$	$\log_B n + \log_2 B$

Incremental Search Tree Updates

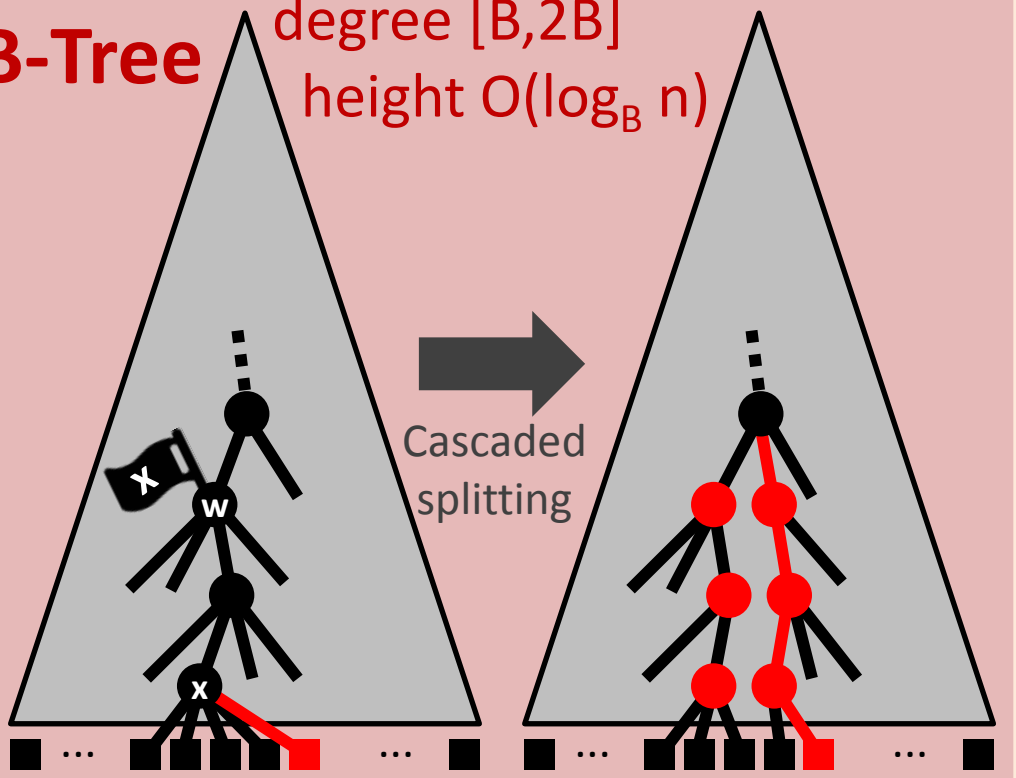


Displacement path w→x
≡ flip all colors on path + all red siblings (flag w with x)

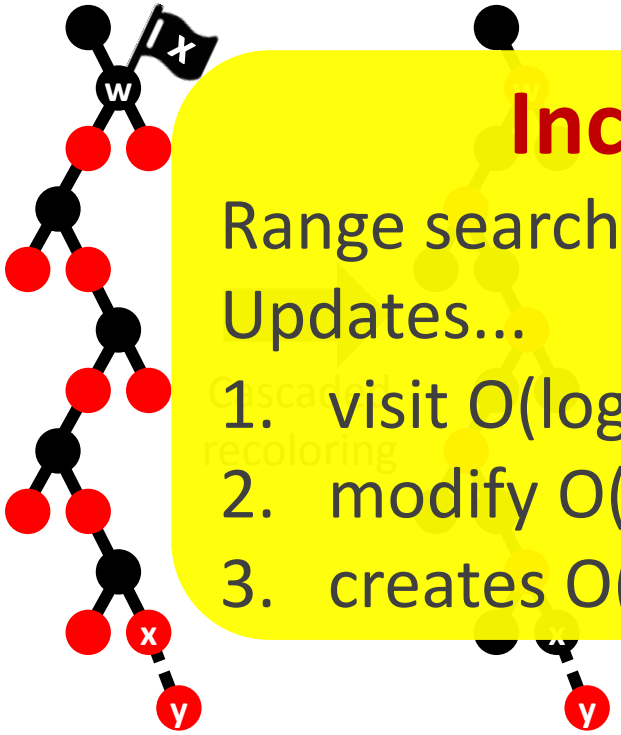
Driscoll et al. 1986

B-Tree

degree [B,2B]
height $O(\log_B n)$

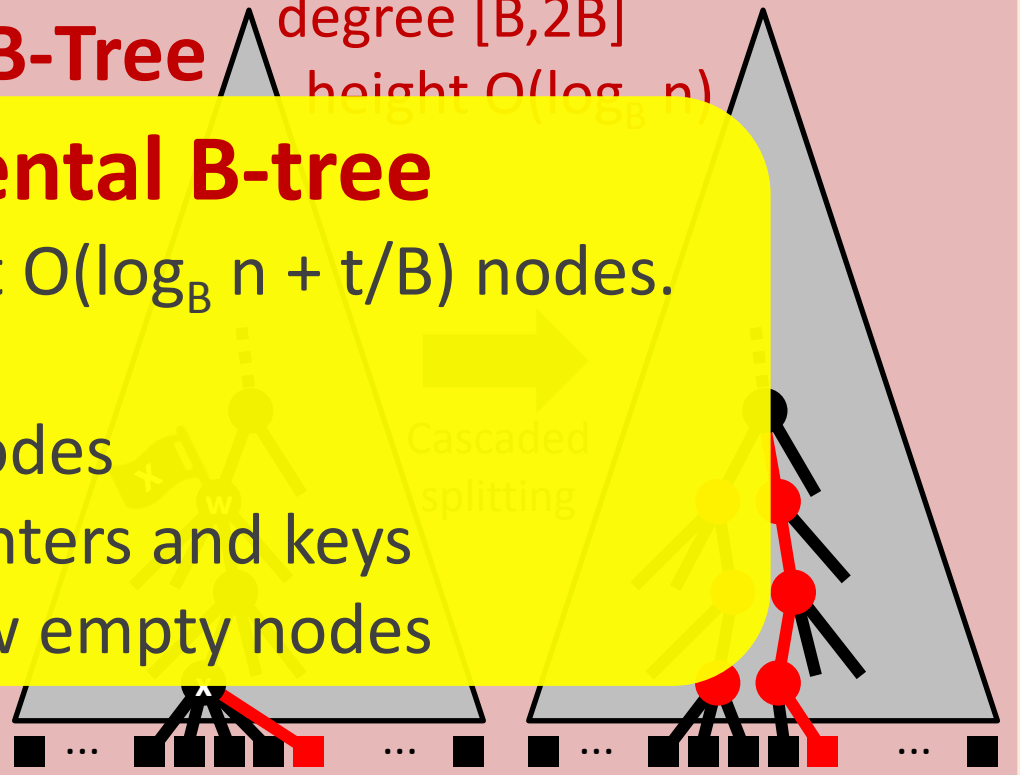


Incremental Search Tree Updates



B-Tree degree [B,2B]

height $O(\log_B n)$



Incremental B-tree

Range searches visit $O(\log_B n + t/B)$ nodes.

Updates...

1. visit $O(\log_B n)$ nodes
2. modify O(1) pointers and keys
3. creates O(1) new empty nodes

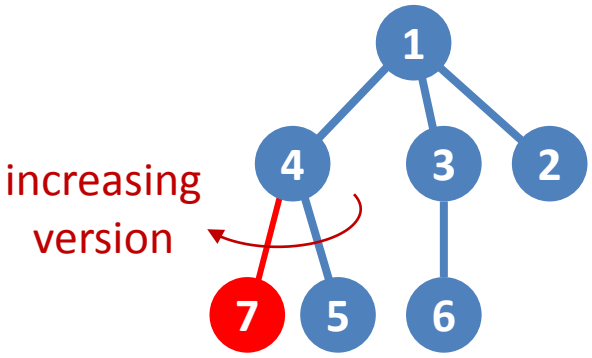
Displacement path w→x

≡ flip all colors on path + all red siblings (flag w with x)

Driscoll et al. 1986

Full persistence : Node Splitting Technique

Driscoll, Sarnak, Sleator, Tarjan 1986



Version tree (numbers = version ids)

preorder traversal

Version list
(order maintenance data structure)

Dietz, Sleator 1987

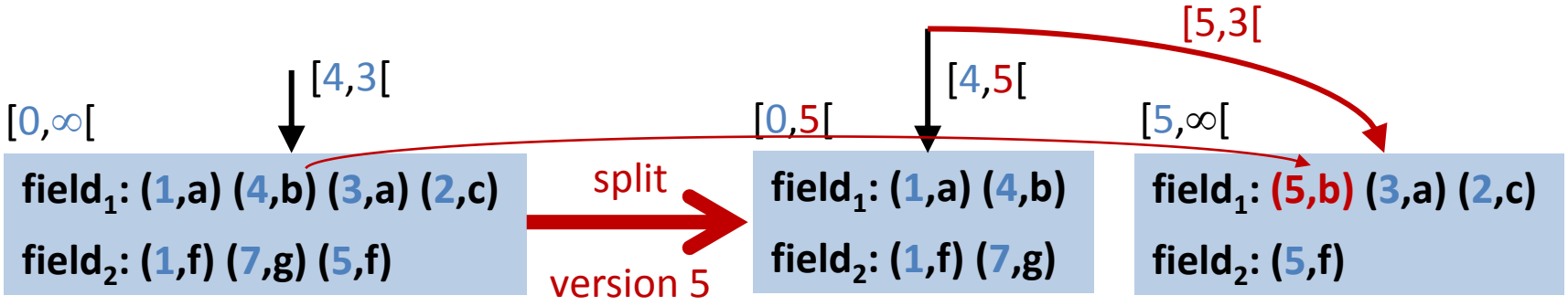
Fat node

Queries = binary search among versions

field: (1,x) (7,z) (5,x) (6,y) (2,x)

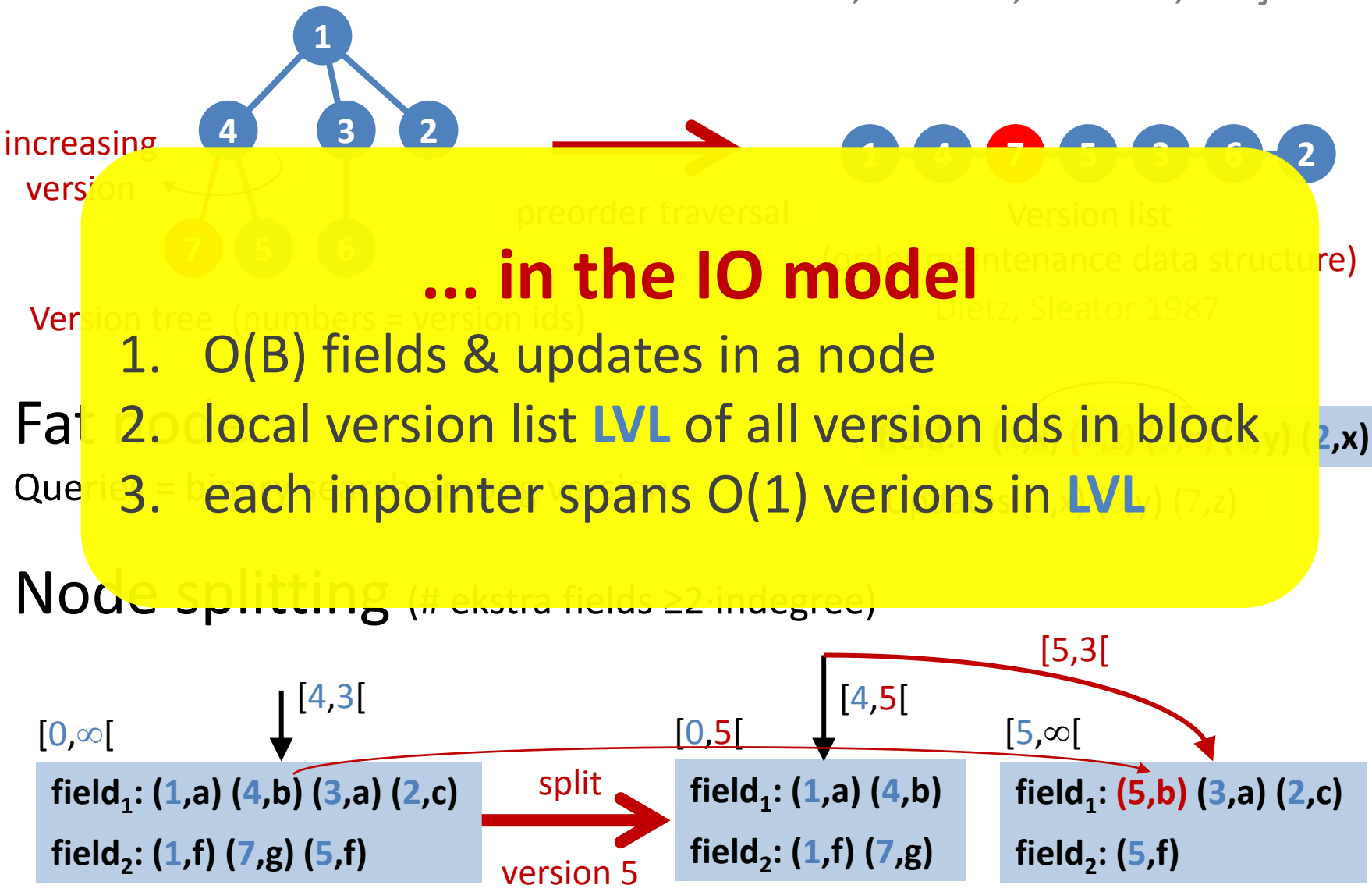
Updates (1,x) (6,y) (7,z)

Node splitting (# ekstra fields ≥2·indegree)



Full persistence : Node Splitting Technique

Driscoll, Sarnak, Sleator, Tarjan 1986



Main result

Fully persistent B-tree with updates in $O(\log_B n + \cancel{\log_2 B})$?
IOs, searches in $O(\log_B n + t/B)$ IOs, using space $O(m/B)$

Thank You