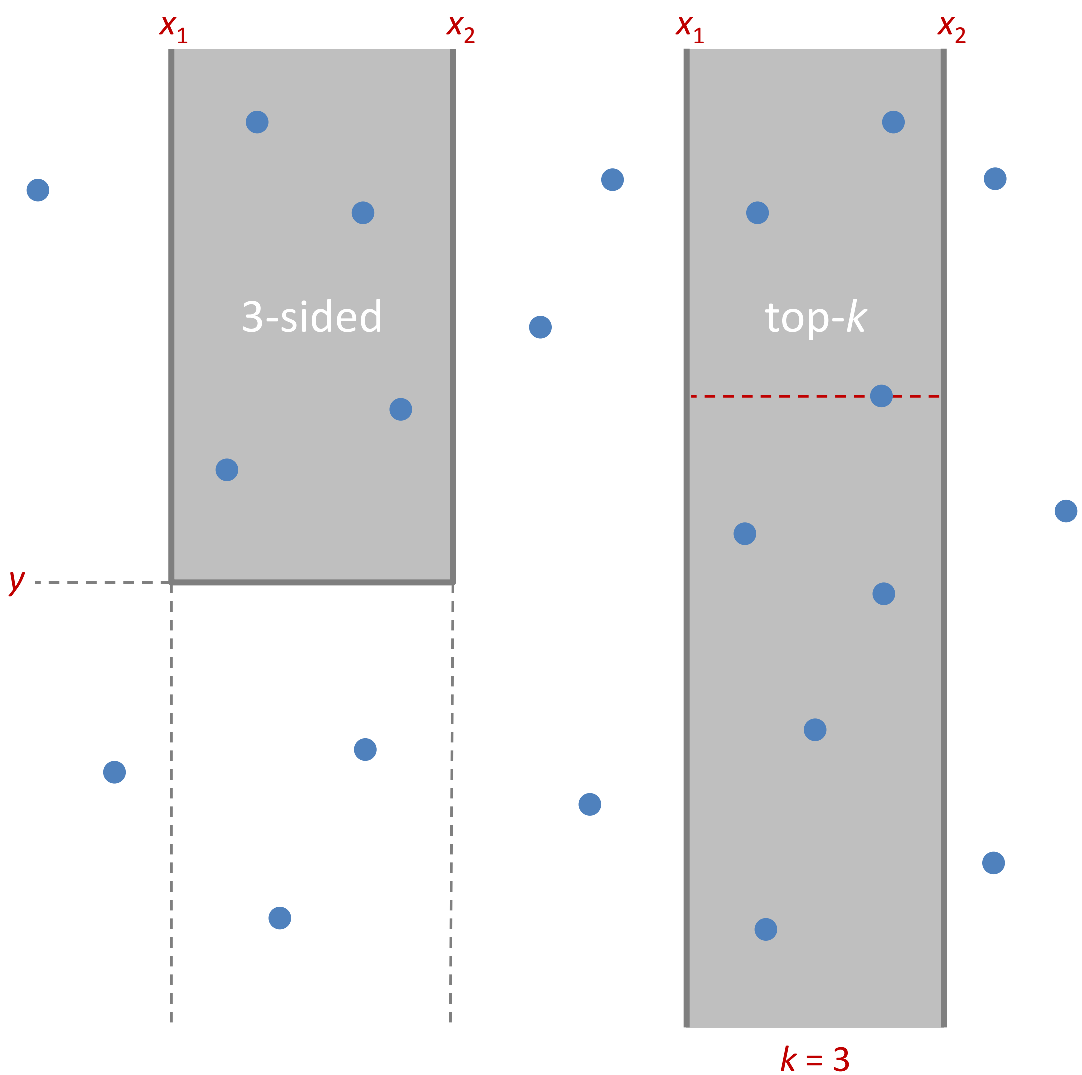
External Memory Three-Sided Range Reporting and Top-*k* Queries with Sublogarithmic Updates

arxiv.org/abs/1509.08240



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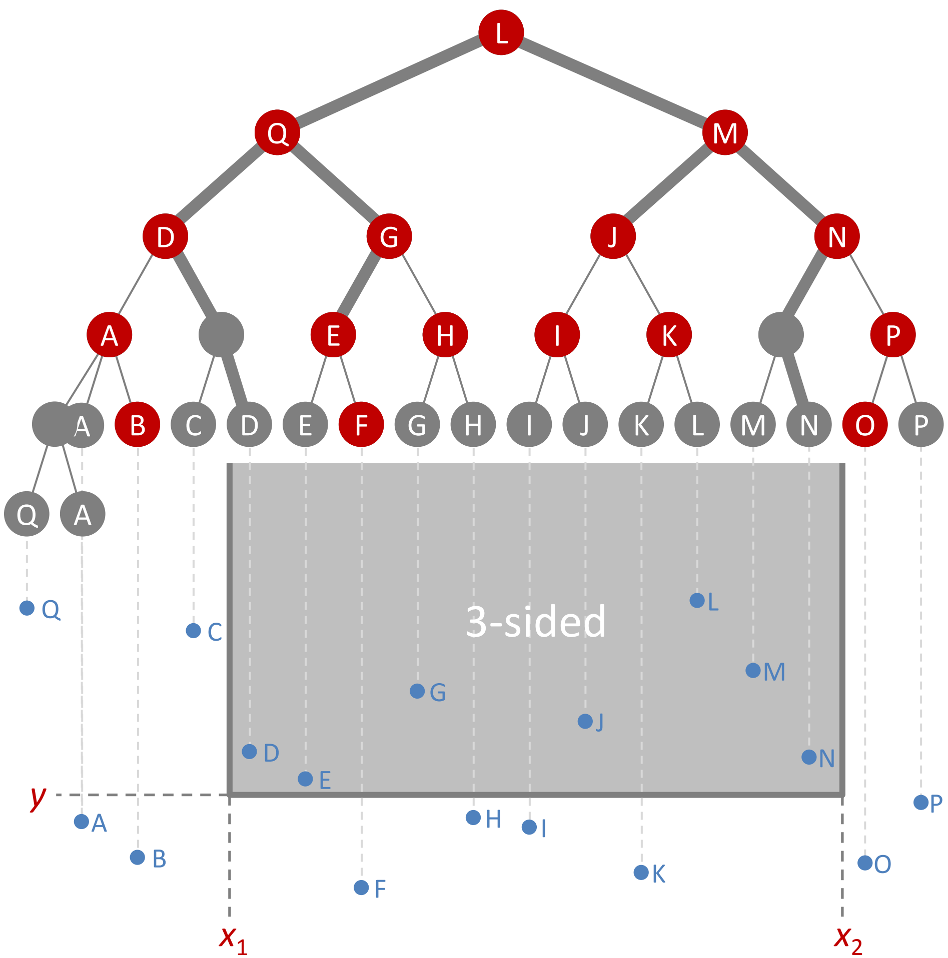
Internal Memory – Priority Search Trees

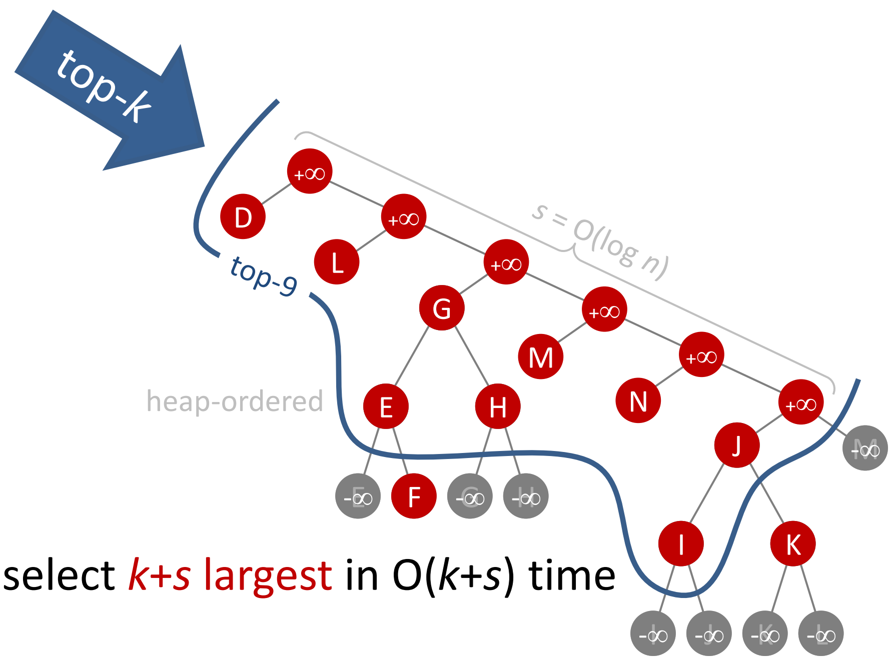
McCreight 1985
Frederickson 1993

Properties

- leaves x-sorted
- point p stored on leaf p-to-root path
- y-values satisfy heap-order

Updates O(log n)
3-sided & top-k O(k + log n)

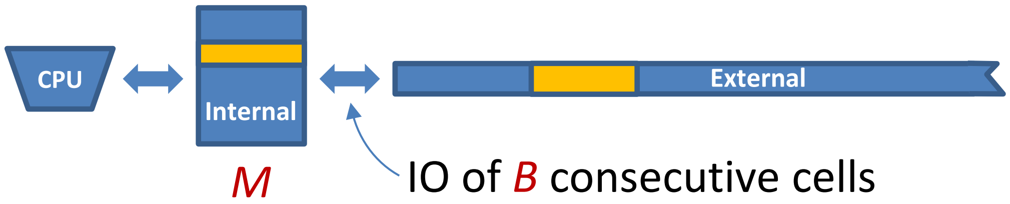




select *k*+*s* largest in O(k+s) time

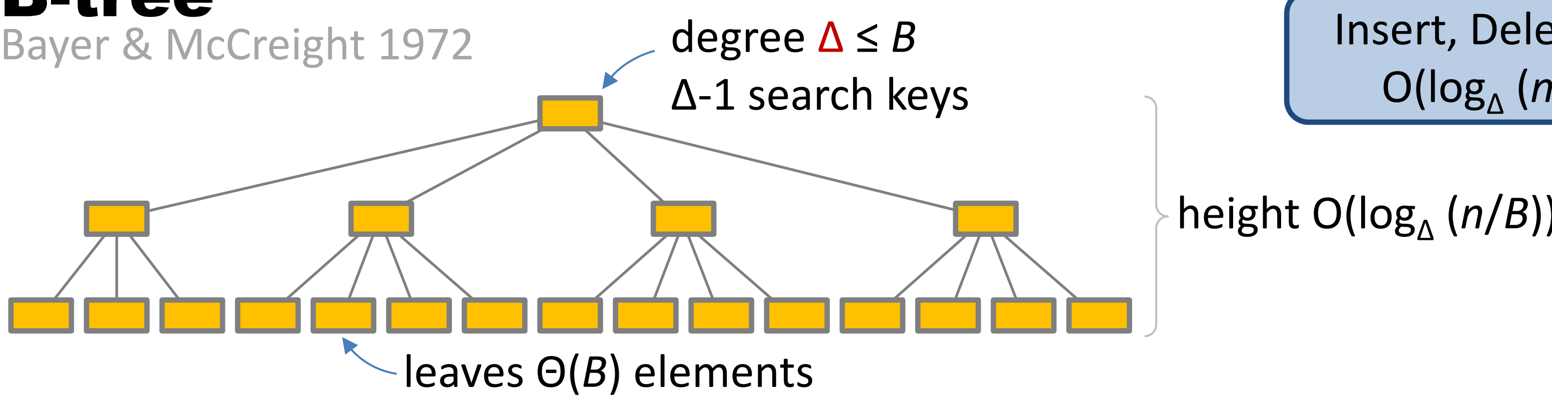
External Memory Model

Aggarwal & Vitter 1988



B-tree

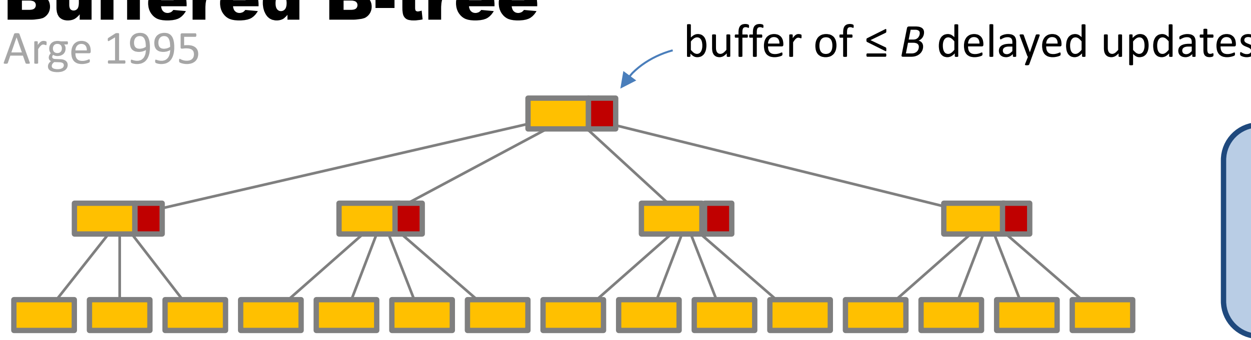
Bayer & McCreight 1972



Insert, Delete, Search
$O(\log_\Delta (n/B))$ IOs

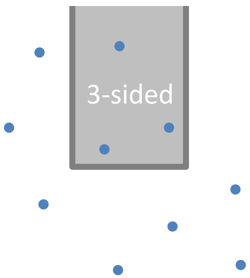
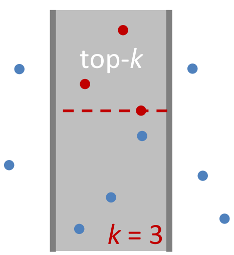
Buffered B-tree

Arge 1995



Search $O(\log_\Delta (n/B))$ IOs
Updates amortized
$O(\Delta/B \cdot \log_\Delta (n/B))$ IOs

Results

		Updates	Query
	Ramaswamy , Subramanian 1995	$O_A(\log n \cdot \log B)$	$O(\log_B n + k/B)$
	Subramanian, Ramaswamy 1995	$O_A(\log_B n + (\log_B n)^2/B)$	$O(\log_B n + k/B + \log^{**} B)$
	Arge et al. 1999	$O(\log_B n)$	$O(\log_B n + k/B)$
	NEW	$O_A(1/(\varepsilon B^{1-\varepsilon}) \cdot \log_B n)$	$O_A (1/\varepsilon \cdot \log_B n + k/B)$
	Afshani et al. 2011	(static)	$O(\log_B n + k/B)$
	Sheng, Tao 2012	$O_A((\log_B n)^2)$	$O(\log_B n + k/B)$
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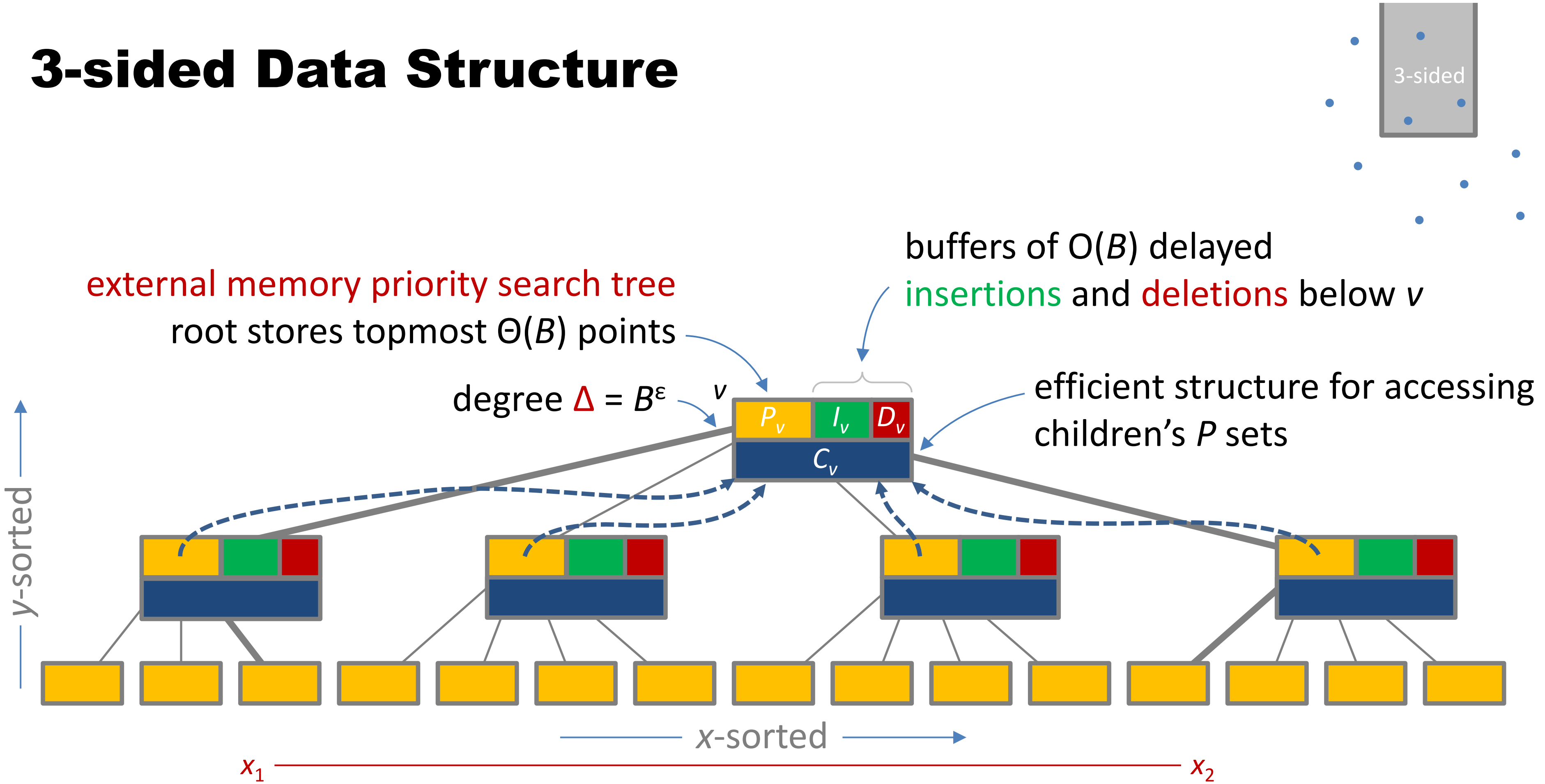
O_A = amortized

NEW result : Combination of Arge 1995, Arge et al. 1999, Frederickson 1993, Blum et al. 1973

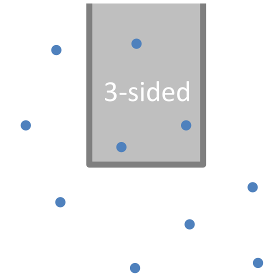
3-sided Data Structure

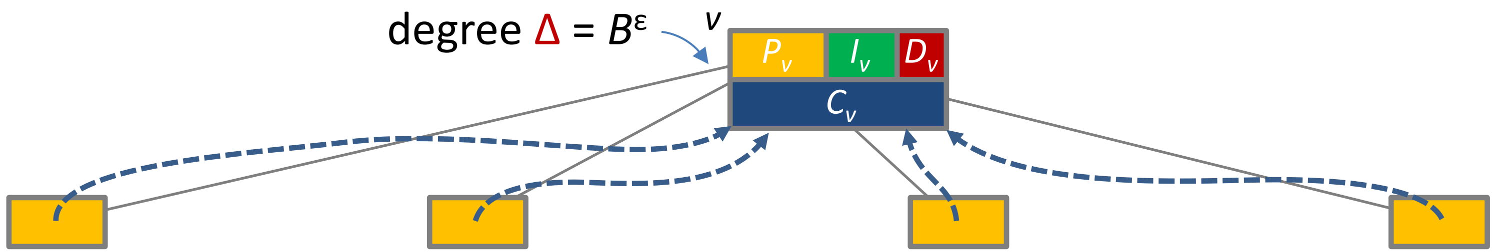


- Insertions / deletions : Update root P_v or add to delayed update buffer I_v / D_v
- Update buffer overflow : Flush recursively to child with most updates ($\geq B^{1-\varepsilon}$)
- Leaf overflow : split leaf, and recursively split ancestors of degree Δ+1
- Underflowing point buffer P_v : pull elements recursively from children using C_v
- 3-sided query : i) Identify nodes to visit using C_v structures. ii) flush updates down from ancestors of visited nodes. iii) report from nodes using P_v, C_v and update buffers

Child Structure C_v

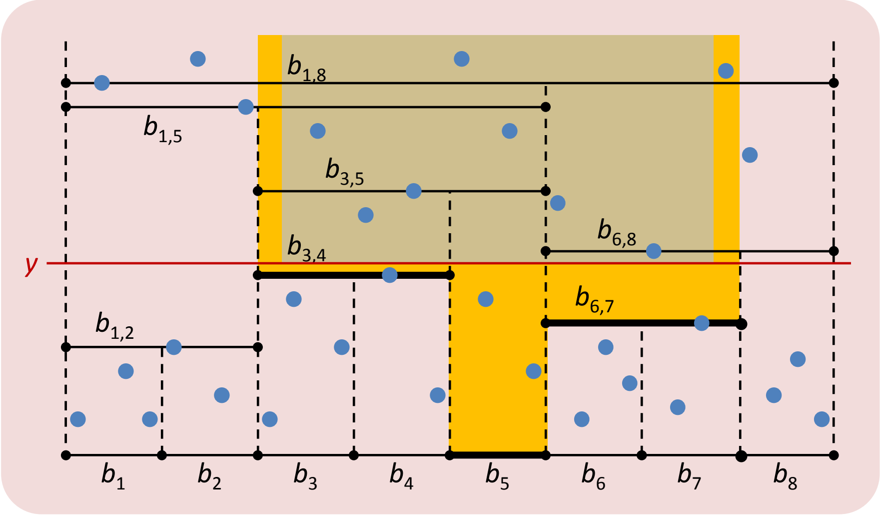
Arge et al. 1999



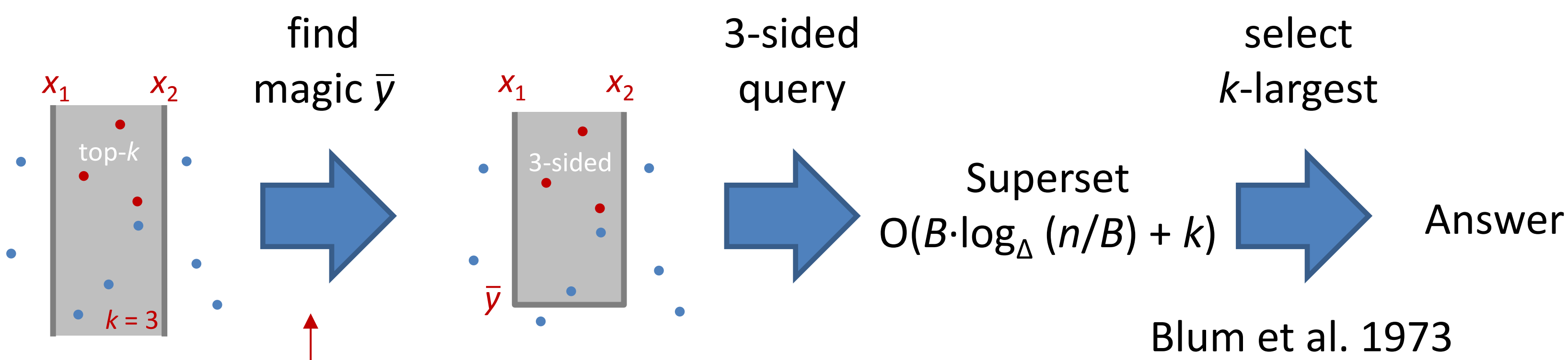


Insert / delete s points : $O(1 + s/B^{1-\varepsilon})$ IOs
3-sided query : $O(1 + k/B)$ IOs
y-samples for range $[x_1, x_2]$: $O(1)$ IOs (new)

- Capacity : $B^{1+\varepsilon}$
- Insetion /deletion buffer $O(B)$ points
- $O(B^{\varepsilon})$ blocks
- Catalog block
- y-samples block (new)



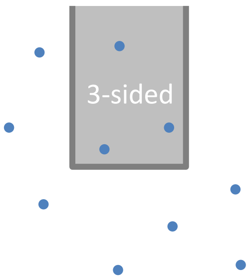
Top-k – Overall Approach



All steps require $O(\log_\Delta (n/B) + k/B)$ IOs
The 3-sided query is amortized

Construct (on demand) a **binary heap** over every $\Theta(B)$'th element in the C_v structures – and select the $\Theta(\log_\Delta (n/B) + k/B)$'th element using Frederickson 1993

Summary – The End

		Updates	Query
	Ramaswamy , Subramanian 1995	$O_A(\log n \cdot \log B)$	$O(\log_B n + k/B)$
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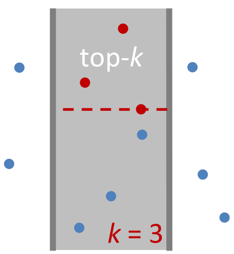	Afshani et al. 2011	(static)	$O(\log_B n + k/B)$
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O_A = amortized

Open problem : Remove amortization ?