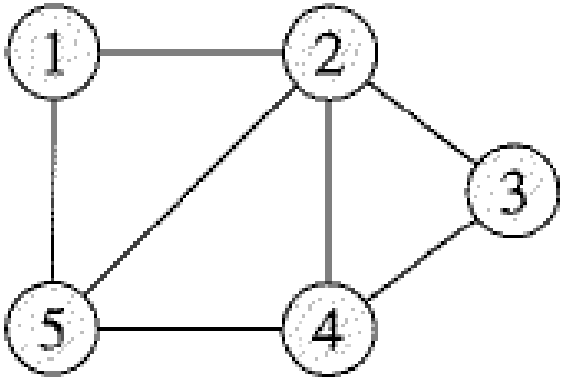
Algoritmer og Datastrukturer 2

Gerth Stølting Brodal

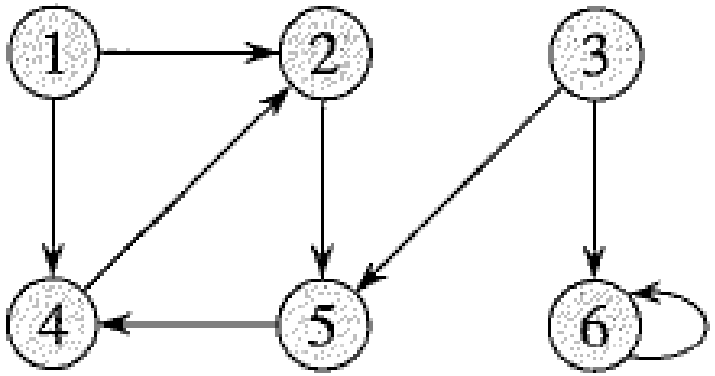
Graf repræsentationer, BFS og DFS
[CLRS, kapitel 22.1-22.3]

AARHUS UNIVERSITET

Grafer



Uorienterede grafer



Orienterede grafer

***G* = (*V*,*E*)** graf med knuder *V* og kanter *E*

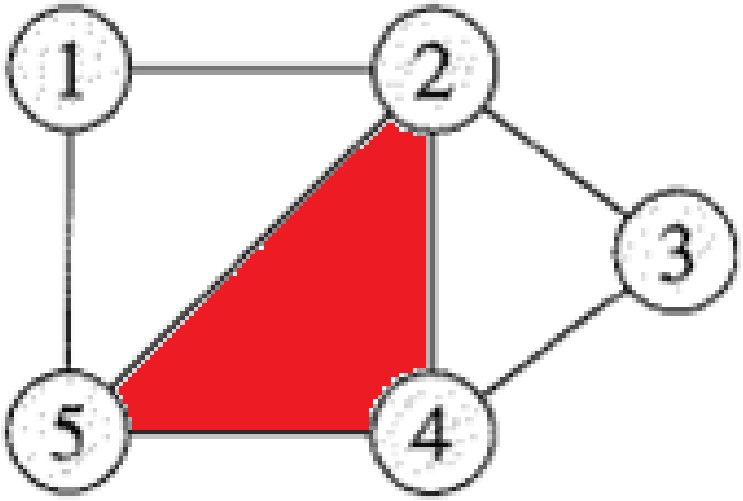
E : **{*u*,*v*}** kant mellem *u* og *v* i en uorienteret graf og

(*u*,*v*) en orienteret kant fra *u* til *v*.

n = |*V*| = antal knuder

m = |E| = antal kanter (forbindelser mellem knuder)

Planar Grafer - Eulers formel



$V = 5$

$E = 7$

flader = 4

For en sammenhængende planar graf gælder:

Eulers formel: $|V| - |E| + \#\text{ flader} = 2$

Korollar: $|E| \leq 3|V| - 6$

(for $|V| \geq 3$, ingen selvløkker, ingen parallelle kanter)

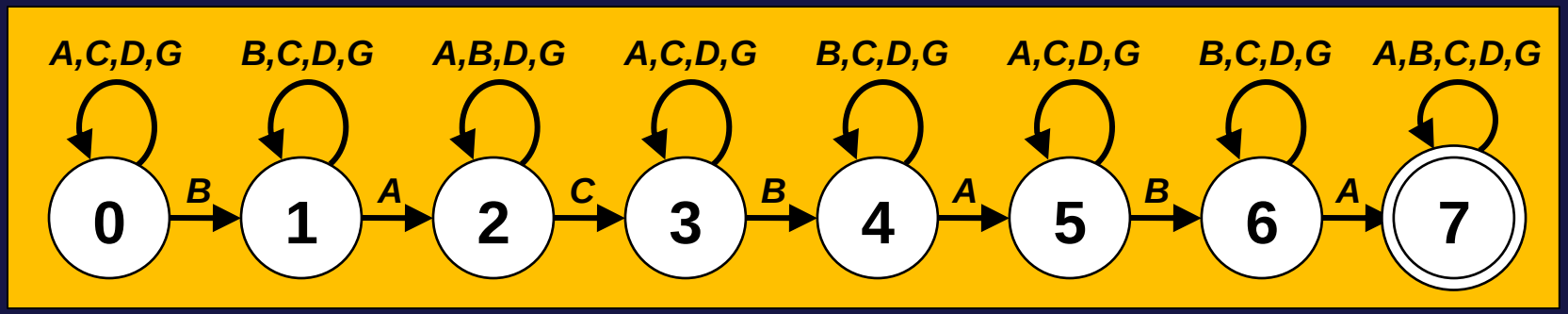
Hvilken løsning finder den grådige algoritme?

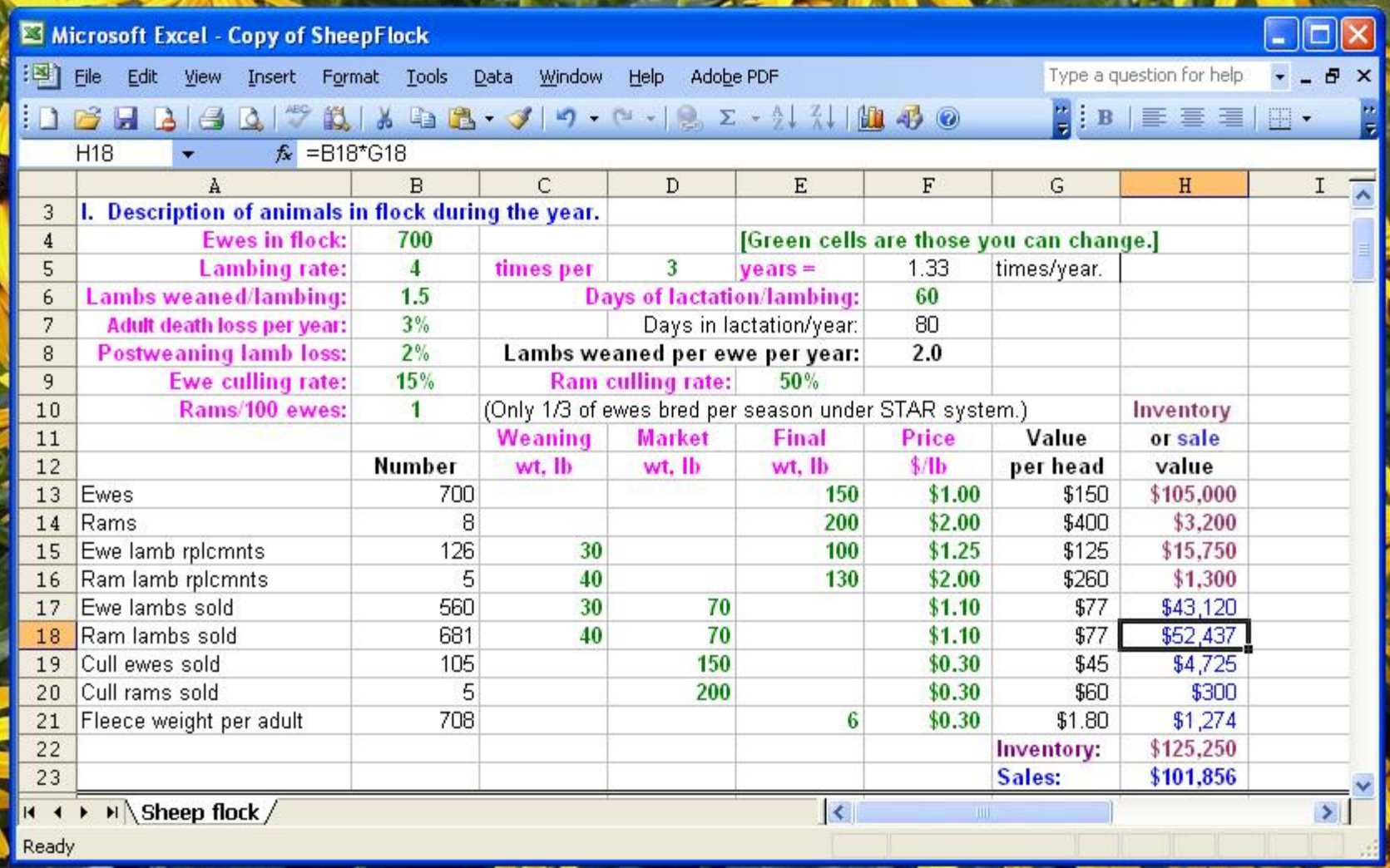
a) ***ABABGACBABAD***

b) ***ABABGACBABAD***

c) ***ABABGACBABAD***

d) Ved ikke





Microsoft Excel - Copy of SheepFlock

File Edit View Insert Format Tools Data Window Help Adobe PDF

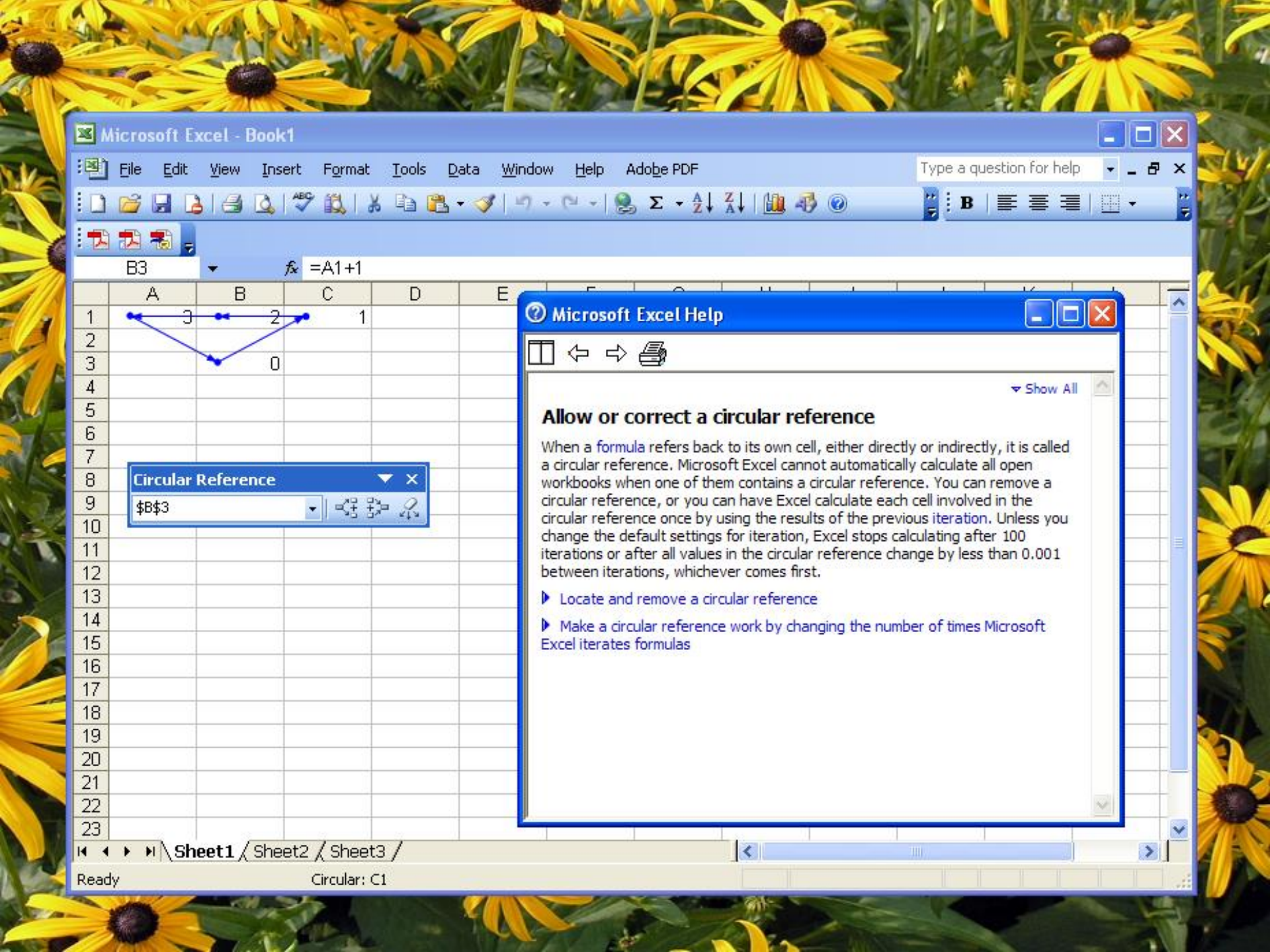
Type a question for help

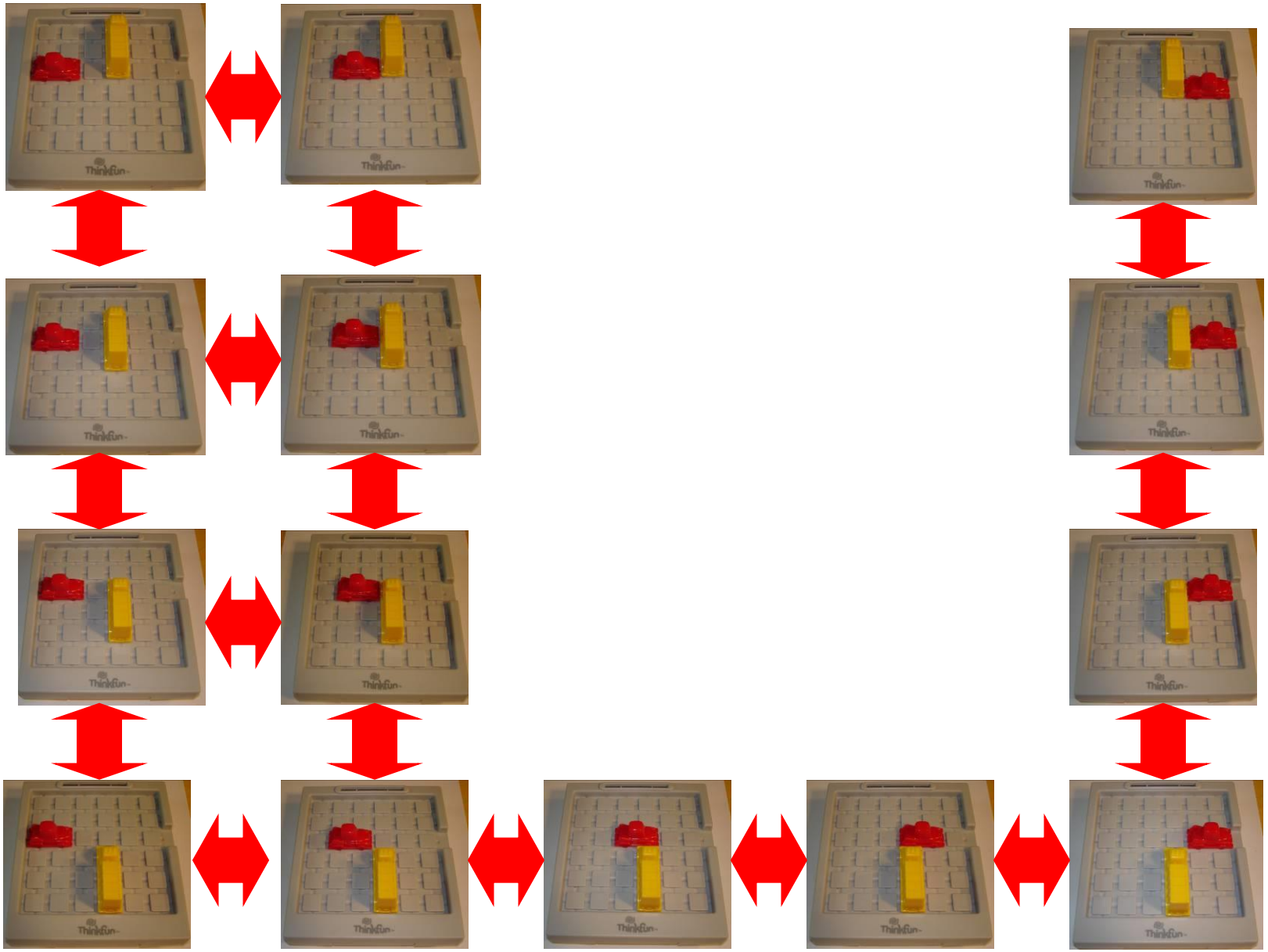
H18 =B18*G18

	A	B	C	D	E	F	G	H	I
3	I. Description of animals in flock during the year.								
4	Ewes in flock:	700			[Green cells are those you can change.]				
5	Lambing rate:	4	times per	3	years =	1.33	times/year.		
6	Lambs weaned/lambing:	1.5			Days of lactation/lambing:	60			
7	Adult death loss per year:	3%			Days in lactation/year:	80			
8	Postweaning lamb loss:	2%	Lambs weaned per ewe per year:			2.0			
9	Ewe culling rate:	15%	Ram culling rate:		50%				
10	Rams/100 ewes:	1	(Only 1/3 of ewes bred per season under STAR system.)					Inventory	
11			Weaning	Market	Final	Price	Value	or sale	
12		Number	wt, lb	wt, lb	wt, lb	$/lb	per head	value	
13	Ewes	700			150	$1.00	$150	$105,000	
14	Rams	8			200	$2.00	$400	$3,200	
15	Ewe lamb rplcmnts	126	30		100	$1.25	$125	$15,750	
16	Ram lamb rplcmnts	5	40		130	$2.00	$260	$1,300	
17	Ewe lambs sold	560	30	70		$1.10	$77	$43,120	
18	Ram lambs sold	681	40	70		$1.10	$77	$52,437	
19	Cull ewes sold	105		150		$0.30	$45	$4,725	
20	Cull rams sold	5		200		$0.30	$60	$300	
21	Fleece weight per adult	708			6	$0.30	$1.80	$1,274	
22							Inventory:	$125,250	
23							Sales:	$101,856	

Sheep flock

Ready


Microsoft Excel - Book1
File Edit View Insert Format Tools Data Window Help Adobe PDF
Type a question for help
B3
=A1+1
A B C D E
1 3 2 1
3 0
Circular Reference
B3
Microsoft Excel Help
Show All
Allow or correct a circular reference
When a formula refers back to its own cell, either directly or indirectly, it is called a circular reference. Microsoft Excel cannot automatically calculate all open workbooks when one of them contains a circular reference. You can remove a circular reference, or you can have Excel calculate each cell involved in the circular reference once by using the results of the previous iteration. Unless you change the default settings for iteration, Excel stops calculating after 100 iterations or after all values in the circular reference change by less than 0.001 between iterations, whichever comes first.
Locate and remove a circular reference
Make a circular reference work by changing the number of times Microsoft Excel iterates formulas
Sheet1 Sheet2 Sheet3
Ready
Circular: C1

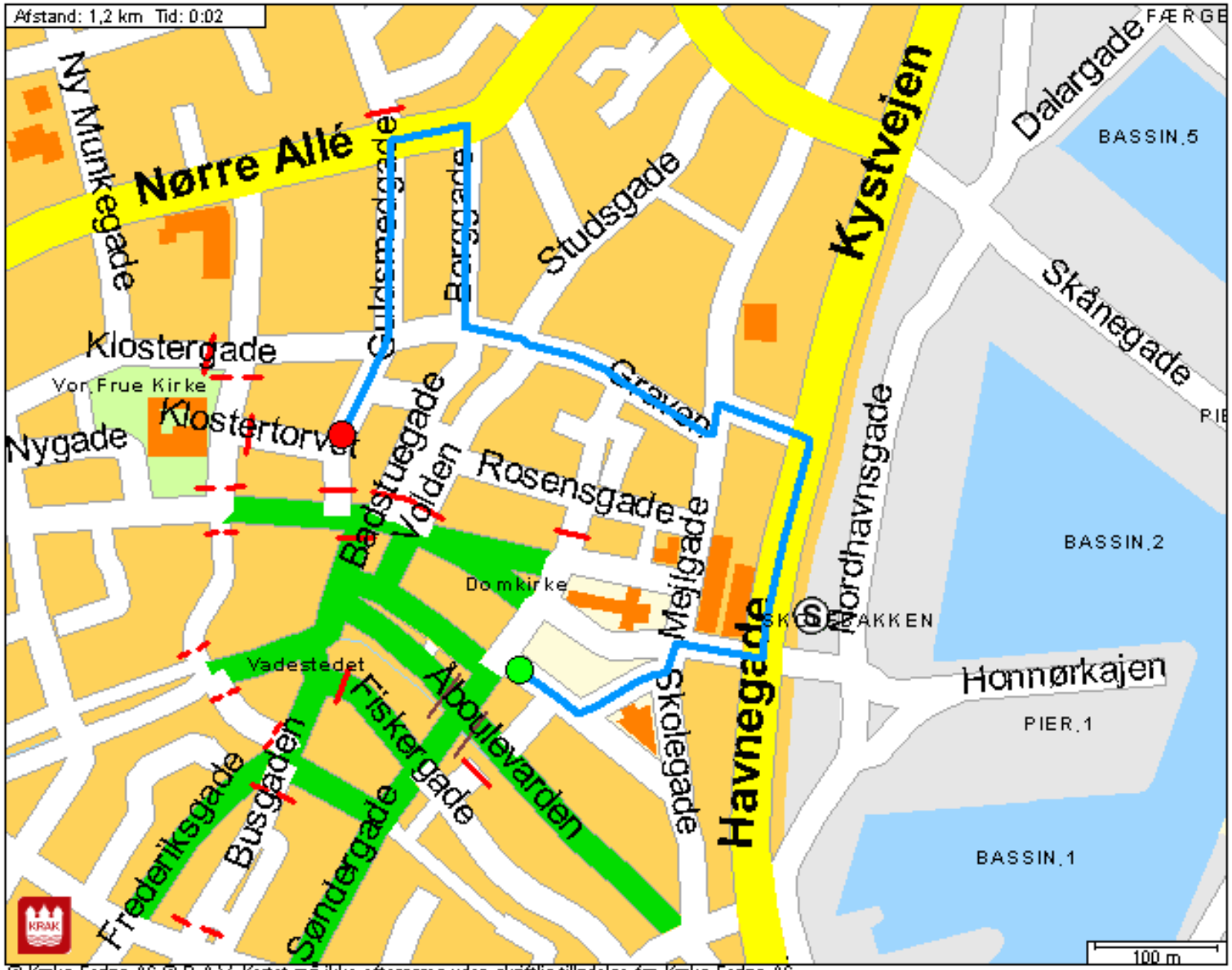


Rute på kort

Fra Kannikegade 1 , 8000 Århus C

Til Guldsmedgade 1 , 8000 Århus C

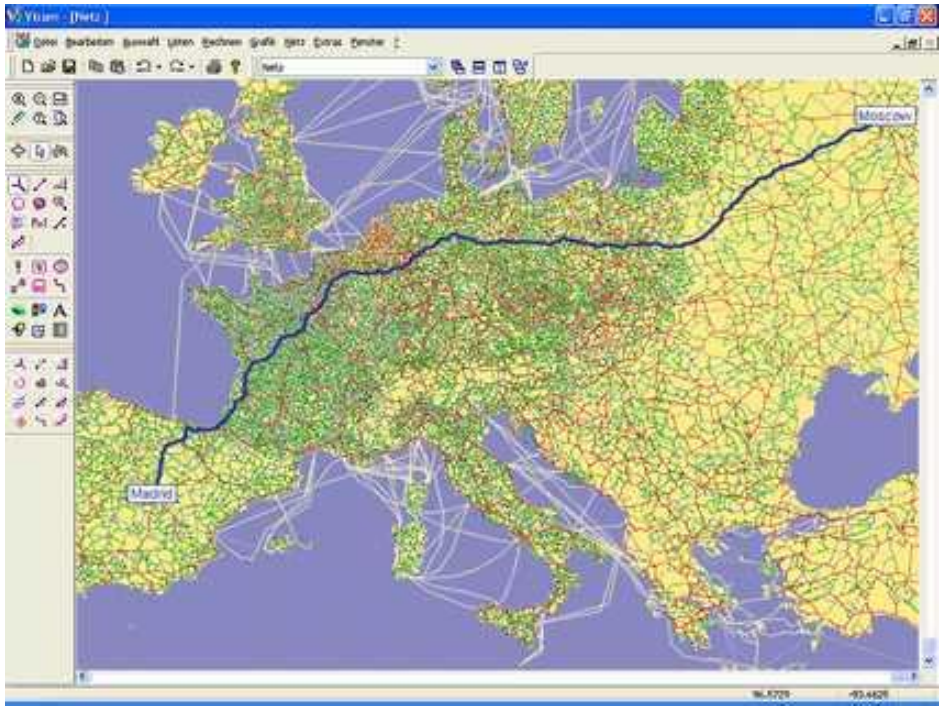
Via

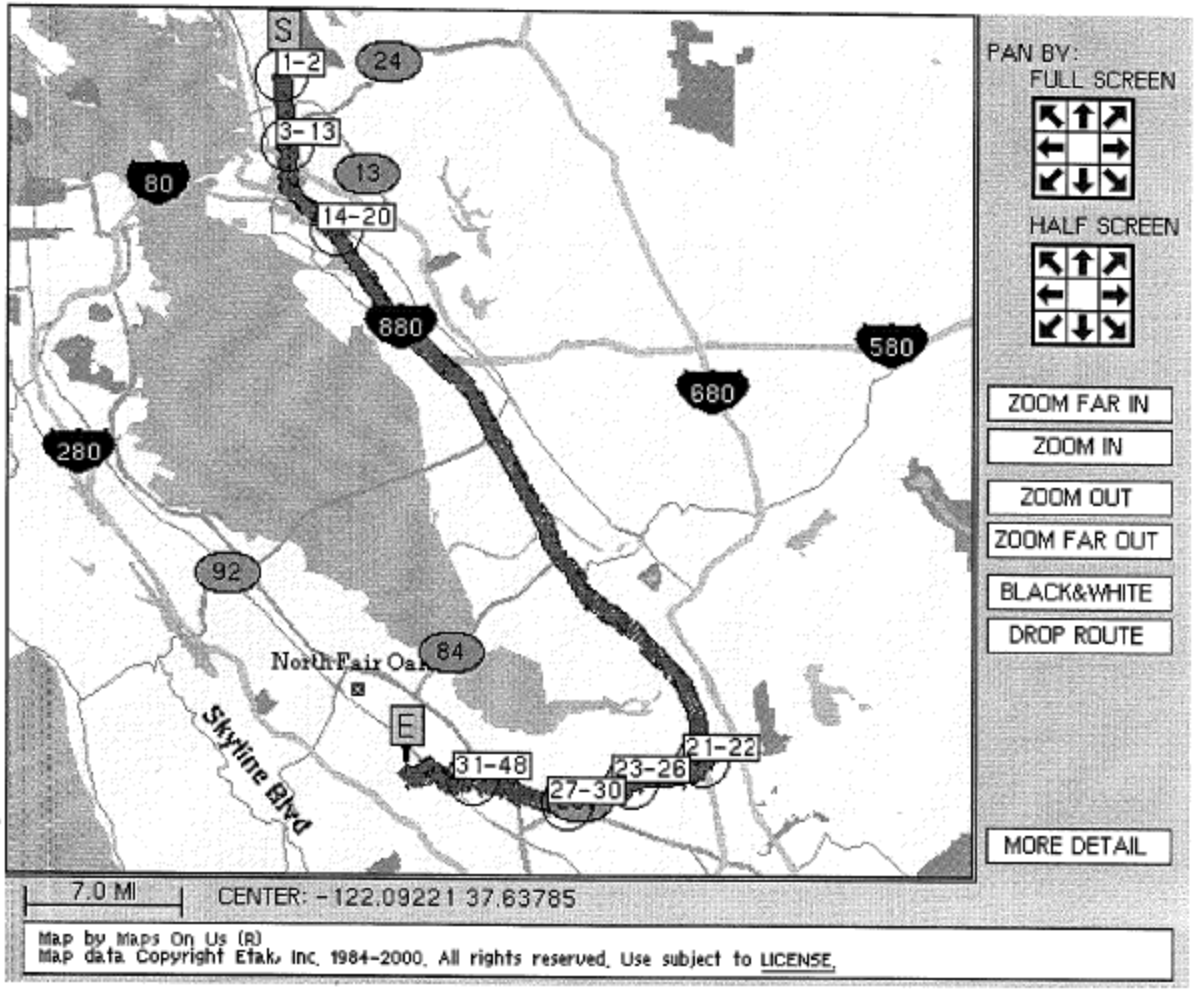


Kort over Vest-Europa

- **18.029.721 knuder**
- **42.199.587 orienterede kanter**

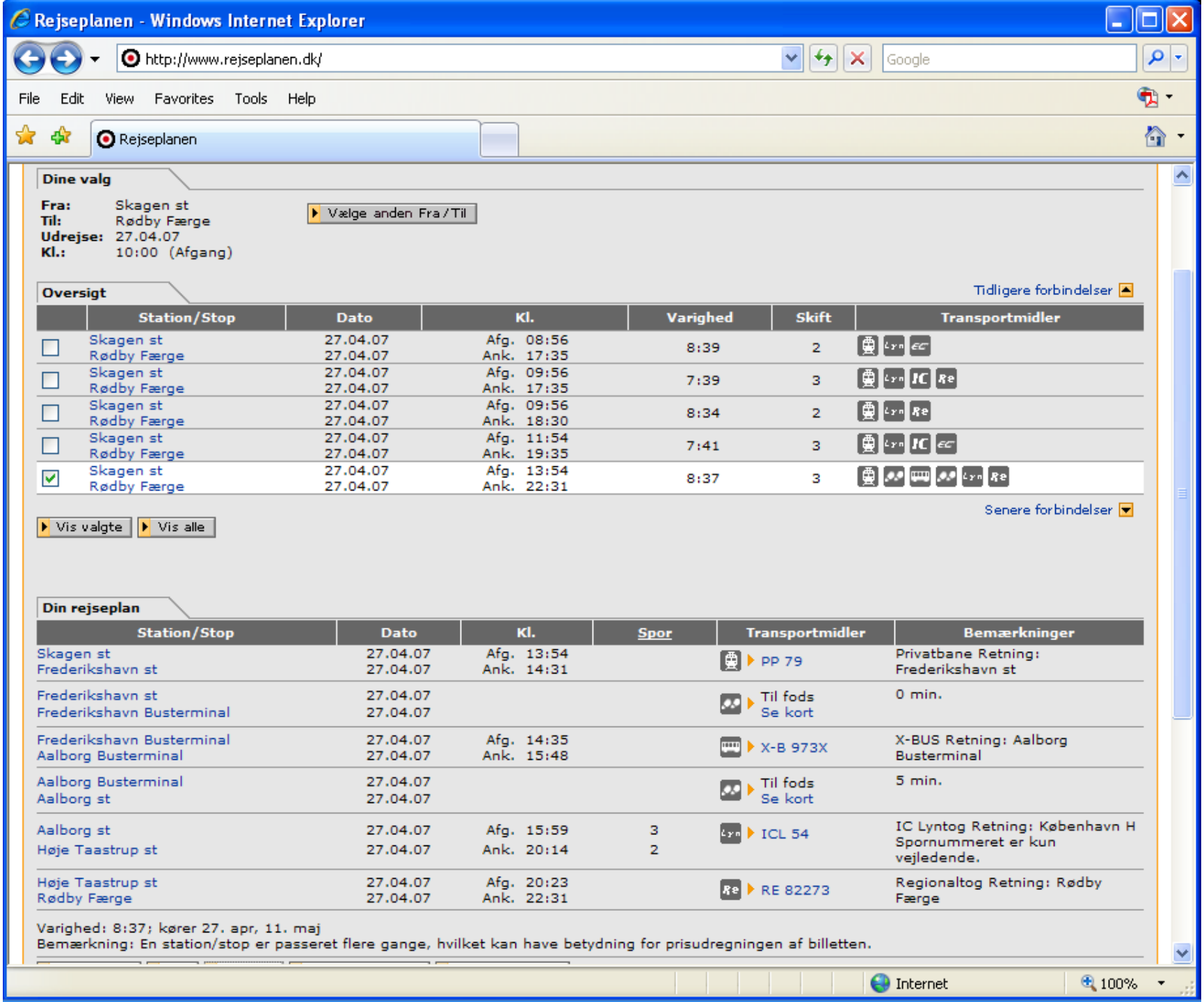
traffic mobility logistics.



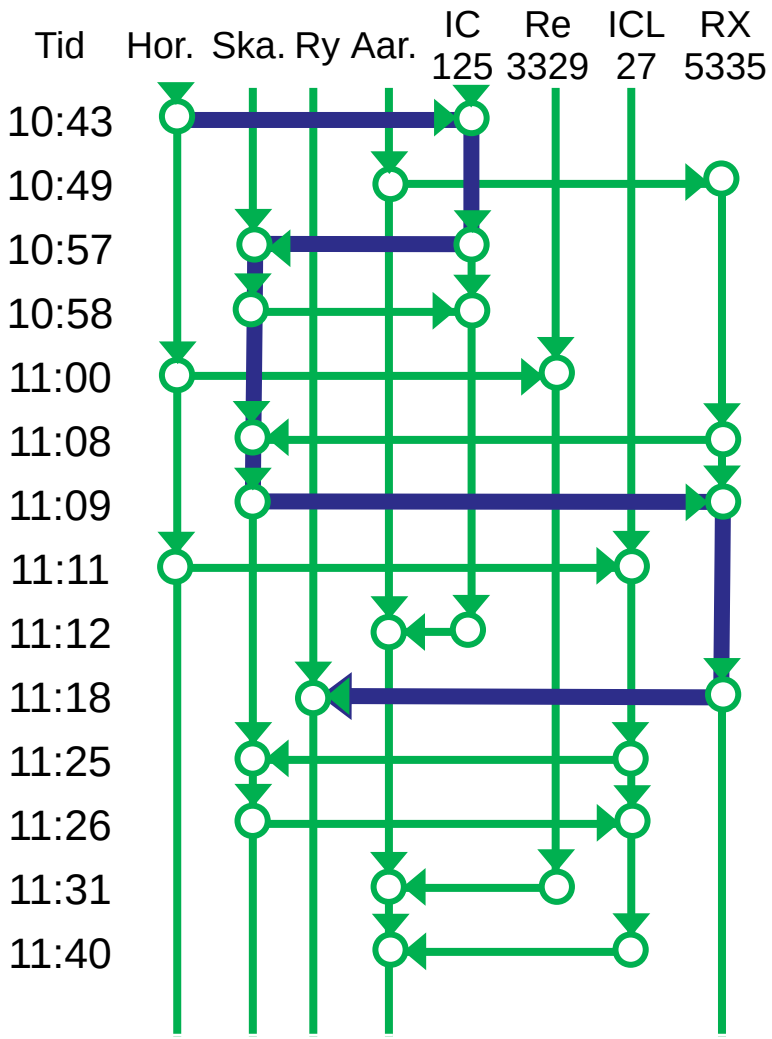


"However, because of the size of the routing data, we have to use heuristics when planning routes. As a result, sometimes a Favor Highways route will be slightly faster than the Fastest route."

— MapsOnUs


Rejseplanen - Windows Internet Explorer
http://www.rejseplanen.dk/
Google
File Edit View Favorites Tools Help
Rejseplanen
Dine valg
Fra: Skagen st
Til: Rødby Færge
Udrejse: 27.04.07
Kl.: 10:00 (Afgang)
Vælge anden Fra / Til
Oversigt
Tidligere forbindelser
Station/Stop | Dato | Kl. | Varighed | Skift | Transportmidler
Skagen st Rødby Færge | 27.04.07 27.04.07 | Afg. 08:56 Ank. 17:35 | 8:39 | 2
Skagen st Rødby Færge | 27.04.07 27.04.07 | Afg. 09:56 Ank. 17:35 | 7:39 | 3
Skagen st Rødby Færge | 27.04.07 27.04.07 | Afg. 09:56 Ank. 18:30 | 8:34 | 2
Skagen st Rødby Færge | 27.04.07 27.04.07 | Afg. 11:54 Ank. 19:35 | 7:41 | 3
Skagen st Rødby Færge | 27.04.07 27.04.07 | Afg. 13:54 Ank. 22:31 | 8:37 | 3
Senere forbindelser
Vis valgte
Vis alle
Din rejseplan
Station/Stop | Dato | Kl. | Spor | Transportmidler | Bemærkninger
Skagen st Frederikshavn st | 27.04.07 27.04.07 | Afg. 13:54 Ank. 14:31 | | PP 79 | Privatbane Retning: Frederikshavn st
Frederikshavn st Frederikshavn Busterminal | 27.04.07 27.04.07 | | | Til fods Se kort | 0 min.
Frederikshavn Busterminal Aalborg Busterminal | 27.04.07 27.04.07 | Afg. 14:35 Ank. 15:48 | | X-B 973X | X-BUS Retning: Aalborg Busterminal
Aalborg Busterminal Aalborg st | 27.04.07 27.04.07 | | | Til fods Se kort | 5 min.
Aalborg st Høje Taastrup st | 27.04.07 27.04.07 | Afg. 15:59 Ank. 20:14 | 3 2 | ICL 54 | IC Lyntog Retning: København H Spornummeret er kun vejledende.
Høje Taastrup st Rødby Færge | 27.04.07 27.04.07 | Afg. 20:23 Ank. 22:31 | | RE 82273 | Regionaltog Retning: Rødby Færge
Varighed: 8:37; kører 27. apr, 11. maj
Bemærkning: En station/stop er passeret flere gange, hvilket kan have betydning for prisudregningen af billetten.
Internet
100%

Rejseplan (Horsens til Ry)

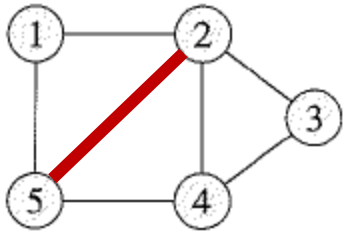


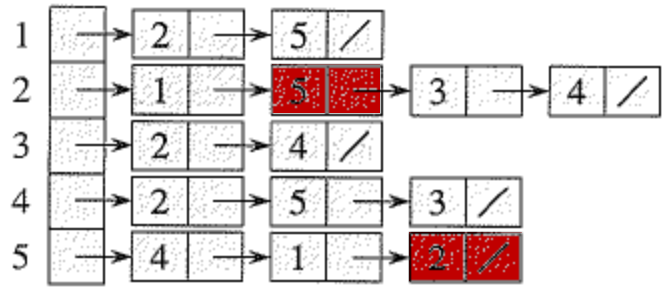
Tog	Ank	Afg	Station
		10:43	Horsens
IC125	10:57	10:58	Skanderborg St
	11:12		Aarhus H
		11:00	Horsens
Re3329	11:31		Aarhus H
		11:11	Horsens
ICL27	11:25	11:26	Skanderborg St
	11:40		Aarhus H
		10:49	Aarhus H
RX5335	11:08	11:09	Skanderborg St
	11:18		Ry St

uddrag af køreplaner

Algoritme
Find tidligste knude for Ry der kan nås fra en given start-knude i Horsens

Graf repræsentationer: Incidenslister og incidensmatricer



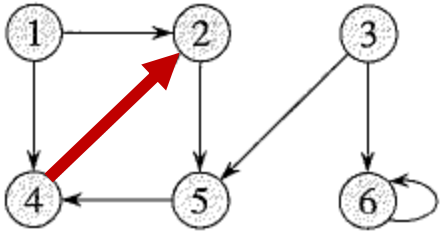


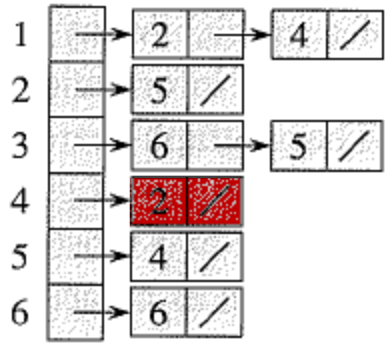
	1	2	3	4	5
1	0	1	0	0	1
2	1	0	1	1	1
3	0	1	0	1	0
4	0	1	1	0	1
5	1	1	0	1	0

Uorienterede grafer

Plads O(n+m)

Plads O(n^2)





	1	2	3	4	5	6
1	0	1	0	1	0	0
2	0	0	0	0	1	0
3	0	0	0	0	1	1
4	0	1	0	0	0	0
5	0	0	0	1	0	0
6	0	0	0	0	0	1

Orienterede grafer

Bredde først søgning (BFS)

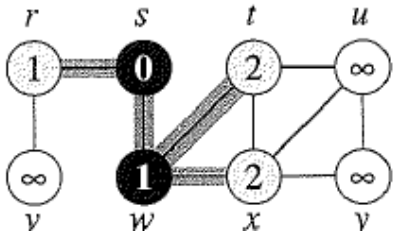
```
BFS(G, s)
 1  for each vertex u ∈ G.V − {s}
 2      u.color = WHITE
 3      u.d = ∞
 4      u.π = NIL
 5  s.color = GRAY
 6  s.d = 0
 7  s.π = NIL
 8  Q = ∅
 9  ENQUEUE(Q, s)
10  while Q ≠ ∅
11      u = DEQUEUE(Q)
12      for each v ∈ G.Adj[u]
13          if v.color == WHITE
14              v.color = GRAY
15              v.d = u.d + 1
16              v.π = u
17              ENQUEUE(Q, v)
18      u.color = BLACK
```

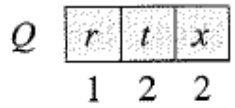
u.color:
WHITE = knuderne endnu ikke besøgt
GRAY = knuderne i køen Q
BLACK = knuderne besøgt

$u.d$ = afstand til s

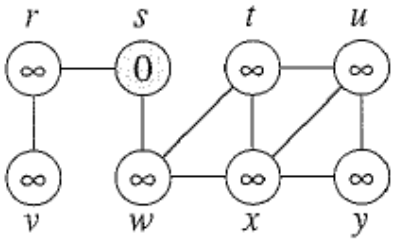
$u.\pi$ = faderen til u i BFS træet

Q = kø af grå knuder (som er forbundet til sorte knuder)

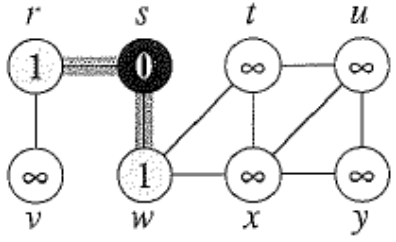




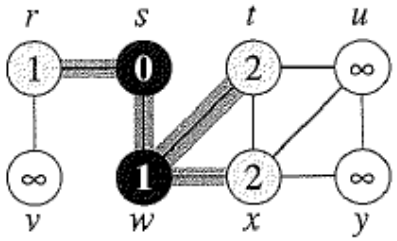
Tid $O(n+m)$

(a) 

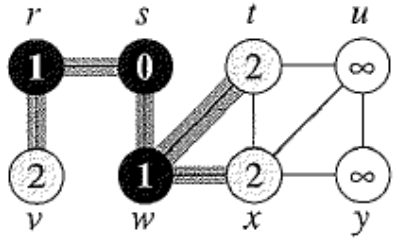
Q | s | 0

(b) 

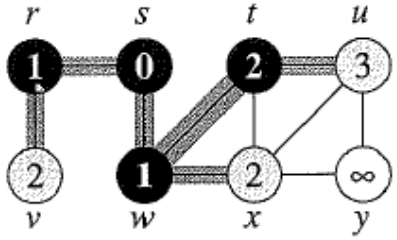
Q | w | r | 1 1

(c) 

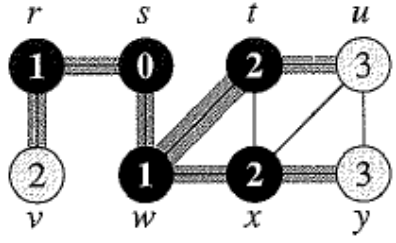
Q | r | t | x | 1 2 2

(d) 

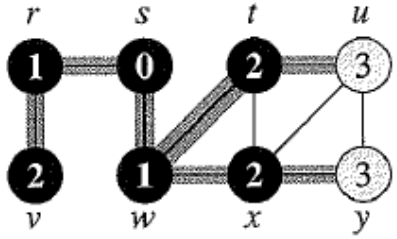
Q | t | x | v | 2 2 2

(e) 

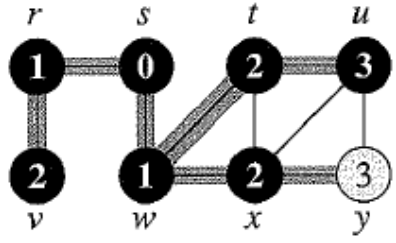
Q | x | v | u | 2 2 3

(f) 

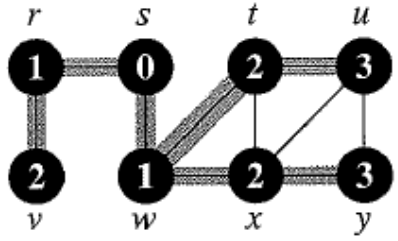
Q | v | u | y | 2 3 3

(g) 

Q | u | y | 3 3

(h) 

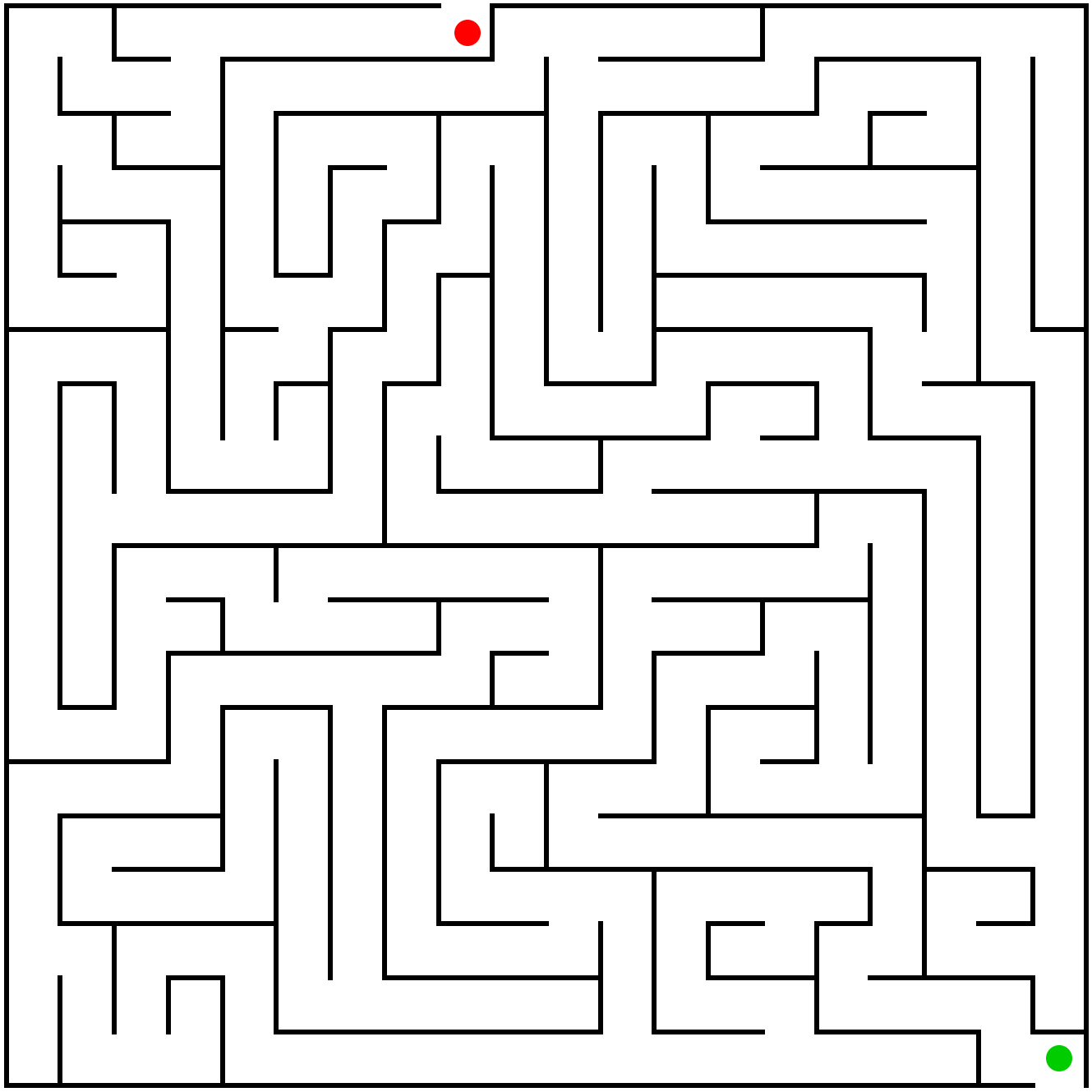
Q | y | 3

(i) 

Q $\emptyset$

BFS : Udskrivning af sti fra s til v

```
PRINT-PATH(G, s, v)
1   if v == s
2       print s
3   elseif v.π == NIL
4       print "no path from" s "to" v "exists"
5   else PRINT-PATH(G, s, v.π)
6       print v
```



Dybde Først Søgning (DFS)

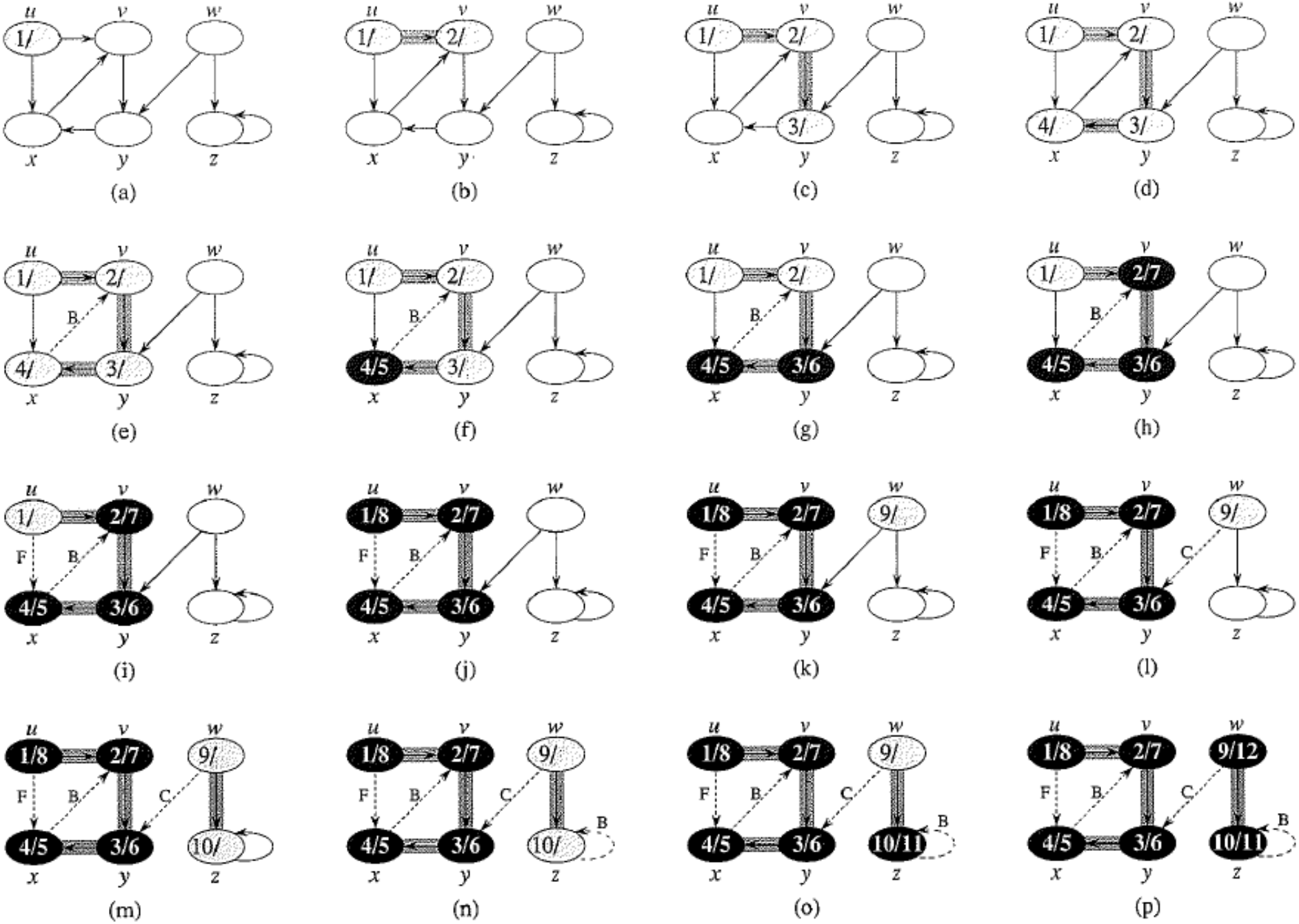
```
DFS(G)
1  for each vertex u ∈ G.V
2      u.color = WHITE
3      u.π = NIL
4  time = 0
5  for each vertex u ∈ G.V
6      if u.color == WHITE
7          DFS-VISIT(G, u)

DFS-VISIT(G, u)
 1  time = time + 1                  // white vertex u has just been discovered
 2  u.d = time
 3  u.color = GRAY
 4  for each v ∈ G.Adj[u]            // explore edge (u, v)
 5      if v.color == WHITE
 6          v.π = u
 7          DFS-VISIT(G, v)
 8  u.color = BLACK                  // blacken u; it is finished
 9  time = time + 1
10  u.f = time
```

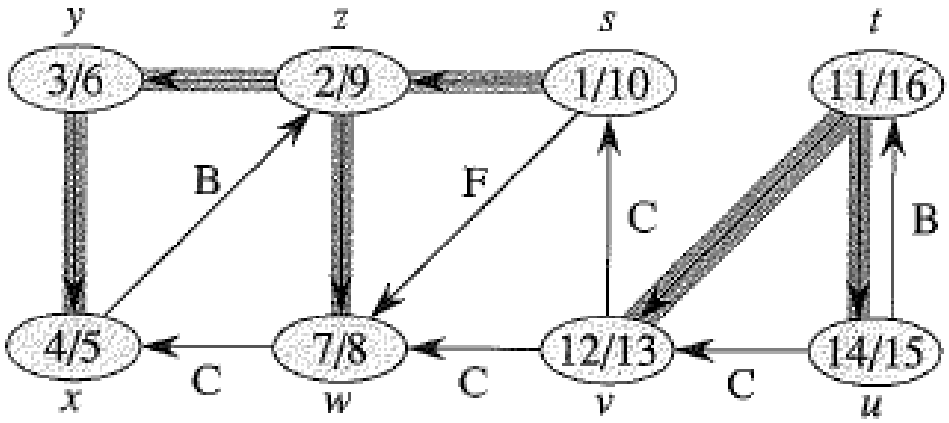
u.color
WHITE = knuderne endnu ikke besøgt
GRAY = knuder på rekursionsstakken
BLACK = knuderne besøgt

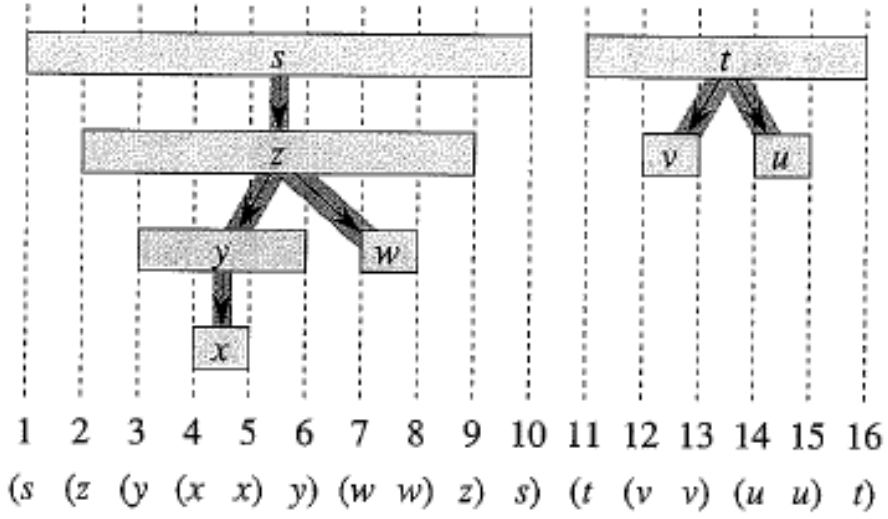
$u.\pi$ = faderen til u i DFS træet

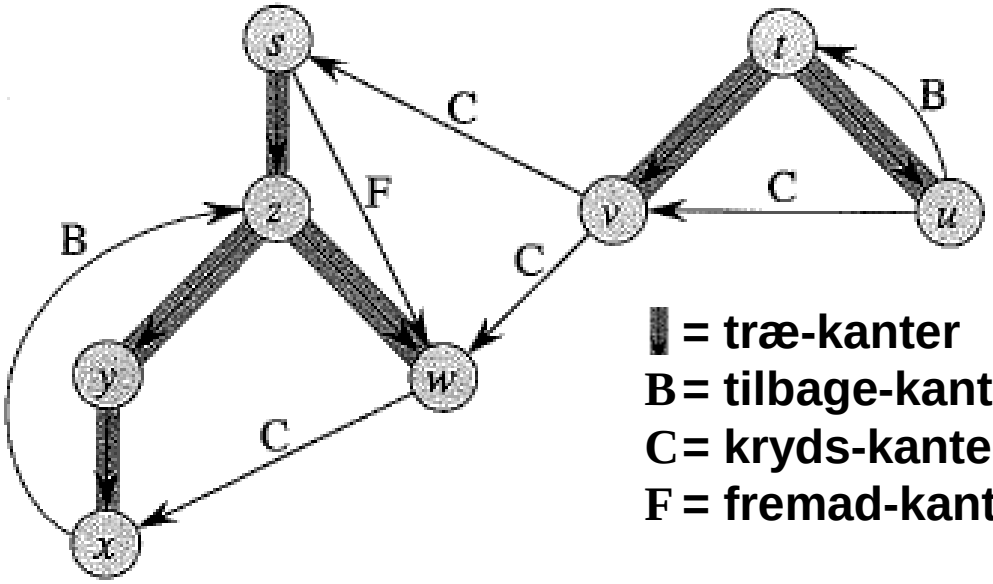
$u.d$ = "discover time" for u
$u.f$ = "finishing time" for u

Tid $O(n+m)$


u v w
1/
x y z
(a)
1/ 2/
(b)
1/ 2/ 3/
(c)
1/ 2/ 4/ 3/
(d)
B
(e)
4/5 3/
(f)
4/5 3/6
(g)
2/7
(h)
F
(i)
1/8 2/7 4/5 3/6
(j)
9/
(k)
C
(l)
10/
(m)
(n)
10/11
(o)
9/12
(p)







▮ = træ-kanter
B = tilbage-kanter
C = kryds-kanter
F = fremad-kanter

BFS og DFS anvendelser

BFS Finde afstande til startknuden
(afstand = antal kanter, f.eks. RushHour)

DFS *Topologisk sortering*
Stærke sammenhængskomponenter
(næste forelæsning)