Algoritmer og Datastrukturer 1

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Elementære Datastrukturer [CLRS, kapitel 10]

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[CLRS, Del 3] : Datastrukturer

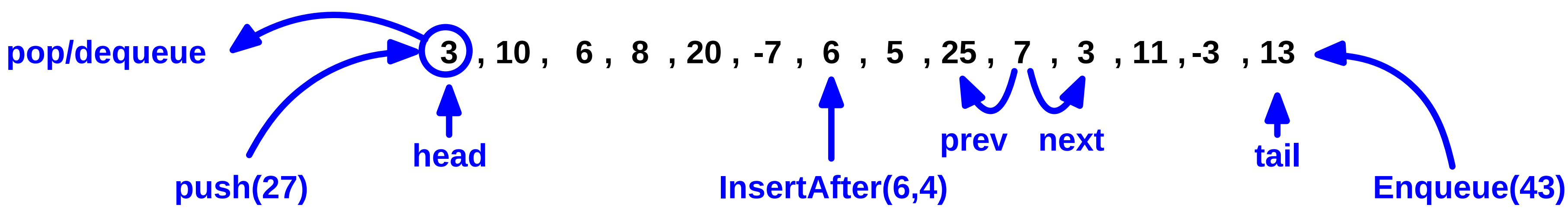
Opretehold en struktur for en **dynamisk** mængde data

Abstrakte Datastrukturer for Mængder

			-Min-prioritetskø	-Max-prioritetskø	- Ordbog
Forespørgsel	Minimum(S)	pointer til element	●		
	Maximum(S)	pointer til element		●	
	Search(S, x)	pointer til element			●
	Member(S, x)	TRUE eller FALSE			
	Successor(S, x)	pointer til element			
	Predecessor(S, x)	pointer til element			
Opdateringer	Insert(S, x)	pointer til element	●	●	●
	Delete(S, x)	-			●
	DeleteMin(S)	element	●		
	DeleteMax(S)	element		●	
	Join(S_1, S_2)	mængde S			
	Split(S, x)	mængder S_1 og S_2			

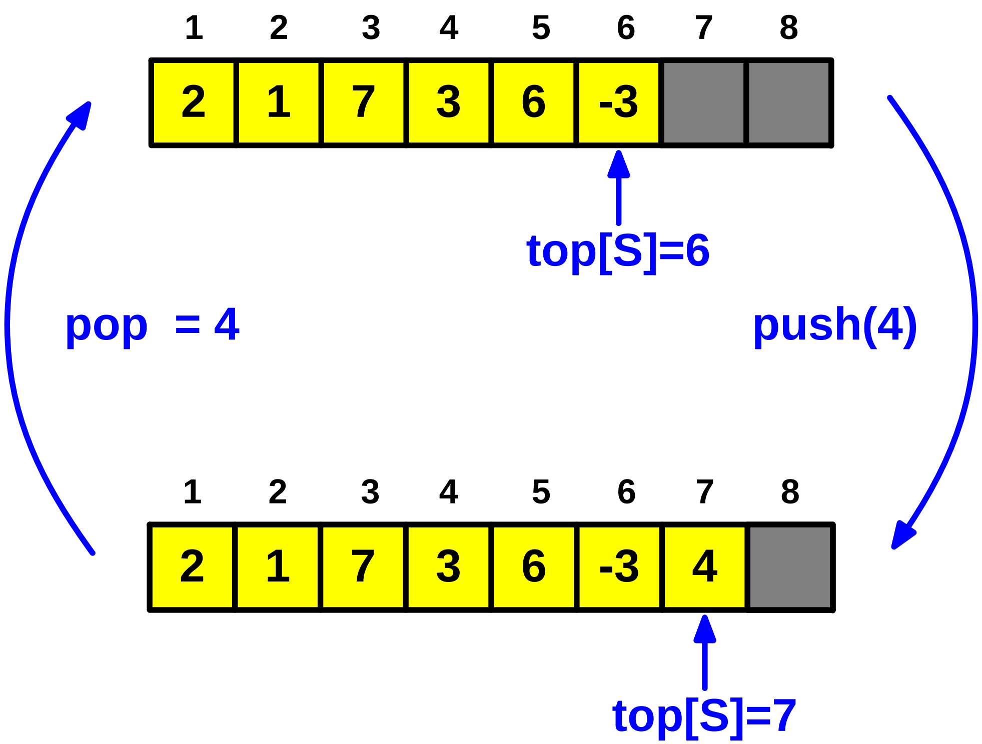
Abstrakte Datastrukturer for Lister

			-Stak	-Kø
Forespørgsel	**Empty(S)**	TRUE eller FALSE	●	●
	Head(S), Tail(S)	pointer til element		
	Next(S, x), Prev(S, x)	pointer til element		
	Search(S, x)	pointer til element		
Opdateringer	**Push(S, x)**	-	●	
	Pop/Dequeue(S)	element	●	●
	Enqueue(S, x)	-		●
	Delete(S, x)	Element		
	InsertAfter(S, x, y)	pointer til element		



Stak

Stak : Array Implementation



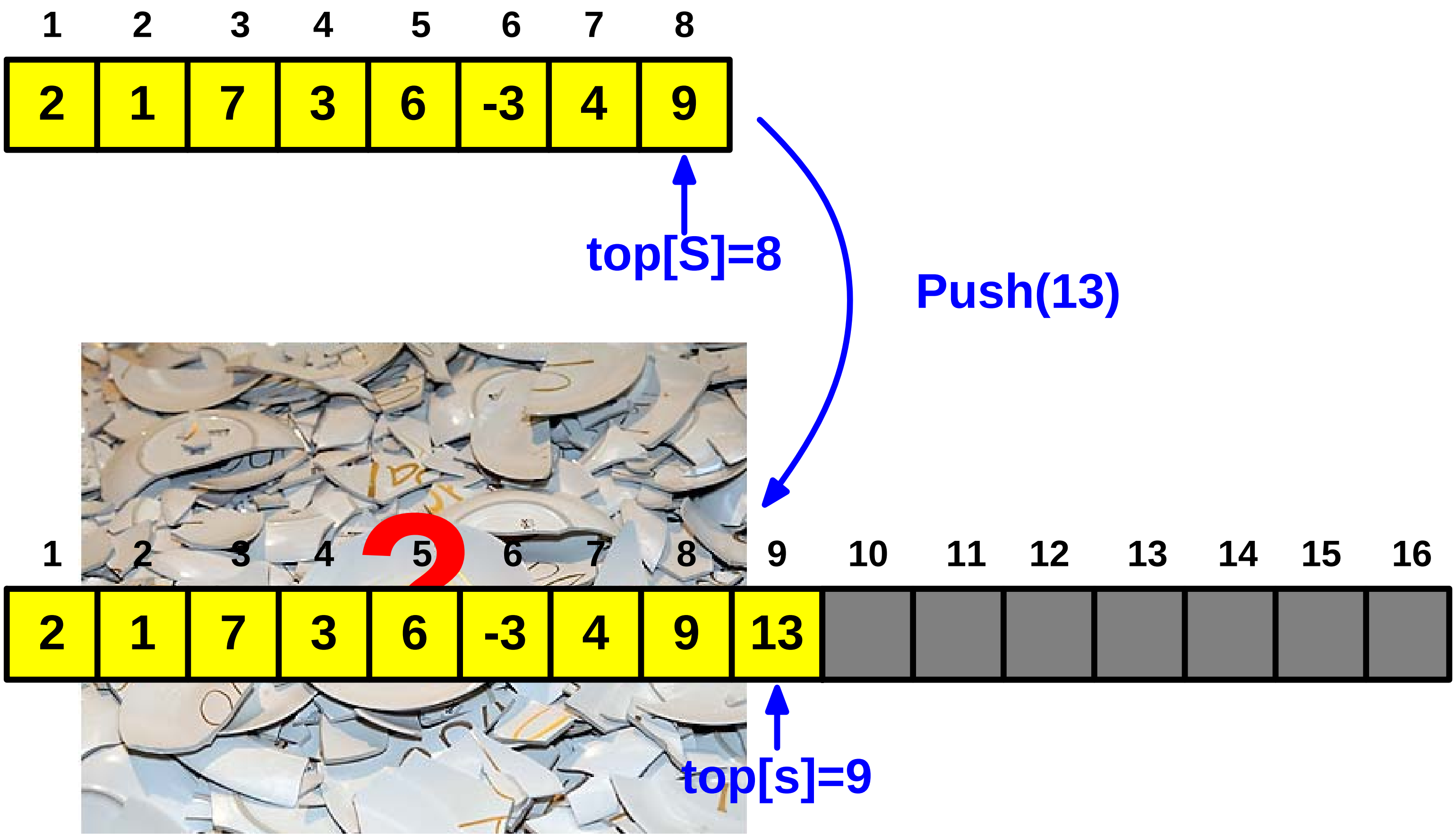

```
STACK-EMPTY(S)
if top[S] = 0
    then return TRUE
    else return FALSE

PUSH(S, x)
top[S] ← top[S] + 1
S[top[S]] ← x

POP(S)
if STACK-EMPTY(S)
    then error "underflow"
    else top[S] ← top[S] − 1
         return S[top[S] + 1]
```

Stack-Empty, Push, Pop : $O(1)$ **tid**

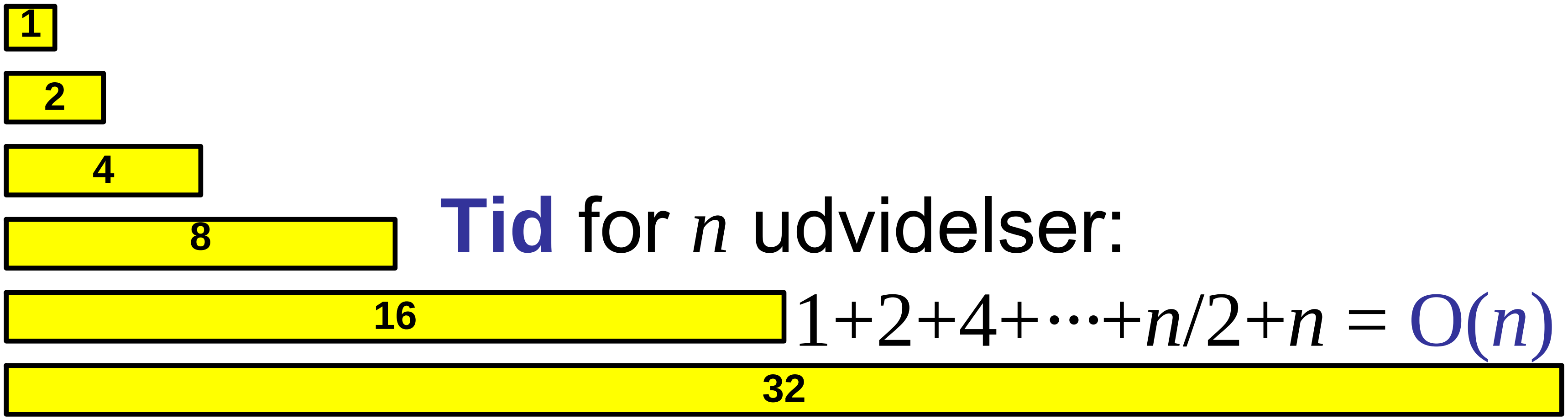
Stak : Overløb



Array fordobling : O(*n*) tid

Array Fordobling

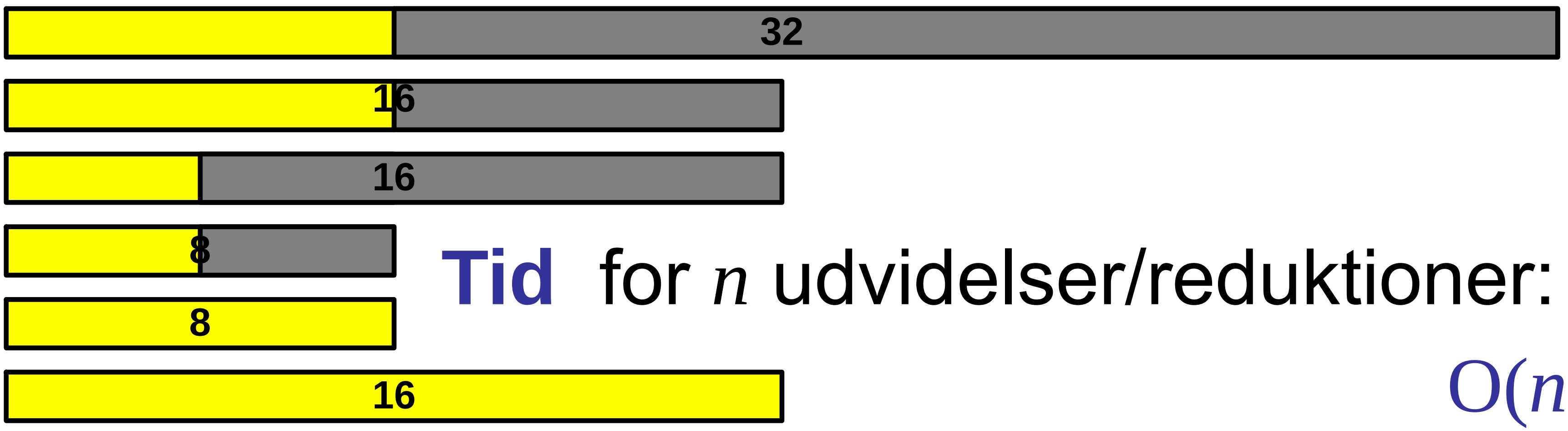
Fordoble arrayet når det er fuld



Tid for n udvidelser:

$$1+2+4+\cdots+n/2+n = O(n)$$

Halver arrayet når det er <1/4 fyldt



Tid for n udvidelser/reduktioner:

$$O(n)$$

Array Fordobling + Halvering
– en generel teknik

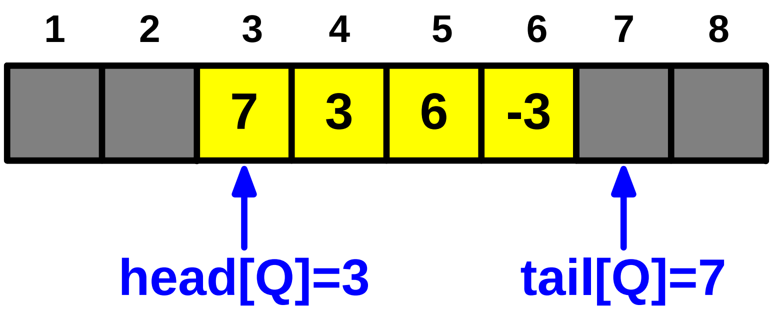
Tid for *n* udvidelser/reduktioner er $O(n)$

Plads ≤ 4 · aktuelle antal elementer

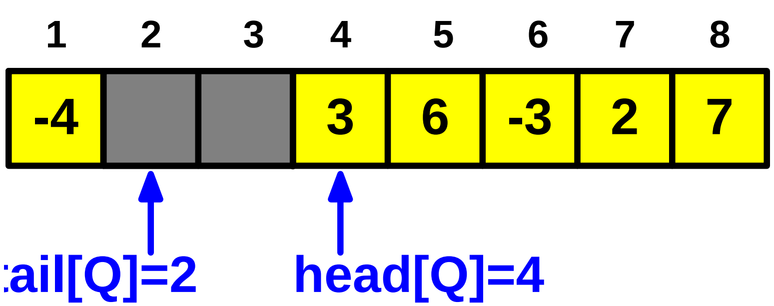
Array implementation af Stak:
n push og pop operationer tager $O(n)$ tid

Kø

Kø : Array Implementation



Enqueue(2)
Enqueue(7)
Enqueue(-4)
Dequeue = 7

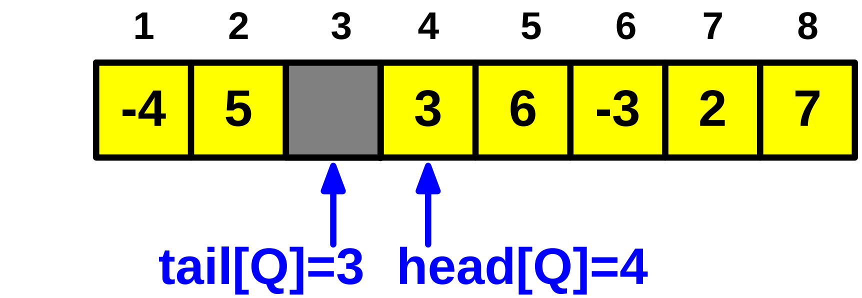



```
ENQUEUE(Q, x)
Q[tail[Q]] ← x
if tail[Q] = length[Q]
    then tail[Q] ← 1
    else tail[Q] ← tail[Q] + 1
```

```
DEQUEUE(Q)
x ← Q[head[Q]]
if head[Q] = length[Q]
    then head[Q] ← 1
    else head[Q] ← head[Q] + 1
return x
```

Enqueue, dequeue : O(1) **tid**

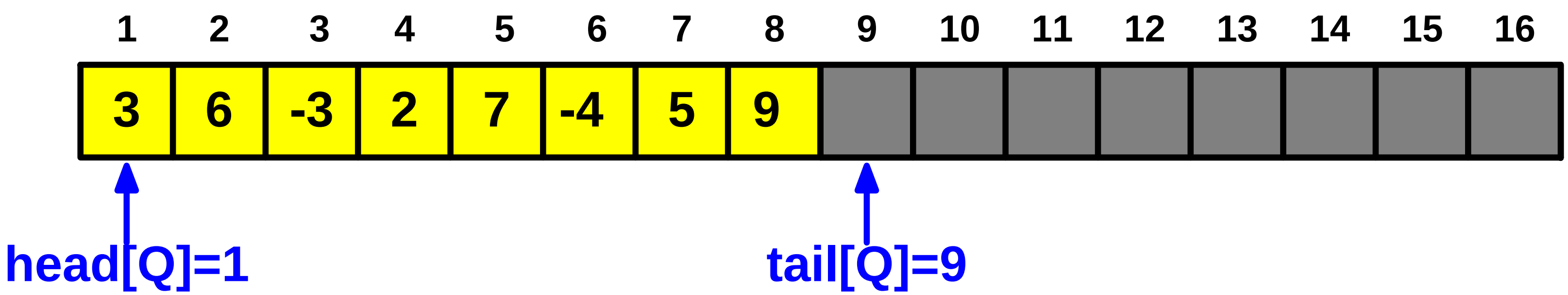
Kø : Array Implementation



Empty : tail[Q]=head[Q] ?

Overløb : array fordobling/ halvering

Enqueue(9)



Array implementation af Kø:
n **enqueue og dequeue operationer tager** $O(n)$ **tid**

Arrays (med Fordobling/Halvering)

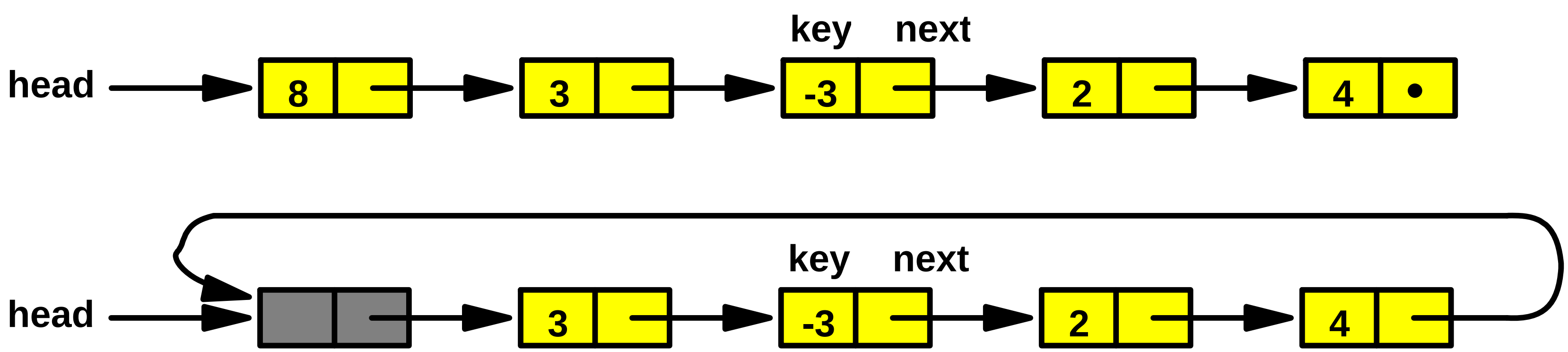
Stak	**Push(*S*, *x*)**	O(1)*
	Pop(*S*)	O(1)*
Kø	**Enqueue(*S*, *x*)**	O(1)*
	Dequeue(*S*)	O(1)*

*** Worst-case uden fordobling/halvering**
Amortiseret ([CLRS, Kap. 17]) med fordobling/halvering

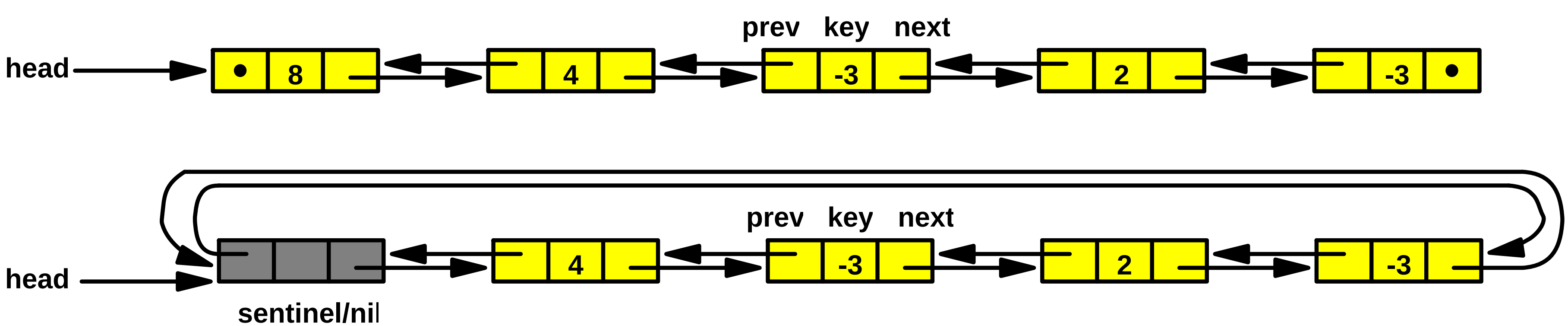
Kædede lister

Kædede Lister

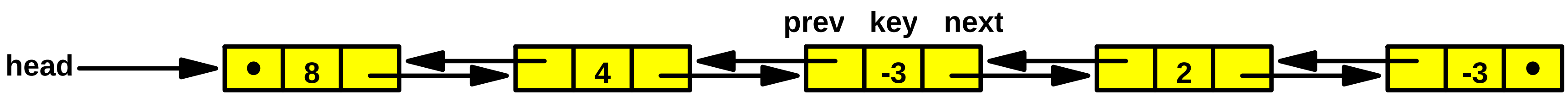
Enkelt kædede (ikke-cyklisk og cyklisk)



Dobbelt kædede (ikke-cyklisk og cyklisk)



Dobbelt Kædede Lister



LIST-INSERT(L, x)

```
next[x] ← head[L]
if head[L] ≠ NIL
    then prev[head[L]] ← x
head[L] ← x
prev[x] ← NIL
```

LIST-SEARCH(L, k)

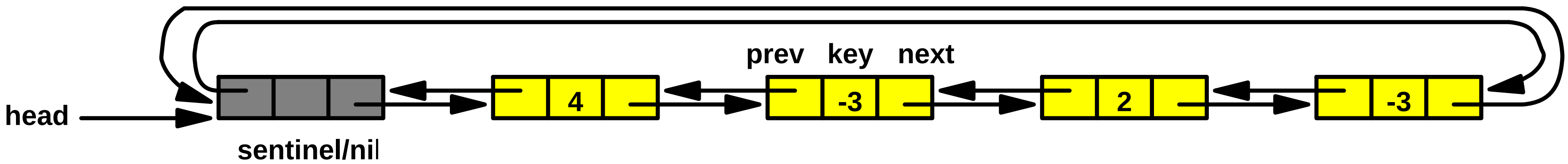
```
x ← head[L]
while x ≠ NIL and key[x] ≠ k
      do x ← next[x]
return x
```

LIST-DELETE(L, x)

```
if prev[x] ≠ NIL
    then next[prev[x]] ← next[x]
    else head[L] ← next[x]
if next[x] ≠ NIL
    then prev[next[x]] ← prev[x]
```

List-Search	$O(n)$
List-Insert	$O(1)$
List-Delete	$O(1)$

Dobbelt Kædede Cykliske Lister



List-Insert′(L, x)

```
next[x] ← next[nil[L]]
prev[next[nil[L]]] ← x
next[nil[L]] ← x
prev[x] ← nil[L]
```

List-Search′(L, k)

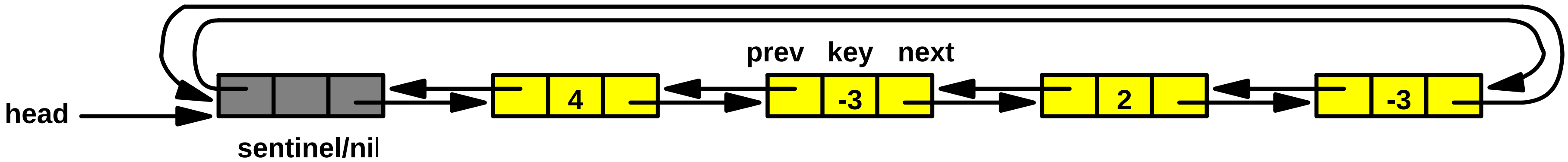
```
x ← next[nil[L]]
while x ≠ nil[L] and key[x] ≠ k
      do x ← next[x]
return x
```

List-Delete′(L, x)

```
next[prev[x]] ← next[x]
prev[next[x]] ← prev[x]
```

List-Search’	O(n)
List-Insert’	O(1)
List-Delete’	O(1)

Dobbelt Kædede Cykliske Lister



Stak	Push(S, x)	O(1)
	Pop(S)	O(1)
Kø	Enqueue(S, x)	O(1)
	Dequeue(S)	O(1)

Dancing Links

Donald E. Knuth, Stanford University

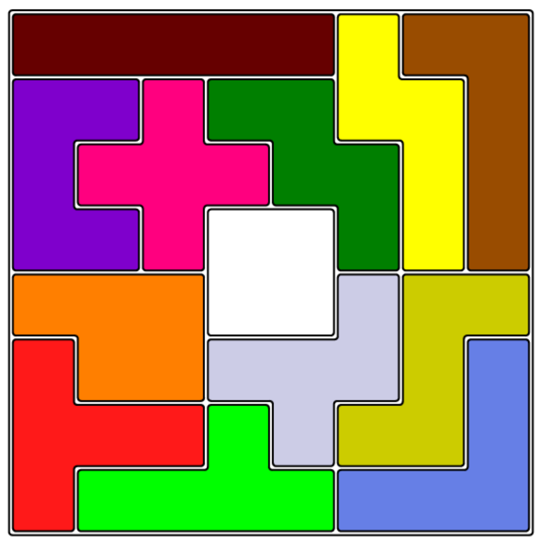
My purpose is to discuss an extremely simple technique that deserves to be better known. Suppose x points to an element of a doubly linked list; let $L[x]$ and $R[x]$ point to the predecessor and successor of that element. Then the operations

$$L\big[R[x]\big] \leftarrow L[x], \qquad R\big[L[x]\big] \leftarrow R[x] \tag{1}$$

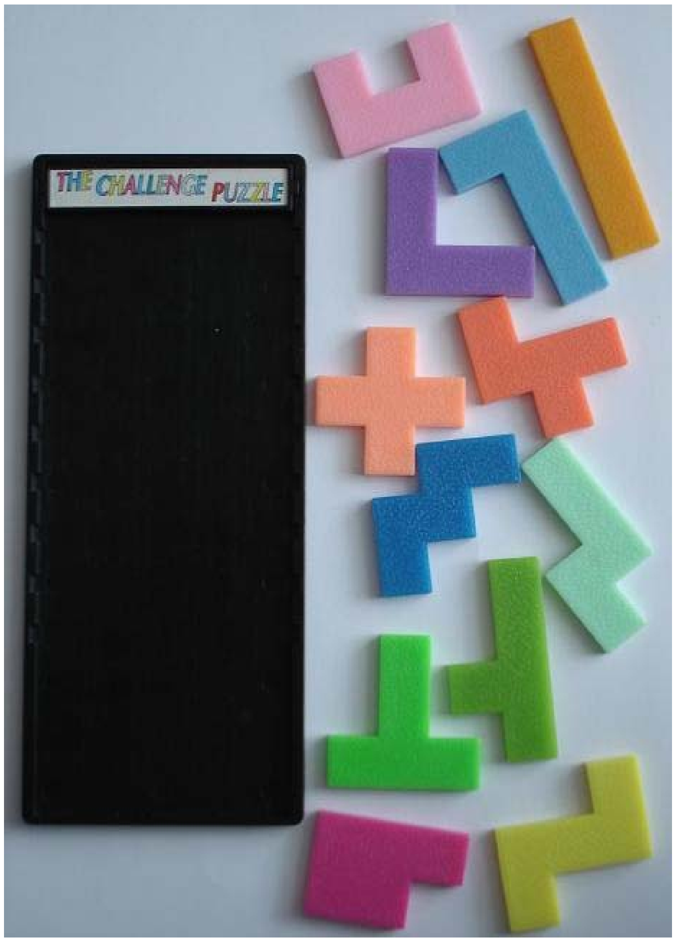
remove x from the list; every programmer knows this. But comparatively few programmers have realized that the subsequent operations

$$L\big[R[x]\big] \leftarrow x, \qquad R\big[L[x]\big] \leftarrow x \tag{2}$$

will put x back into the list again.



”The Challenge Puzzle”



?

”The Challenge Puzzle”

```
L := Tomt bræt
B := Alle brikker
Solve(L,B)

procedure Solve(Delløsning L, Brikker B)
   for alle b i B
      for alle orienteringer af b  (* max 8 forskellige *)
         if b kan placeres i nederste venstre fri then
             fjern b fa B
             indsæt b i L
             if |B|=0 then
               rapporter L er en løsning
             else
               Solve(L,B)
             fi
             slet b fra L
             genindsæt b i B
         fi
```

Nederste-
venstre fri

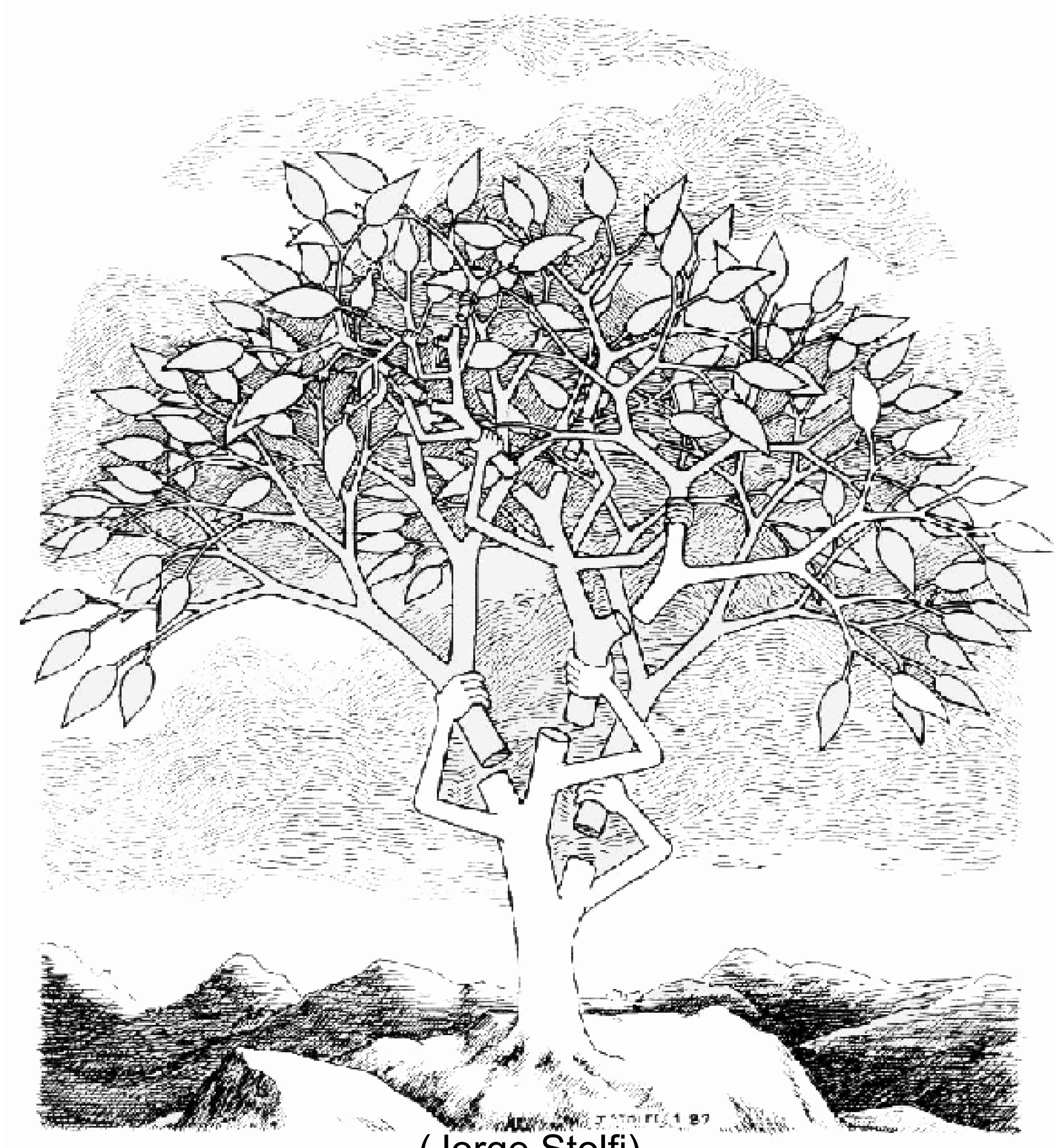
Før

Efter

”The Challenge Puzzle”

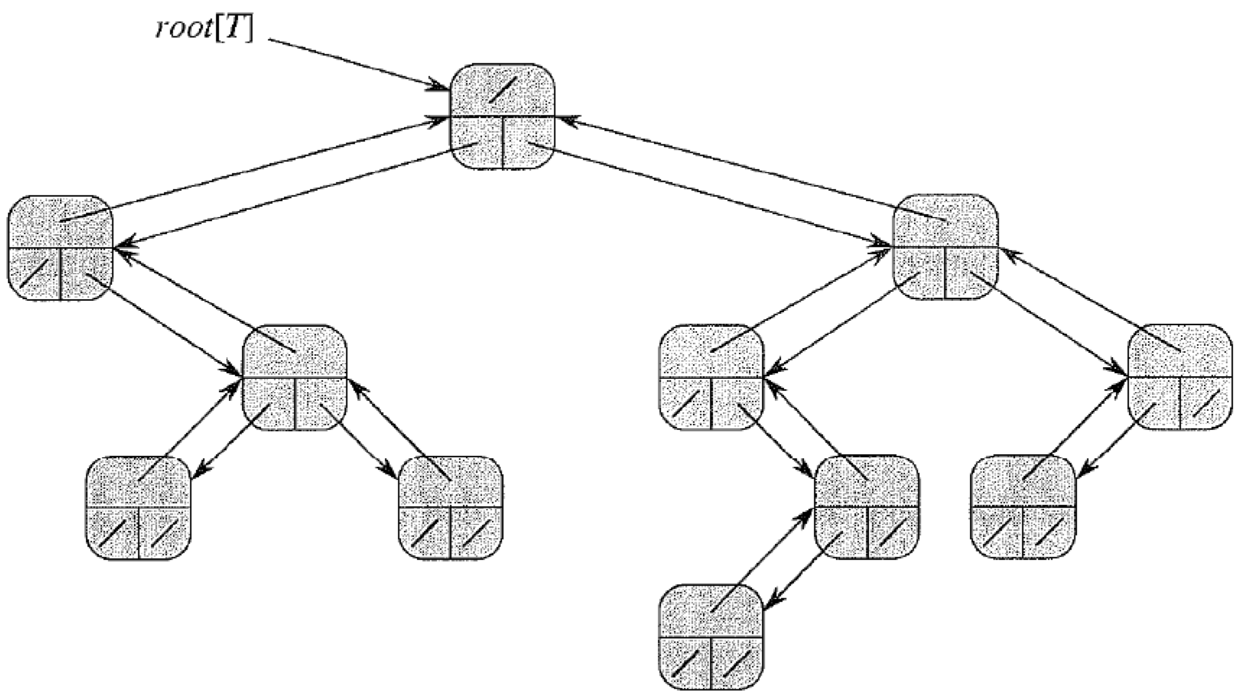
4.040 løsninger

Solve placerer
8.387.259 brikker



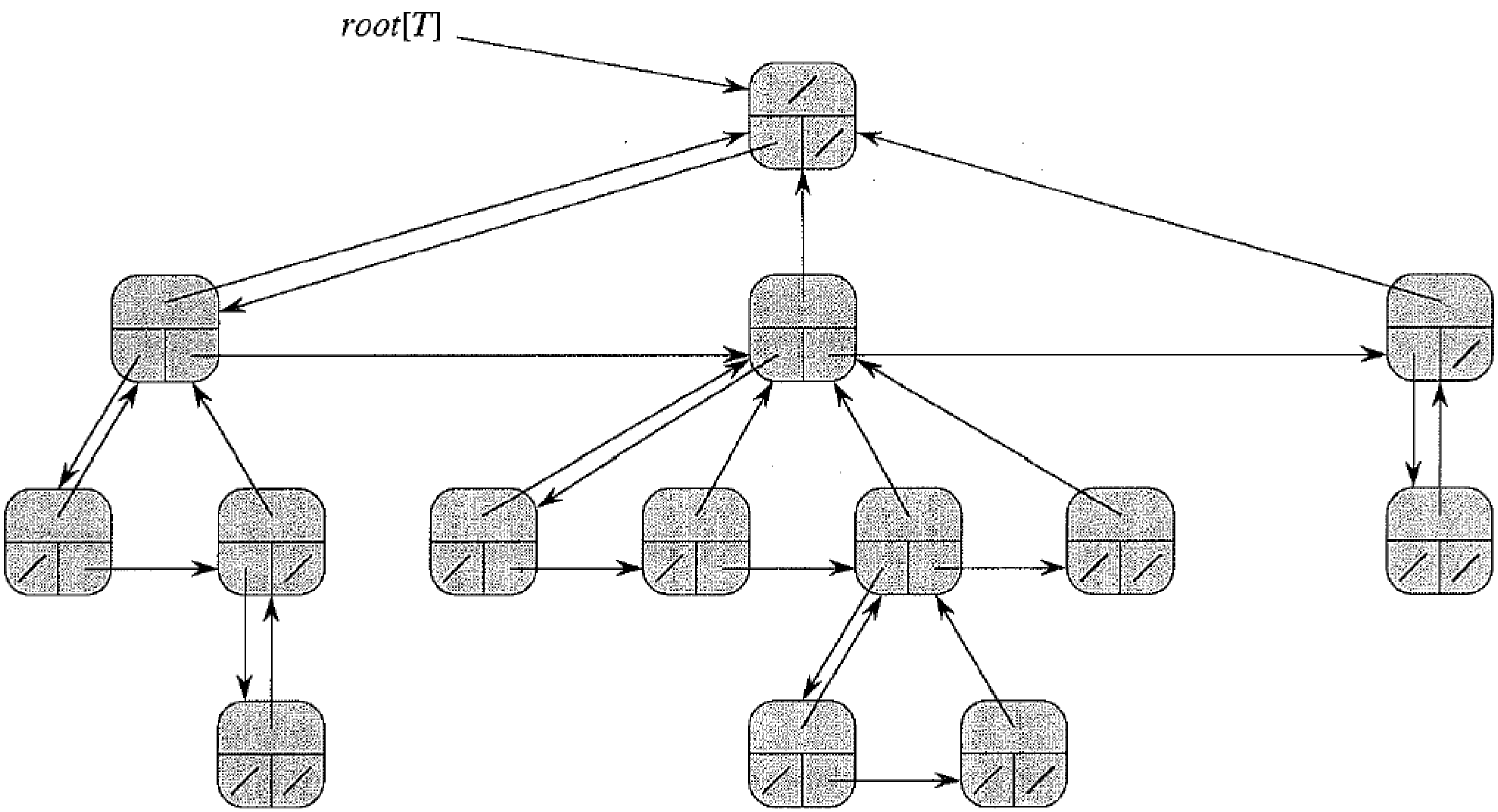
(Jorge Stolfi)

Binær Træ Repræsentation



Felter: **Left**, **right**, **parent**

Træ Repræsentation



Felter: **Left**, **right sibling**, **parent**