## Homework Exercises for Lecture 1

A.1-1 (i) Compute the VC-dimension of the space dual to $\left(\mathbb{R}^{2}, L\right)$ where $L$ is the set of all the lines.
(ii) Compute the VC-dimension of $\left(\mathbb{R}^{2}, S\right)$ where $S$ is the set of all the line segments.
(iii) Compute the VC-dimension of the set system dual to the one defined in (ii).
A.1-2 Consider a set $R$ of $n$ disjoint regions in $\mathbb{R}^{2}$ with simply connected boundaries (in other words, each $r \in R$ is the set of all the points contained inside a closed curve). Let $C$ be a set of curves in $\mathbb{R}^{2}$. We say two curves $c_{1}, c_{2} \in C$ are equivalent if the set of regions crossed by them is identical.
(i) What is the maximum size of $C$ if we know no two curves in $C$ are equivalent?
(ii) Show that if we add an additional assumption to (i) that the curves are disjoint then $|C|=O\left(n^{4}\right)$.

