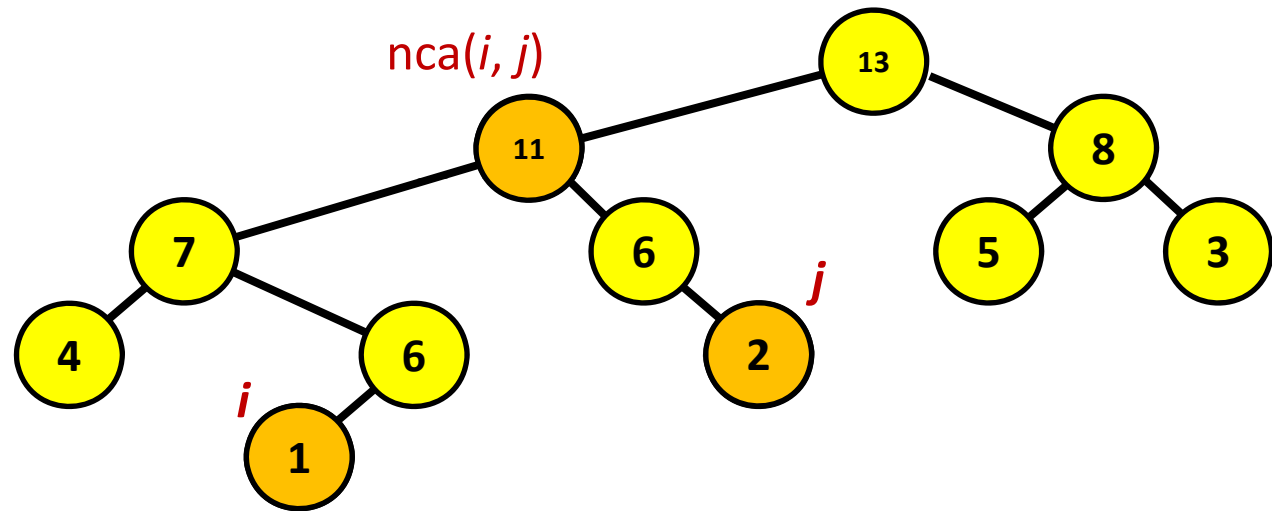


Nearest Common Ancestors (NCA)

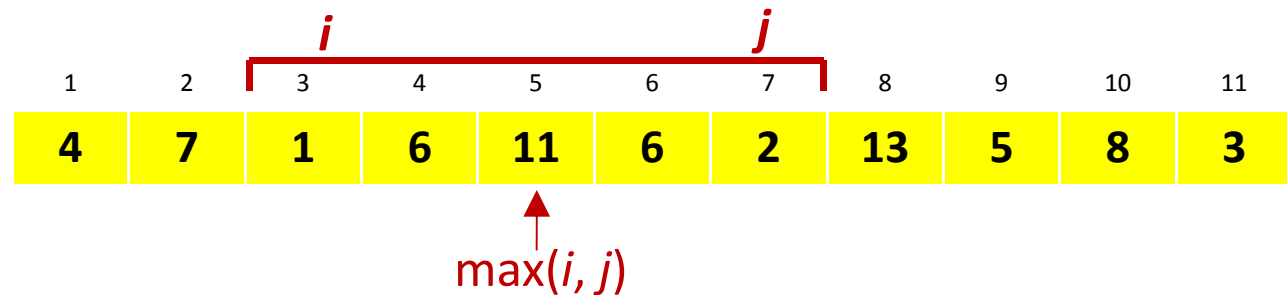
Org. [D. Harel, R.E. Tarjan, *Fast algorithms for finding nearest common ancestors*, SIAM J. on Comp. 13 (2): 338–355, 1984]

Preprocessing Time vs Query Time ?

Cartesian Tree
[Vuillemin 1980]

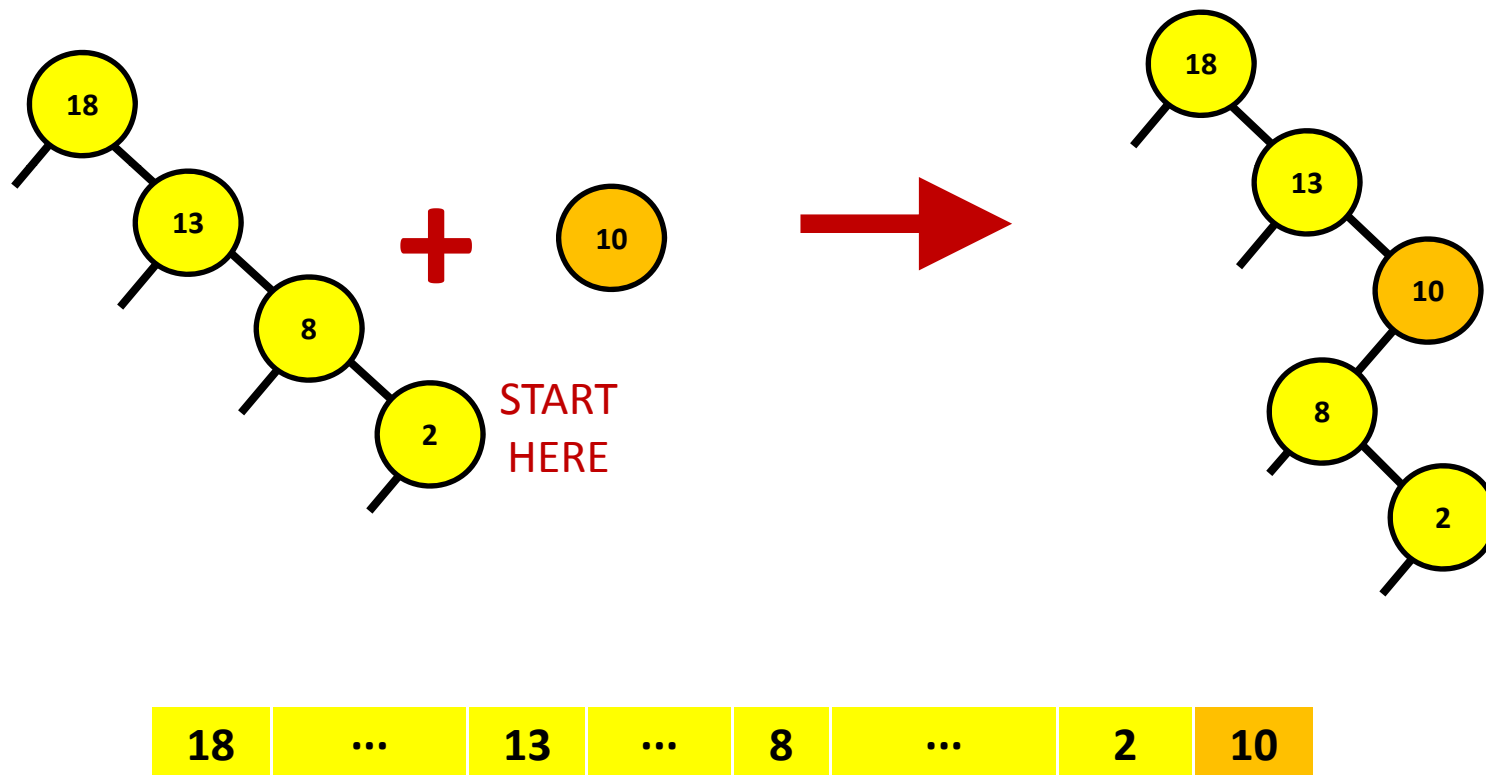


Discrete Range Maximum



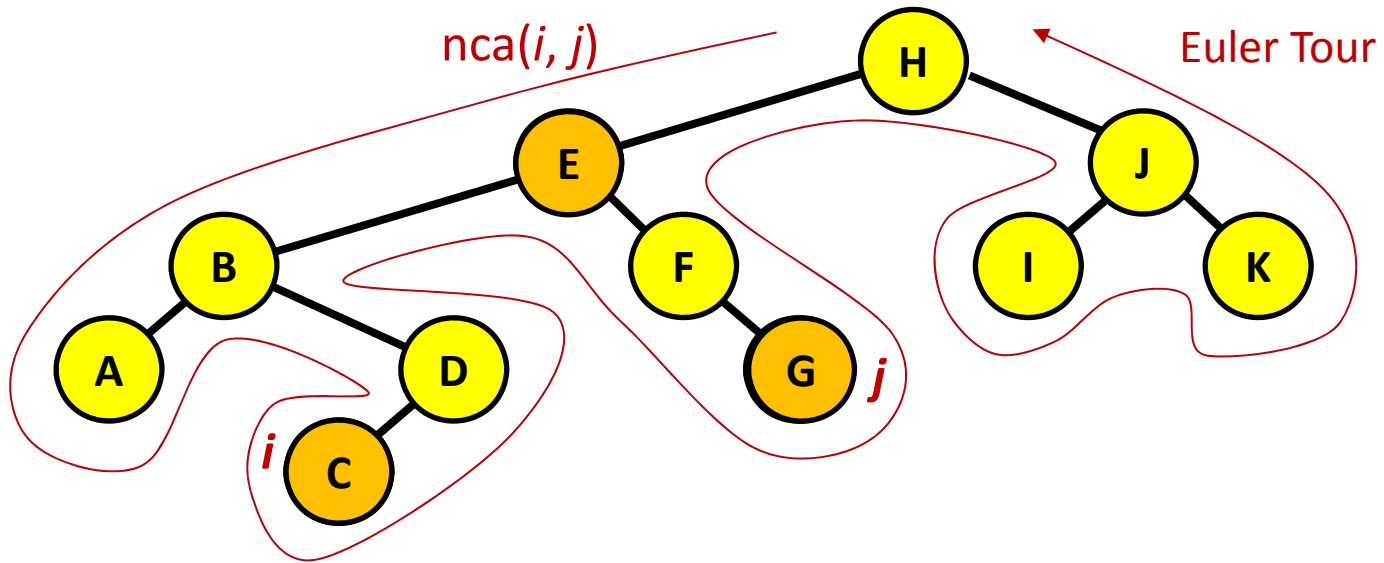
Cartesian Tree Construction

- Incremental construction left-to-right



- $O(n)$ time

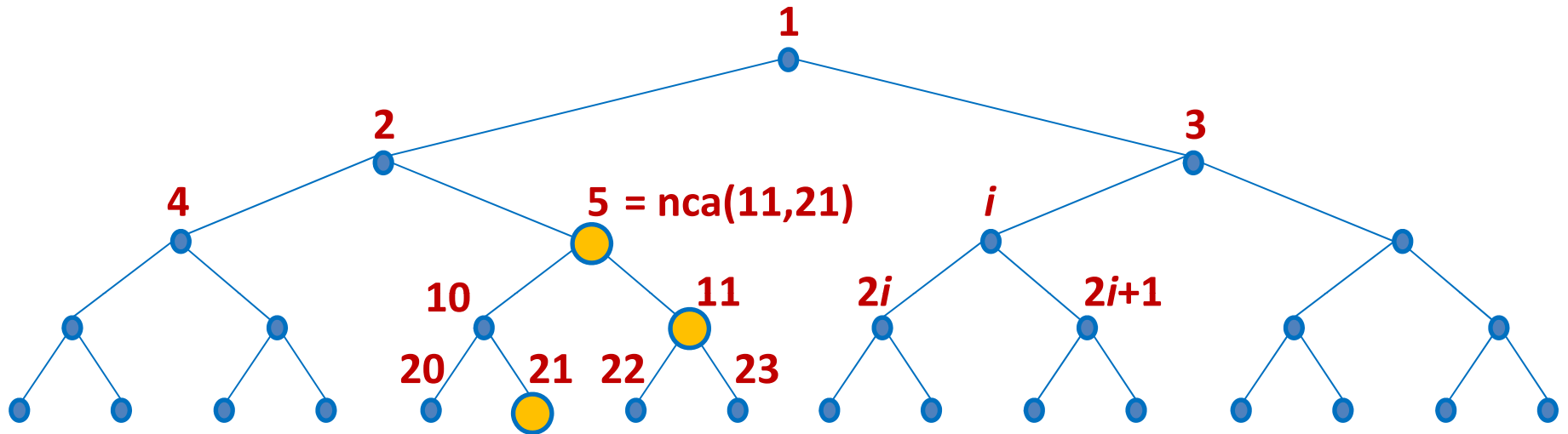
Reduction: NCA \Rightarrow Discrete Range Maximum



						<i>i</i>			$nca(i, j)$	<i>j</i>											
node	H	E	B	A	B	D	C	D	B	E	F	G	F	E	H	J	I	J	K	J	H
depth	1	2	3	4	3	4	5	4	3	2	3	4	3	2	1	2	3	2	3	2	1

$\underbrace{\hspace{15em}}_{\text{minimum depth}}$

NCA on Perfect Binary Trees



$11 = 1011_2$

$21 = 10101_2$

$nca(21,21) = 5 = 101_2 = \text{lcp}(\mathbf{1011}_2, \mathbf{10101}_2)$

longest common prefix

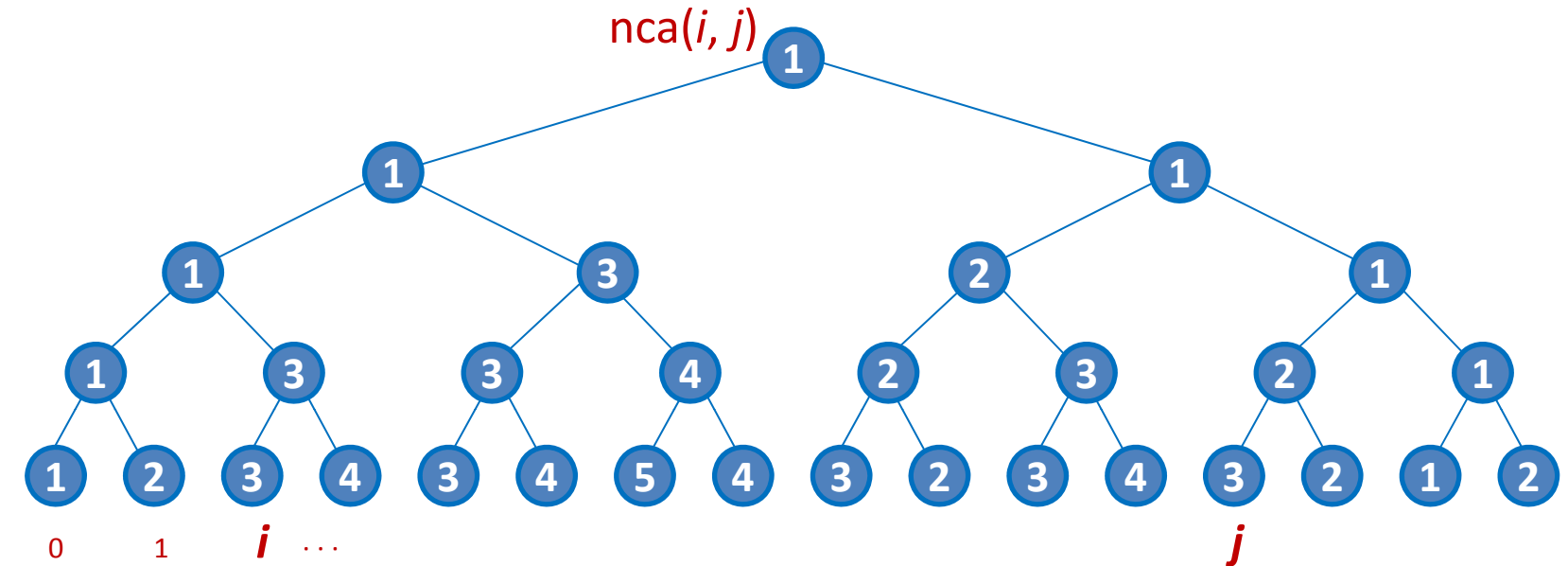
proc lcp(x, y)

if $y < x$ **then** swap (x, y)

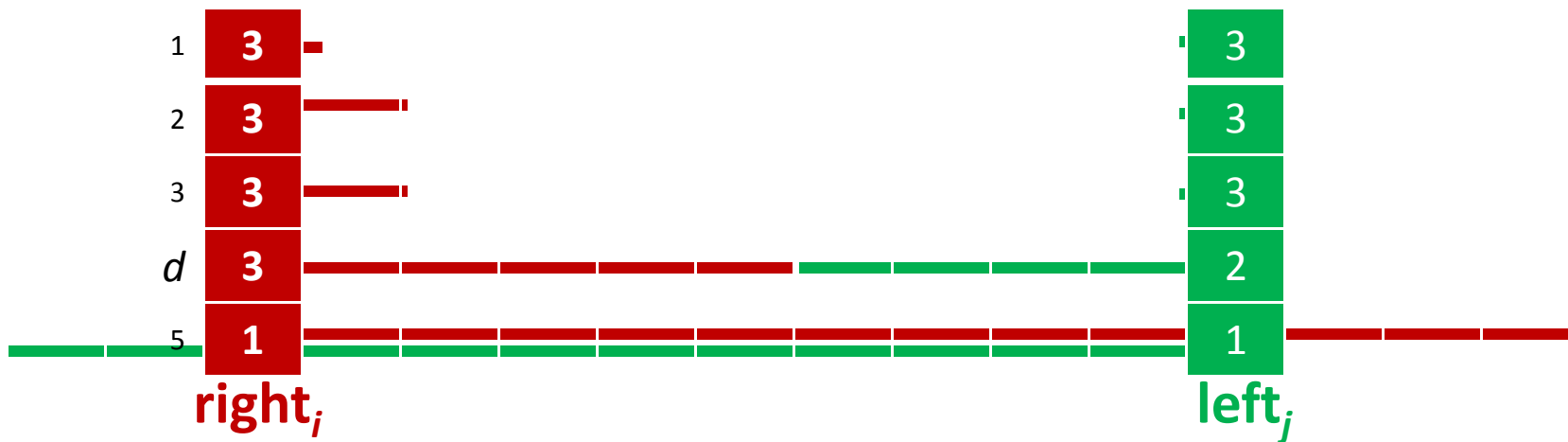
return $x \gg (\text{msb}(x \text{ XOR } (y \gg (\text{msb}(y) - \text{msb}(x))))$

position of most significant bit $\neq 0$

Discrete Range Minimum - $O(n \cdot \log n)$ Space



1	2	3	4	3	4	5	4	3	2	3	4	3	2	1	2
---	---	---	---	---	---	---	---	---	---	---	---	---	---	---	---

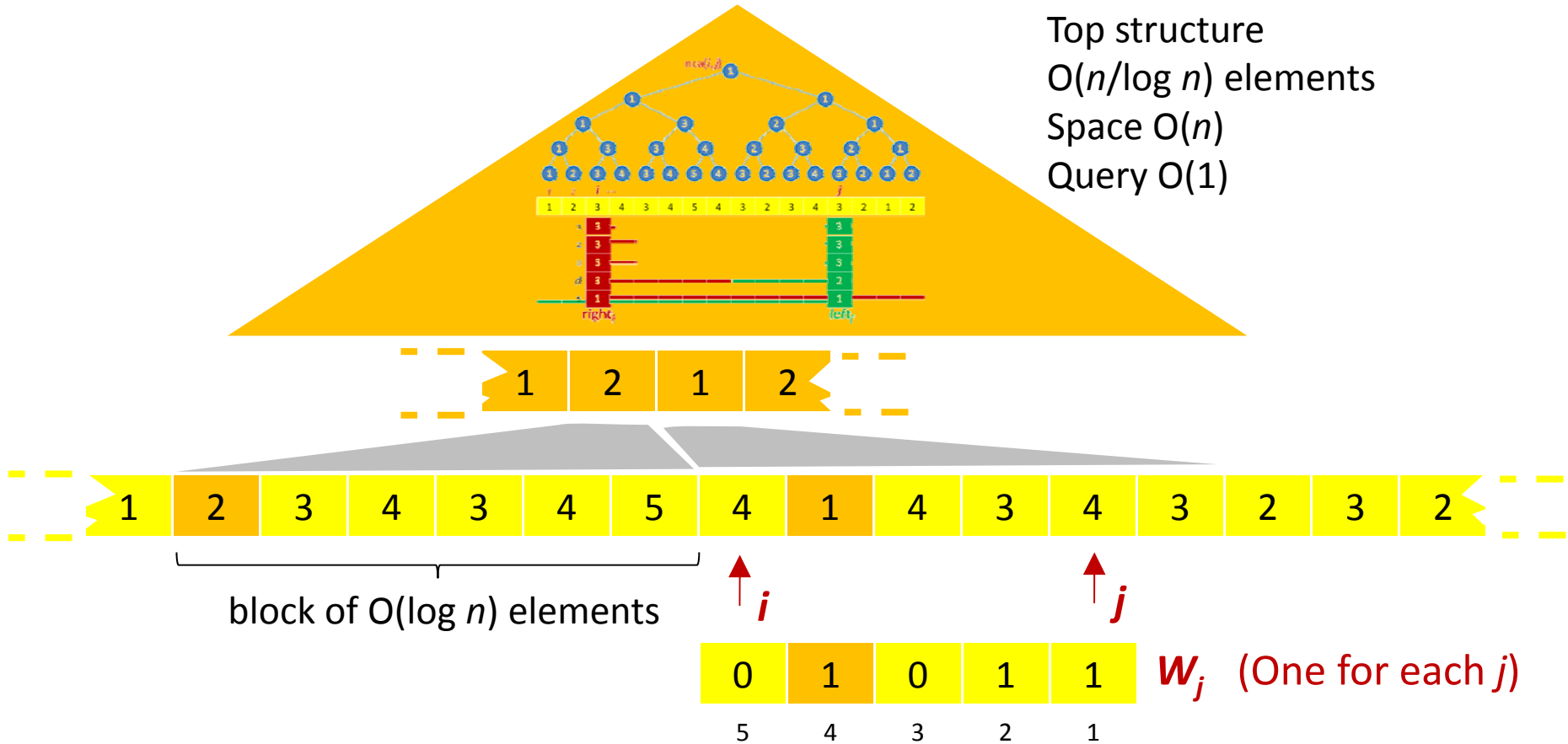


$$\text{drm}(i, j) = \min(\text{right}_i(d), \text{left}_j(d))$$

$$d = \text{msb}(i \text{ XOR } j)$$

Blocked solution – $O(n)$ space

Top structure
 $O(n/\log n)$ elements
 Space $O(n)$
 Query $O(1)$



Block query: $j+1 - \text{msb}(W_j \text{ AND } ((1 \ll (j-i+1)) - 1))$
 General query: 1 top query + 2 bottom queries

$O(n)$ Preprocessing Time $O(1)$ Query Time

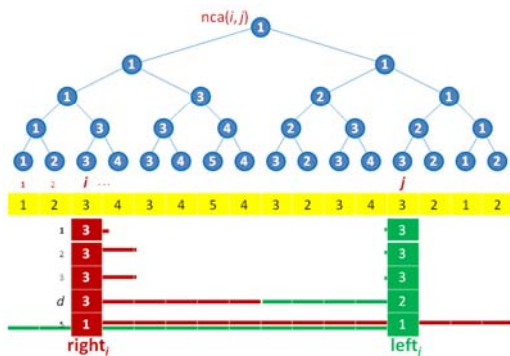
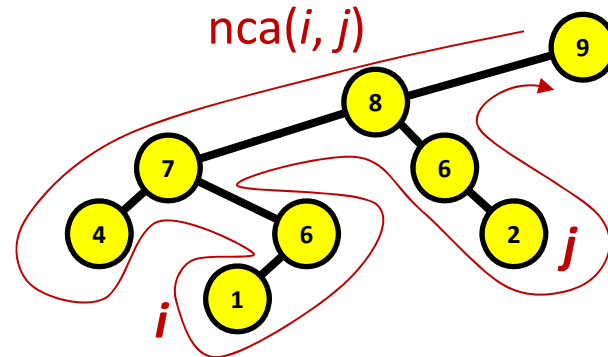
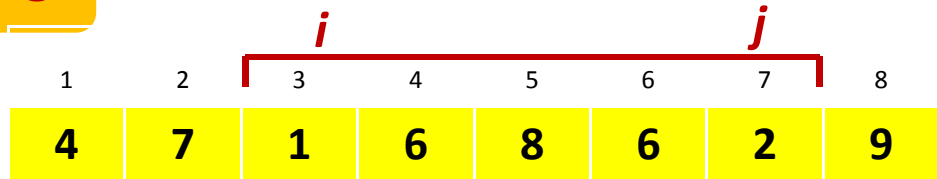
Summary...

General Discrete Range Searching

Cartesian Tree

NCA

Discrete Range Max on Depth Array



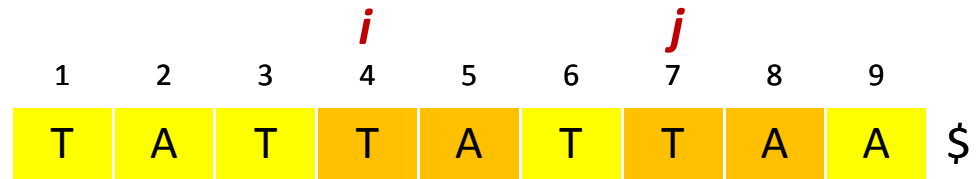
" $O(n \cdot \log n)$ " solution on $O(n/\log n)$ blocks
 $O(\log n)$ size blocks

$O(n)$ Preprocessing Time

$O(1)$ Query Time

An NCA application...

- Preprocess a string



- Query

Length of longest common substring starting at positions *i* and *j*

Build suffix tree + NCA query

